

# The Transformation problem after Kliman and Wright Are we approaching a solution?

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Abstract

The paper tries to contribute to a stock-taking of the debate on the transformation problem. The starting points are two recent contributions Andrew Kliman (2007) and Ian Wright (2011). Both authors regard themselves as trying to show that a consistent theory of the relationship between human effort (labour) and prices can be constructed, i.e. the transformation problem is – or can be solved. The authors differ in their methodological approach. Kliman points to the necessity of escaping from static equilibrium modelling, while Wright argues that the root of the transformation problem is not simultaneism, but a conceptual mistake regarding how capitalist's consumption has been treated in the linear algebra models. The paper holds that both authors have valid points – and that together with other insights accumulated in the decades since Steedman's "Marx after Sraffa" (1977) – we are approaching a solution. But only in the sense that the labour theorist could have a fruitful discussion of what criteria, theoretical/empirical facts a solution must satisfy. After Kliman (2007) no one can argue that Marx was inconsistent in the sense that there are only non-logical interpretations of his texts, so that Marx needs to be corrected "logically". The paper argue – as Kliman (2007) also points out – that there are textual support for both static and dynamic approaches in Das Kapital, but that the static is clearly not Marx' fundamental model. That means that phenomena like increasing returns to scale, real-world money – not numéraires, different technologies producing the same good with different organic compositions etc. That is the kind of phenomena that must be part of a real solution of the TP. From this perspective labour value theorists still have a lot of work to do.

Introduction

The Transformation problem (TP) still haunts economist inspired by Marx. One of Marx's formulations of the problem is:

"This law clearly [market price = value] contradicts all experience based on appearance. Everyone knows that a cotton spinner, who, reckoning the percentage on the whole of his applied capital, employs much constant and little variable capital, does not, on account of this, pocket less profit or surplus value than a baker, who relatively sets in motion much variable and little constant capital. For the solution of this apparent contradiction, many intermediate terms are as yet wanted, as from the standpoint of elementary algebra many intermediate terms are wanted to understand that 0/0 may represent an actual magnitude."<sup>1</sup>

So far no solution to this problem has become broadly accepted. No one has so far in outline or detail given us the "many intermediate steps that are wanted to understand that 0/0 may represent an actual magnitude.

The question is then if there has been any progress on the issue since Ian Steedman's "Marx after Sraffa (1977)" made it impossible to try to reconcile Marx and Sraffa in the way that Sweezy, Seaton, Meek, Dobbs other Marxist of those generations had tried to do.

In my opinion there has been progress. There has been a lively and interesting debate over the last 35 years. Rereading the response to Steedman's book. the debate that followed my opinion is that it contains very many insights. There was from the LTV'ers a varied response, new interpretations and approaches. I will argue that some of the "intermediate terms" has been outlined. But mostly in the negative sense that the "poverty of [linear] algebra" (Shaikh, 1981) has become increasingly clear

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<sup>1</sup> <http://www.marxists.org/archive/marx/works/1867-c1/index.htm>

and that the traditional Bortkiewicz-Steedman models are an *interpretation*, that is – there is no need to “correct” Marx or to free him from “inconsistencies”. Quite another matter is if one thinks that the solution Marx gave is a satisfactory one.

Another area where there has been progress is the role of money. For example real novelty of “The New Interpretation” (Dumenil, Foley, Lipietz) is the relationship between the wage and the commodities. Another step in the same (right) direction is of the monetary expression of labour time (MELT). The stochastic approach of Farjoun and Machover (1983) among many other very valuable insights also sketches another relationship between money and labour content.

There has in the decades since “Marx after Sraffa” emerged several new approaches to TP. If we compare this to the development in the General Equilibrium Theory camp or in the “Analytical Marxism” camp, I think clearly one can say that the LTV-camp is the most vibrant one, there is a productive debate, although the TP (or O/O problem) is still with us.

There has recently been published some new, serious attempts to address the TP. The paper will focus in particular on Andrew Kliman’s book “Reclaiming Capital” (2007) and papers by Ian Wright, especially the latest “A category mistake in the classical labour theory of value: identification and resolution”.<sup>2</sup> I will try to relate these two contributions to each other and to the preceding literature contributions on some *selected* points.

Needless to say this paper cannot discuss the alternative approaches. The points are not necessarily the one that I regard as the most important, but the ones that I managed to write about. My interest in the TP goes back to the early eighties, but for example French literature from the late seventies, early eighties I have not been able to revisit, so it will not be discussed at all, although I learned a lot from it at that time and I am convinced that there is valuable insights with relevance to the solution of the TP. Likewise I will not be able to discuss other approaches that for most Marxian economists are under the radar like, Bryer’s “Accounting solution”<sup>3</sup>, but there is clearly some valuable insights regarding how real life accounting, the stock exchange etc. value firms, calculate rates of profit, allocating joint costs. Another novel approach is Hagendorf’s use of standard micro-economic tools, inspired by the labour theory of Gossen, the works of Kondratieff which I became aware of while finishing this paper. Although I am instinctively sceptical to the use of standard micro-economic theory, it is in my opinion not a neutral tool; there might be some valuable insights in a theory that focuses on marginal labour values<sup>4</sup>.

The reason for using Kliman and Wright as focussing devices is that the two authors are both starting from the position that the labour theory of value (LTV) is fundamentally correct in the sense that labour is the only source of value, but their way of solving the TP is at radically different.

Kliman and the TSSI-school attack the “standard” assumption that Marx made a mistake in not equating input and output prices. They attack physicalism and its twin brother simultaneism. Wright on the other hand is 100% physicalist and simultaneist, but have a Shaikh-like view on how to model capitalist consumption (profits).

Wright’s solution starts from treating capitalist consumption not as a net output, but as a cost of production. Wright argues that – with good textual support from Marx that “all the phases of the commodity are produced simultaneously in the various spheres and branches”. Kliman in his book also points to the fact that there is considerable textual support for simultaneist interpretations by devoting a whole chapter to the question of “Was Marx a simultaneist?”

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<sup>2</sup> [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1963018](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1963018)

<sup>3</sup> [http://www2.warwick.ac.uk/fac/soc/wbs/subjects/accountinggroup/research/working\\_papers/marxs\\_accounting\\_solution\\_to\\_the\\_transformation\\_problem.pdf](http://www2.warwick.ac.uk/fac/soc/wbs/subjects/accountinggroup/research/working_papers/marxs_accounting_solution_to_the_transformation_problem.pdf)

<sup>4</sup> See Hagendorf’s papers at: [http://papers.ssrn.com/sol3/cf\\_dev/AbsByAuth.cfm?per\\_id=1182224](http://papers.ssrn.com/sol3/cf_dev/AbsByAuth.cfm?per_id=1182224)

## On the inconsistencies of Marx after Kliman

Kliman's "Reclaiming Marx' Capital" is of course a milestone in this debate. The terms of the debate can no longer be that Marx was inconsistent, needs to be corrected etc. Kliman's chapter 4, "On Making Marx Make Sense: On Interpretive Method" is a challenge that any "corrector" of Marx' should face squarely. But as Kliman stresses throughout the book: "I am discussing whether Marx' arguments should be judged internally consistent, *not* whether the theoretical conclusion should be accepted as true." (p. 55)

Hopefully we can now leave the charge of inconsistency levelled against Marx' solution to the TP behind us and focus on whether we think his solution - as we interpret it makes sense as an economic model - or to use this excellent German word, is Marx' solution a *Realabstraktion*. That question is still in front of us. It will be interesting to see if the "correctors" are willing to continue the debate as a debate between different *interpretations*. That would be a big step ahead, because to portray Marx fundamentally a physicalist and simultaneist - that is a hard one to play. Not the least because the inherent static nature of Sraffa's models make it impossible to claim that the results of the static model(s) is relevant for such a dynamic system as capitalism. I think they will experience the same problem as the economists in the GET camp, as Stiglitz formulated it almost twenty years ago:

The work of the past decade has made, I think, a fundamental and lasting contribution to economic analysis. The contributions have been both negative and positive: we have learned that much of what we believed before is of only limited validity; that the traditional competitive equilibrium analysis, though having the superficial appearance of generality - in terms of superscripts and subscripts - is indeed not very general; the theory is not robust to slight alterations in the informational assumptions. It is but a special - and not very plausible - 'example' among the possible set of informational assumptions which can be employed to characterise an economy. (Stiglitz, 1985, p. 21)

I am convinced that the Sraffa models are not robust to "slight" and realistic alterations in the assumptions underlying these models. Can they cope with non-convergent profit rates? With multiple techniques producing the same good? With real money and not an artificial numeraire? Seeing is believing - and we have not seen it yet. The limits of Sraffian, neo-Ricardian economics seems to me fairly clear by now. It's limits was pointed in the first round of answers to Steedman. The conceptual issues of calculation versus causal determination is well covered in Shaik (1981) and more mathematical/empirical issues by several authors in Mandel and Freeman (1984).

### Was Marx a Simultaneist?

This is the title of chapter six in Kliman's "Reclaiming Marx's Capital" (RMC). The fact that he devotes a chapter to this discussion shows in itself that it is a issue that has to be discussed. In the opening pages of the chapter Kliman writes:

"Before turning the texts, it is important to be clear about what remains to be shown and what does not. I shall not be able to prove conclusively that this or that passage is absolutely incompatible with simultaneism. All texts of all sorts permit multiple interpretations - which do not imply that all interpretations are equally acceptable. Nor shall I be able to show that simultaneist interpretations perform more poorly on a case-by-case basis, taking each contested passage in isolation. When constructed in this fashion, I find many passages to be simply ambiguous. (Some passages even seem to support the dual-system interpretation - when taken out of context. One only needs to read "the quantity of labour needed to produce" and then, disregarding whether this makes coherent sense of *Capital* as a whole.) And although I am convinced that simultaneist interpretations perform more poorly when one consider the contested passages as a whole and in relation to the rest of *Capital*, I

am sure I shall not be able to demonstrate even this much to the satisfaction of simultaneist interpreters" (p. 90).

In my opinion the problem here is not that the "simultaneist interpreters" can argue that there is quite some textual evidence for a simultaneist interpretation of Marx. Static equilibrium linear equilibrium will never capture a dynamic reality – whatever the textual support in Marx works. The problem is - let me call it *tension* in Marx text between brilliant insights in the dynamics of competition, of technological change etc. – and his *models* that are rather static.

Smith, Ricardo and Marx often argued on the basis of models of a capitalist economy in long run equilibrium. They used this kind of static models in order to get rid of all accidental fluctuations. Marx certainly knew that such an equilibrium state would never materialise. There is an abundance of passages where he underlines the ever changing, restless nature of capitalism. But in my opinion it is very little doubt that Marx thought that long run equilibrium models were useful to illustrate and/or prove some of his major results.

"We have thus demonstrated that different lines of industry have different rates of profit, which correspond to differences in the organic composition<sup>5</sup> of their capitals and, within indicated limits, also to their different periods of turnover; given the same time of turnover, the law (as a general tendency) that profits are related to one another as the magnitudes of the capitals, and that, consequently, capitals of equal magnitude yield equal profits in equal periods, applies only to capitals of the same organic composition, even with the same rate of surplus-value. These statements hold good on the assumption which has been the basis of all our analyses so far, namely that the commodities are sold at their values. *There is no doubt, on the other hand, that aside from unessential, incidental and mutually compensating distinctions, differences in the average rate of profit in the various branches of industry do not exist in reality, and could not exist without abolishing the entire system of capitalist production. It would seem, therefore, that here the theory of value is incompatible with the actual process, incompatible with the real phenomena of production, and that for this reason any attempt to understand these phenomena should be given up.*"<sup>6</sup> (Emphasis added)

What had happened if Marx had used in his numerical example of the transformation to PoP – not a uniform, but a very narrow distribution of profit rates? Just to remind the reader that a state where all profit rates are uniform will never be realised? Because the problem is that as soon as you say that it is generally OK to use static equilibrium models. Then to conclude that comparative statics is legitimate as the main method of economic analysis is a very small step to take. Having decided to use comparative statics – input prices must be equal to output prices for mathematical reasons. If not – way too many unknowns, not  $n$  prices, but 2 times  $n$ , input and output prices and only  $n$  behavioural (price) equations. To put it another way, the road to Bortkiewicz has a starting point by Marx. Below I will discuss if input and output prices they are equal in *reality* – a question Kliman very, very consciously do not discuss in his book, but that must be addressed in order to solve the TP.

Kliman argues – and quotes Freeman (1984, p. 232) that an average of a fluctuating variable generally is different from the equilibrium magnitude of that variable. That is a valid point. The problem is that Marx (and the other classics) – as I read them – consciously wanted to discuss how the economy works, the rate and distribution of profit, wages etc. on the basis of long term equilibrium, that is a position where the variables have got to their average size as a moving average over the last five or ten years, but their true, natural, precisely their equilibrium magnitudes. That is when all movements of capital between sectors of the economy, all accidental temporal changes due to weather, wars, cheating, arbitrage, changes in preferences and technology has disappeared. Marx'

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<sup>5</sup> In my opinion this is a contestable translation. One might argue from the German text that it is the different "lines of industry" that corresponds to different organic compositions.

<sup>6</sup> <http://www.marxists.org/archive/marx/works/1894-c3/ch08.htm>

average in this context I read as the empirical proxy of the equilibrium value, the unbiased estimator for the equilibrium magnitude so to speak.

The point is not that Marx was an equilibrium theorist – he clearly was not. That is so clear that if you want to see it – you realise it - as did Bortkiewicz when he accused Marx of being a “successivist”. But if you do not want to see it, then there is some textual support for the point of view that Marx’ models were intended to be equilibrium models in the same manner as Ricardo’s. That they were not conceived or understood as equilibrium models in the extreme static way as they are in modern economics. They were seen as simplifications of a more general and fundamentally dynamic model. But as the intellectual history of the TP has shown us – such models have their inherent dangers. There is no obvious way to make them dynamic, and even less any guarantee that the results are still valid. But I find it hard to understand the history of the TP if Marx had been 100 % clear that long run equilibrium models were taboo.

“Embodied” labour, (re)valuation and money

A theoretical point where I think there has been quite some progress is the less “physicalist” approach to the TP . All the new interpretations that have emerged over the last three decades view workers wages as a sum of money and not any longer as a bundle (“vector”) of goods. But still in my opinion we are still not quite “there” when it comes to integrate money into the the models that we use to try to solve – or at least to throw light on the TP.

There has been written a lot of insightful papers and books on the role of money in Marx theory of capitalism. What I extract from these works – and from Marx himself of course - is the following value for Marx is indivisible from exchange, from valorisation, the social recognition- of the private profit oriented efforts. To put it short, value takes the form of price. But still too often in the TP literature one call values which is actually just more or less “averaged” – socially necessary measures of productive effort in hours. On the other hand there is a tendency regard to regard “costs” not as values but as some kind prices only, just an accounting figure.

Everybody also knows that there is a lot of “averaging” going on. Or in other words the productive efforts from the micro individual level, the firm, the firms producing products that compete directly on the market, to sectors. Let’s take a simple example. The lazy carpenter uses 200 hrs. to make a table, the very eager carpenter uses 100 hrs. That number of hours is very objective. Dependent on demand conditions the sum of money received for the two tables could be anything from zero to – way above the “normal price”. But the lazy and the eager carpenter is just an illustration of firms with different technologies, one producing a machine in 200 hrs. and the other using only 100 hrs. What is the “value” of these two machines. That cannot be decided without studying the valuation processes in the relevant markets, wholesale and retail etc. to final consumption.

One major problem with the linear algebra models is that these models cannot deal with different technologies producing the same product. I’ll return to that below. But first another issue that I think throws light on another aspect of transformation from one set of prices to another. I intentionally leave the question of what input prices.

(Re)valuation and simultaneism

In Kliman’s RMC, in the chapter “Was Marx a simultaneist” Kliman has under the subtitle “Evidence of Simultaneous Valuation in Marx’s Works?” a thorough discussion of the question of revaluation. Kliman analysis the textual evidence in detail and while he admits it might give some support for the simultaneist position, he finds that on a closer reading it does not. But in my opinion Kliman is too generous on this point. He really have to stretch the interpretations of Marx’s text to avoid a simultaneist interpretation for at one important temporal reasons. If the rate of profit is going to have any meaning for capitalists and Marxian theorists the period over which it is calculated must be at least ... a year. In the case of “heavy” fixed capital – it might be ten, fifteen, twenty years. The

investment in fixed capital and also “circulating” constant capital is done in monetary terms. So if the market price of fixed capital and/or circulating capital changes – the principal of the loan and the rate of interest do not change accordingly. So neither capitalists nor Marxian theorists can escape from the fact that *value* always has a monetary expression. Bills have to be paid. That the good that you bought has been, due to technological development or temporary changes in demand in/decreased in price (=value) does not change your payments to the bank. If these changes is not some random fluctuation, like weather, fashions etc. it is due to technological development, but there is no room in the LAME for technological changes. Can the technical coefficients change – yes, but you only got two different equilibria, without the real –life *temporal* coherence. There are a few authors with an accounting background/interest that have pointed to the way such changes are handled. What happens if your firms fixed capital is made completely “morally obsolete”. You cannot sell it no matter how low the price. It has no value. But if you have borrowed the money, the bank tends to have a very historical valuation attitude. What happens – that is an open question. If you are a lucky capitalist you can go on as before, produce the same commodities, has the same rate of profit as before. That the computers you bought a year ago no longer have a market value might be no problem besides that simultaneous valuation predicts a much higher profit rate than you actually have on the you advanced.

I think that socially necessary labour time is a simultaneous concept. It means that the market price of goods can undergo abrupt and irreversible changes in their value that will also change the price/value of goods already produced, fixed capital goods, intermediate and consumer goods, the law SNLT is simultaneous and ruthless. When Marx as Kliman substantiates with several relevant quotes says that goods cannot transfer more value than they had when they entered production etc. that is not in contradiction to SNLT and simultaneous revaluation, but because Marx in these context abstracts from technological change, from bad harvest and similar factors that might change the market price/value of the product. What is even more important is that Marx never loses sight of the human effort that was needed to produce the product. That number of hours will never change and in many of Marx analytical contexts he uses the word value for a number of hours that deterministically will be socially validated in the market place, that will not fall victim of the disorder of the market.

The Farjoun and Machover dissolution of the TP

For me Farjoun and Machover’s book “The Laws of Chaos” is still the best general attempt to renew Marxian economics in the spirit of Marx. They use a statistical approach arguing convincingly that the rate of profit, prices etc. are stochastic variables. The discussion of the impossibility of a uniform rate of profit as a point of reference point for economic reasoning is excellent. It is the distribution of rates of profit that can be observed and discussed – and which reflects the “order out of disorder” character of the atomistic, uncoordinated, competitive character of a capitalist market economy. Farjoun and Machover (F&M) uses the paradigm of statistical mechanics to make a series of insights and observations for example regarding the rate of exploitation. Regrettably it is beyond the scope of this paper to even give a short review of the main results of the “Laws of Chaos” – and why it has not had the impact I think it deserves.

F&M devotes a chapter to the “Dissolution of the Transformation Problem”. Since F&M – correctly in my opinion – points out that theoretically there is no reason to expect that rates of profit will be uniform, which is also empirically verified; their main critique of Marx is not that he is inconsistent and needs to be corrected in the Bortkiewicz sense, but that he painted himself into a corner with the uniform rate of profit assumption. Marx “assumed that one can and should proceed in theory *as though* rates of profit are already uniform, and that it is possible to understand the internal logic of the relations between prices, profits etc. by positing an ideal situation in which these variables are replaced by their averages”. (p. 128).

Their judgment on Marx solution is that:

“Marx was unable to carry out the modifications of his model [price = value of Capital 1] in mathematical detail and generality; he merely gave a few numerical examples, incompletely worked out. But if one attempts to carry out this programme mathematically, one encounters a grave difficulty” (p. 129)

F&M formulates Marx' two models in their stochastic terms which in itself for me sheds new light on what the problem really is in a better way than the traditional linear algebra model. Given this stochastic formulation one important result is that in a stochastic sense F&M's key concept of labour content is the best predictor of a commodity's price:

“In other words, the generally rejected model of the first volume of *Capital* happens to point to the same results, concerning the specific price of non-labour commodities, for which we have argued in chapter V. So from our point of view of our own theory, the model leads, *in this particular but important case*, to a broadly correct conclusion.” (p. 133)

In the chapter on the transformation problem F&M, through a series of steps, deduces that the surplus rate of profit is equal to the expected average rate of profit “approximately” (p. 136). F&M's comment on their result:

“it displays the social meaning of profits, by expressing the average rate of profit (or a very good approximation to it) in terms of deep-level economic quantities, labour-values, rather than in terms of prices which are ephemeral. The implication of the ideas presented in this chapter should by now be clear. The transformation ‘problem’ is best forgotten; but new and ideas, of the kind we have attempted to develop in chapter V, ought to be marshalled to bring about a modern – and, necessarily unorthodox – reconstruction of Marxian economic theory.” (p. 136)

F&M also make some interesting points regarding the organic composition in other chapters of their book pointing out that while classical and Sraffian models make a very strong prediction about the profit rate - that is a single number, that competition should make profit rates converge etc. - these models make no prediction about the organic composition. F&M argues that there are “good common-sense – reasons” to suggest that the organic composition has a fairly narrow distribution. If a firm has very little capital costs compared to wages there is “little reason for the firms's existence as a capitalist business – the workers might just as well operate as freelancers or as independent producers” (p. 170). If the firm is very capital-intensive, and as a consequence has a highly automated production process, F&M argues that it might be regarded as a product and run the risk of being integrated in another firm.

The stochastic approach of F&M on the other hand makes testable predictions about the organic composition (p. 167 ff.) F&M substantiates their theoretical approach with some empirical data (p. 1985). The data shows that the organic composition is a fairly narrow and stable, comparing data from 1947 and 1975.

Despite these results the TP has not been forgotten. Again it is beyond the scope of this paper to discuss why the F&M approach has not become more dominant in Marxist economics. It is by no means dead. There was a 25<sup>th</sup> anniversary conference in 2008<sup>7</sup>. The book – while out of print – is still cited, but their dissolution of the TP has not been widely accepted. One reason might be that it is an “approximately” dissolution. Although the TP was the starting point for writing the “Laws of Chaos”<sup>8</sup>, in chapter six they do not spell the implications out at length. If the reader is not familiar with the mode, standard deviation and variance of the Gamma distribution it would be OK to get the implications spelled out. Are Marx' famous equalities, prices equals values and surplus value equals

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<sup>7</sup> <https://sites.google.com/site/iwright/probabilisticpoliticaleconomy> - abstracts are downloadable

<sup>8</sup> See Foreword, p. 9

profits, satisfied? And if only approximately – what does that mean in the sense of political economy? Because political economy, that is economics is not about technicalities, but about justice, about relationships between humans. It is about who is entitled to what share of what is produced<sup>9</sup>. But also on the ideological level it would have been desirable to spell out the implications of the results at length. The question of inconsistency of the need to correct Marx has loomed and is looming so large in the debate that when ones proposes a new approach one needs to relate that (dis)solution of the TP to the various positions in the debate.

It might be the case that a probabilistic approach has too little to say about individual prices. We are also – from the point of view of social justice – interested in guidance about what is the “correct” price for individual products?

Is a refutation also a solution?

As stated above I think Kliman has made very important contribution on the question of researchers attitude towards a scientific work on the question of interpretation and consistency so that the discussion hereafter will be a debate about interpretations of Marx and their empirical relevance I think there is a difference between stating that judging from the totality of Marx work a set of general Marxian principles like input prices do not need to be identical to output prices. Quite another thing is to formulate a refutation of the Bortkiewicz-type of model by the means of a Bortkiewicz-type model, by just taking away one of the rigid assumptions, for example that input prices equals output prices.

In Reclaiming Marx Capital (RMC) uses a small numerical example to refute Bortkiewicz proof of inconsistency (p. 150, table 8.2):

Period 1	Dept	Reve- nue.	Const.	Var	Surpl.	Value	Profit	PoP	Rate of s-value s/(c+v)	PoP Profit rate $\pi/(c+v)$	“Money”
	I		280	72	48	400	88	440	13,6 %	25,0 %	352
	II		80	96	64	240	44	220	36,4 %	25,0 %	176
	III		40	72	48	160	28	140	42,9 %	25,0 %	112
	Sum		400	240	160	800	160	800	25,0 %	25,0 %	640
Period 2											
	I	66	308	66	54	428	102	476	14,4 %	27,3 %	374
	II	44	88	88	72	248	48	224	40,9 %	27,3 %	176
	III	30	44	66	54	164	30	140	49,1 %	27,3 %	110
	Sum	140	440	220	180	840	180	840	27,3 %	27,3 %	660
[...]											[...]
Period 14											
	I	88	378	74	46	498	88	540	10,2 %	19,5 %	452
	II	40	108	99	61	268	40	247	29,6 %	19,5 %	207
	III	25	54	74	46	174	25	153	35,9 %	19,5 %	128
	Sum	153	540	247	153	940	153	940	19,5 %	19,5 %	787

<sup>9</sup> In my paper to the 25th anniversary conference entitled “Labour content and skills: social justice or statistical pragmatism?” I discuss the a bit too “pragmatic”, “approximate” approach to question of social justice.



In this table – from period 1 to 14 the three equalities hold. Sum of values = sum of prices of production (PoP), sum of surplus value = sum of profits and value rate of profit = PoP rate of profit. Simple reproduction occurs. The physical quantities are the same in all periods, the sum of variable capital and surplus labour is always 400, and technology is constant. There is an “inflation” in the price of constant capital, the same physical units are bought at these values at 400 and end up costing 440. The values start at 800 in period 1 and end up at 940 in period 14. After 14 periods the system is practically in a steady state with a profit rate that is the same as the rate you would get if you solved the simultaneous equation system a la Bortkiewicz.

The transition from one period to the next is done so that the output prices (PoP) of one period is the input prices of the next period, so to get the new price for constant capital in department 1, the old price, 280, is multiplied by a factor that equals the sum of output constant capital prices, 440, divided by the sum of input constant capital prices, 400. That is  $280 \times (440/400) = 308$ .

As an arithmetic refutation this works, but does it work as a first step towards a convincing solution? For me, not yet. The table raises several questions about how to interpret it. First of all the questions related to money. Kliman says correctly that “Finally, it is worth noting that early critics of his [Marx’s] solution, such as Böhm-Bawerk and Sweezy, did not complain about any mixing and matching of money and labour-time figures, Bortkiewicz explicitly recognized that ‘Marx thought of values and prices in terms of money’.

But where does money come from in this model? If we suppose that in some way or another there was enough money in the “system” to finance the initial amounts of C and V. That sums up to 640 units, not enough to realise any surplus value. This raises Rosa Luxemburg’s problem with how the surplus value is realised. This fundamental question should be answered.<sup>10</sup>

Another deep question is how “the system” figures out how much surplus value that have to be realised? A correct critique of neo-classical Gen. Eq. Theory is that how do the “price-takers” know which prices to take without a Walrasian auctioneer crying out the prices? One might argue that the rate of exploitation has been found by trial and error in the past, so that at this point in history the PoP profit rate has converged to a level where the surplus value is realised. There are several more or less plausible ways of “closing” the model in this respect. But we need a real-life story – if not in the model so at least behind it.

Since the values are constant, there is inflation in the model. As time/period goes by you get less value (products) for the same amount of money. Is there a bank here printing money as the need arises from period to period? Since the system converges to a steady state that is a *logical* possibility, but one needs to substantiate this by pointing to some observable real world phenomena, contemporary or historical.

Laibman in his critique of the original formulation of the above refutation by Kliman and McGlone (1999) states that “In fact computer simulations shows that these numbers do not converge” (Laibman, 2004, p. 10) and claims that there is “an infinite set of regress problem” and hints at that Kliman and McGlone stopped after two periods due to this problem. To me the system seems to converge, actually to the same *alternative* solution as Laibman describes as the “arguably the truly dynamic variant of this model” (p. 10). Laibman also claims that “These numbers, whether defined as quantities of abstract labour time or their money counterparts, corresponds to a real production process involving flows direct and indirect labour.” (p. 10) I would object to the claim that these models are so abstract, so far from the dynamics of capitalism that they must be seen as logical

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<sup>10</sup> See <http://libcom.org/library/finance-realization-problem-rosa-luxemburg-%E2%80%98circuitist%E2%80%99-reappraisal-riccardo-bellfiore-m> for an up to date introduction and reference to the literature about the problem by Ricardo Bellofiore.

exercises and that a lot of analytical work is need to substantiate that they represents – even in a very abstract manner – any real production process.

But be that as it may. The point is that Laibman also recognises the need for a new approach stating in the concluding part that “the tableau method of approach to this problem, originated by Marx and refined by Bortkiewicz, is a rather clumsy one” (p. 13). Laibman calls for methods that both grasp that “Many processes in capitalism are sequential, and constant disruption and transformation is a fact of life. ... There is also simultaneous determination, simultaneous equation models in fact capture one essential aspect of the capitalist economy interdependence among atomistically separated units of control”. (p. 14). I do not think that simultaneous equation models we know of today, Walrasian or Sraffian can throw much light on that interdependence because there is causality in time, there is learning/innovation etc. The Walrasian and Sraffian models are models where the firms are not active price makers. Prices are given either by the auctioneer or by technical coefficients, increasing returns to scale is taboo etc. Laibman also states that “As a methodological too, equilibrium paths are necessary ground for the study of *disequilibrium* dynamics.” (p. 15). That is true, but the fact is that Arrow-Debreu and Sraffa are no paths, it is fixed points. The stability of Arrow-Debreu models have not – and in my opinion cannot be proved. Sraffa’s models are constructed not to generate any paths, equilibrium or disequilibrium.

A category mistake? Non-standard labour values?

In a series of papers Ian Wright (2007, 2009, 2011) has elaborated the concept of “non-standard labour values” and argued that the TP has its root in a category mistake. The mistake is to treat – if I understand it correctly – capitalist consumption out of profits in the same manner as workers consumption. Since it as since Bortkiewicz – and especially after Steedman (1977) been clear that one had to think “outside the box” in order to solve, dissolve or dismiss the TP, at least in its Bortkiewicz’ formulation.

But before discussing Wrights main points a small digression related to the question of using linear production theory. Wrights starts his “Category mistake” paper with a exposition of Marx view on the two aspects of time regarding production, antecedent and coexisting. The antecedent aspect focuses on the necessary sequence in production:

“The part of capital which consists of instruments and materials of labour is as “commodities already created” always a pre-condition in each *particular* branch of production. It is impossible to spin cotton which has not yet been produced, to operate spindles which have yet to be manufactured, or to burn coal which has not yet been brought up from the mine. These always enter the [production] process as forms of existence of *previous labour*. Existing labour thus depends on antecedent labour and not only on coexisting labour, although this antecedent labour, whether in the form of means of labour or materials of labour, can only be of any use (productive use) when it is in contact with living labour as a material element of it.”<sup>11</sup>

The coexisting aspect is described by Marx in the following way:

‘[Raw] cotton, yarn, fabric, are not only produced one after the other and from one another, but they are produced and reproduced simultaneously, alongside one another. What appears as the effect of antecedent labour, if one considers the production process of the individual commodity, presents itself at the same time as the effect of coexisting labour, if one considers the reproduction process of the commodity, that is, if one considers this production process in its continuous motion and in the entirety of its conditions, and not merely an isolated action or a limited part of it. There

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<sup>11</sup> <http://www.marxists.org/archive/marx/works/1863/theories-surplus-value/ch21.htm>

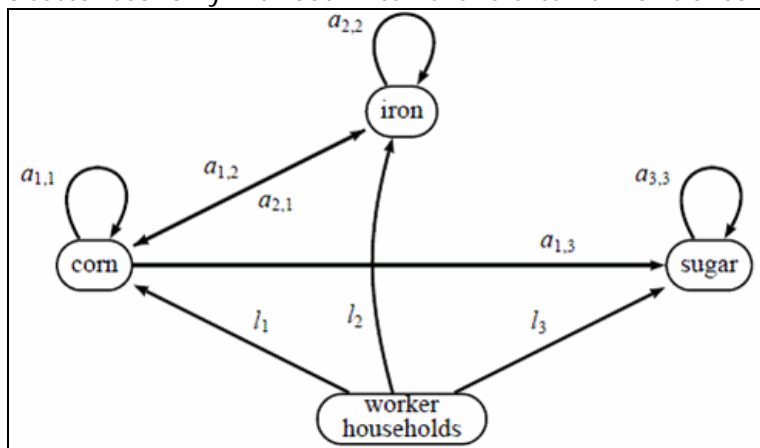
exists not only a cycle comprising various phases, but all the phases of the commodity are simultaneously produced in the various spheres and branches of production.<sup>12</sup>

The point I want to make is that how time is treated in a model is not either antecedent (sequential) or coexisting (simultaneous). Our models should be able to cope with both. Although it is not explicit in Wright's 2011 paper, I interpret it as a justification for using a traditional linear production model for his purpose. Which means that he does not deal with different, coexisting technologies producing at the same time the same product = competition. But that's not the issue just now.

In Fig 1 we see Wright's graphical illustration of a standard linear production model. It shows the inputs to one sector from the other sectors, and the sectors use of its own product.

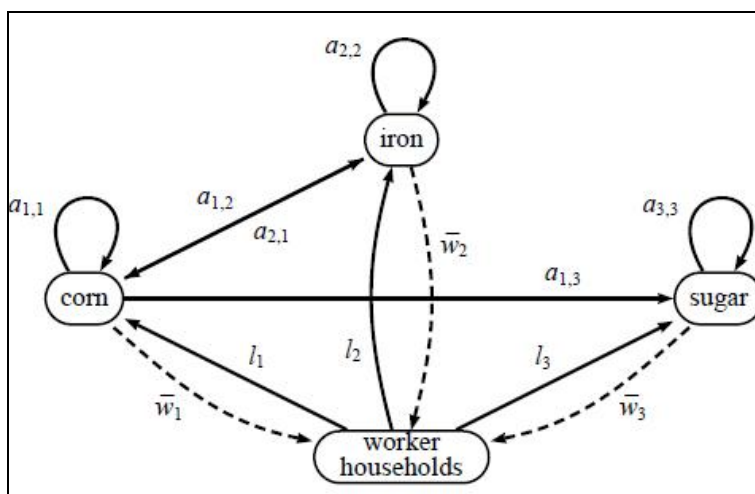
Figur 1 A

3 sector economy with both internal and external flows of commodities, standard labour values



The use of the graph is that one can start in a sector  $i$  and trace the direct and indirect labour required to produce a certain product. Iron has  $l_2$  direct labour, and since the production of iron also need corn and sugar, there are indirect labour  $l_1$  and  $l_3$  also. This is merely physical, no wages or profits has been introduced yet.

Figure 2 A social accounting matrix – workers only economy.



Another name for the economy in Figure 2 is the "early and rude state" of Adam Smith, and "everybody" agrees that in such a "workers only economy" equilibrium prices are proportional to labour values. Since we measure the value of commodities by hours of labour time it does not make sense to ask "what is the value of an hour of labour, measured by labour time" since the real cost of

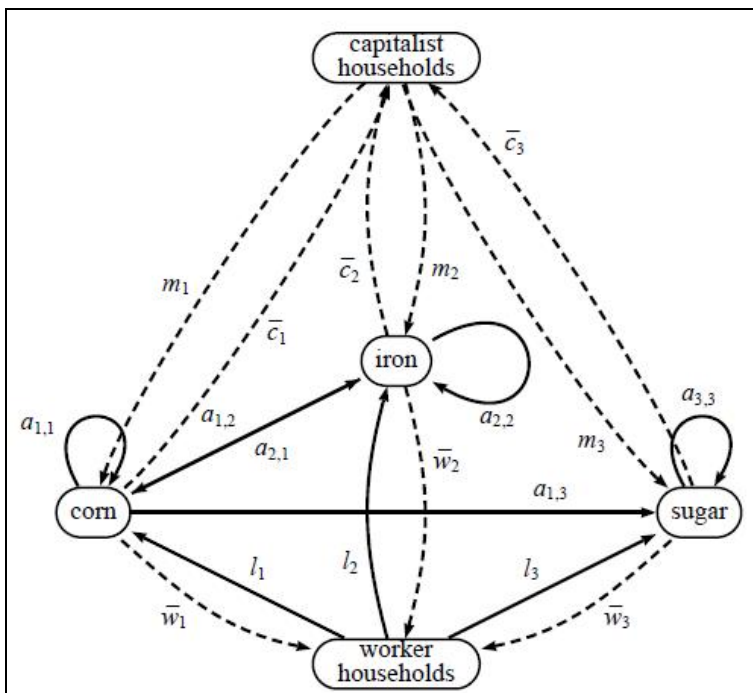
<sup>12</sup> Ibid.

1 hour of labour time, measured by labour time, is one hour. Or to put it another way – the tracing of labour values stops at the sector that supplies labour.

The fact that very few have any objections to the LTV in this early and rude state, but a suddenly jumps to explaining, or determining prices in terms of utility and “the technical conditions of production” throwing the LTV over board. But if the LTV was true in a state without “frozen labour” (capital) how could it suddenly become false when frozen labour enters the picture/economy.

Wright is not among those that throws the LTV overboard when getting from a rude to a civilised state of the economy. Instead his approach is to “introduce capitalist profit income and determine exactly why this simple relationship breaks down. (2011, p. 10). At this stage Wright also introduces money into his model, because “in capitalist conditions production also requires money capital”.

Figure 3 A social accounting matrix with money as a factor of production.



As Wright point out:

“Nobody ‘makes’ money-capital, even in circumstances where money is a commodity. Money capital is not produced, but lent. Hence we assume the supply of money does not incur indirect labour. [...] But although there are no direct labor costs there are indirect labor costs associated with production financed

by money-capital. Capitalists do not advance money-capital for free, either nominally or in real terms. [...]The standard formula for labor-value does not vertically integrate over the input paths corresponding to money-capital. Money-capital inputs are not part of the technique, and are therefore ignored, which is equivalent to treating money-capital inputs as an irreducible terminus on the same footing as the supply of labor (e.g., all the dashed input edges from capitalist households in Figure 3 are not traversed). In consequence, standard labor-values do not count the coexisting labor employed to produce capitalist consumption goods as a real cost of production.” (p. 11).

Wright then asks “Should this labour be counted as a cost?” and the answer is of course what we want to measure and if we want to measure productivity increases independent over time independent of the distribution of income – standard labour values are the right measure. But if we want to measure total labour costs we – according to Wright - cannot use standard labour values since they exclude the costs of reproducing the capitalist class. Wright points out that “... the labour

required to produce capitalist consumption goods is not a cost of *reproducing labour...*" (p. 11, emphasis added) and that standard and non-standard labour costs are identical in the case of a worker only economy and concludes that "If we aim to calculate the *total* coexisting labour required to produce a commodity then we must treat money-capital as a *bona fide* commodity and include its (indirect) labour cost as a real cost of production." (ibid).

Under the subtitle "Dissolution of the transformation problem" Wright points out that "Non-standard labour-values, by definition, include surplus labour as a cost of production. In consequence, they cannot split the working day into necessary and surplus parts. In terms of total labour costs, the whole working day is "socially necessary..." (p. 15).

Before going discussing Wright's dissolution of the TP let's point out – as Wright does – that the key point – that it is capitalist consumption that creates the TP has been pointed out already by Anwar Shaik (1981, 1984)<sup>13</sup>. Shaik argued that the non-conservation of labour value in price is due to the transfer of value out of what Shaik calls "circuit of capital" into capitalists "circuit of revenue" – capitalist consumption.

Could there be any other labour values than total labour values?

The TP is fundamentally about whether commodities sell at a price closely correlated with the human effort (measured in time)? That is the model of the first volume of Capital, but not of the third volume. But let's take a step back – or rather into the factory producing, let's say ten chairs a day. Isn't it then rather obvious that the effort behind each chair containing the effort of living labour and antecedent labour in the form of a share of the costs connected to the machines (wear&tear, rents&replacement) – is the same for each chair? And that in a worker only economy – the price of the chair would correlate closely to the effort (labour) behind each chair? At the usual level of abstraction in the TP-debate it would be a 1:1 correspondence (homogenous labour, no realisation problems, no moral obsolescence etc. etc.)

In a workers-only economy the "labour content", the "embodied labour", the "value" of a chair would be 1:1 with its price. Under feudalism the aristocrat would take four chairs let the workers sell residual six chairs. Under capitalism the extraction of surplus value takes different forms. One of them is very high wages for the executive layer of firms. In fact the worker-only model applies, products sell at their "values" with the modification that wages are not uniform. From a linear algebra point of view – as Wright has shown – the TP then dissolves.

The problem is then transferred to the problem of what wages are "just". A very interesting result in Farjoun and Machover (1983) is that if your wages is twice the average you starting to get some of the surplus value, you are starting to be an exploiter.

Of course shareholding is also a very common way of extracting the surplus – almost always in combination with very high wages for the executive layer of the firm. One might model shareholding as a kind of direct, human effort to the production of the product – and again we are in a workers-only economy. Another way of modelling this would be to use feudalism as a mental model – adapted to a completely monetary capitalist market economy. Let's think of it as two stage procedure. First labour (with a given wage structure, more or less egalitarian) produces the chairs. They are then sold. The price of each chair reflects directly the "costs" (effort, labour of workers and executive). This results in a sum (pile) of money. Due to the power of shareholders in society they are able to get a share of this sum, which does not reflect their productive contribution (labour). Also in this case the chairs sell at their value, from a linear algebra point of view this is a workers-only economy where the TP does not exist.

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<sup>13</sup> Wright (2007), footnote p. 38. Wright only refers to Shaik (1984), the explanation in Shaik (1981) is less technical.

If the solution was so simple why did not Marx propose it?

That is a difficult question, but when I read Marx I often get a feeling that there is a tension between Marx's insights into the dynamic chaos of capitalism, of competition and a Marx that wants to prove his point on the basis of a Ricardian framework. Just to take one example, money. Some parts of Marx' economic writings outlines a theory of money as a number in a computer representing abstract labour as the only "social cost" or fiat money. But now a days money is just more and more pure numbers in databases (files). In other parts, money is a commodity. Another question is wages, as subsistence wages (regulated by natural laws) and wages being dependent on historical and moral factors<sup>14</sup>.

When it comes to value theory, there is a tension between a "substance" theory, the use of "embodied labour", "labour content" leads to think of something that is a characteristic (Eigenschaft) of the thing, when in fact the only thing which is objective in that sense is the amount of human effort. The lazy carpenter uses 200 hours, the eager 100, but only the social context can tell what the price (value) of the two tables will be on the market.

Summing up – criteria for a real solution to the TP

Although Kliman and Wright have made substantial contributions to the debate. Kliman and the TSSI getting us closer to the end of a sterile "inconsistency" and "correcting Marx" research agenda. Wright in making us really rethinks the concepts of the problem and their use and meaning. I still think we have some distance to go before the TP has got a solution that is broadly accepted. From my own rereading of parts of the TP literature I would argue that a solution has to satisfy the following criteria:

1. It must take as a tautological starting point that prices are the monetary form of values. Consequently the sum of prices must be equal to the sum of values no matter if the essence of value is utility, energy or human effort. It must be a single system.
2. It follows tautological from point 1 in the same tautological manner that sum of profits equals sum of values and that the value rate of profit is equal to the price rate of profit.
3. Marx equalities holds, that is the obvious starting point. The real TP problem is to have a real, meaningful, economic story that links human effort to prices and as a result to profits. The theory should preferably be formulated mathematically to make it easier to discuss whether the theory is consistent.
4. The solution must have a theory of money. It cannot be a barter economy with the i'th good is arbitrarily chosen as a numeraire. The theory of money must show how surplus value is realised, when the money capital advanced is the sum of prices of means of production and wages.
5. It must be a solution that do not presuppose that we are in a short-term or long term equilibrium, it must be a theory that models the (dis)equilibrating effects of real life competition among capitals and workers. It should model the "equilibrium levels of disequilibrium".
6. A mathematical solution to the TP (the effort-price problem) must be both general and sufficiently specific, satisfying the following criteria.
  - a. Specific in the way that time must be explicit, the length of the period(s) of profit rate calculation by individual firms (micro) and the economy (macro) must be indicated. Are we dealing with a split second or a decade?
  - b. Input prices need not be identical to output prices, but can be

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<sup>14</sup> From what I have heard about it, Michael Heinrich, Die Wissenschaft vom Wert : die Marxsche Kritik der politischen Ökonomie zwischen wissenschaftlicher Revolution und klassischer Tradition. VSA-Verlag, Hamburg 1991 dicusses this tension in Marx' writings.

c. To handle different technologies (organic compositions) producing the same product. It cannot presuppose that SNLT has established itself, since this is a major transformation from human effort to price, is a transformation problem in itself. How SNLT relates to human effort is of extreme importance when analysing exchange between economies with different technological and/or wage levels.

d. It must handle all types of returns to scale, especially increasing returns to scale, since this is the most prevalent and for profit-maximising firms - the preferred shape of the production function. There are always some fixed/sunk costs, always some learning by doing which leads to increasing returns to scale.

The above criteria does of course not mean that it is forbidden or wasteful to make small, static, linear algebra models to investigate a certain aspect of the TP, to clarify one of the "many intermediate steps" that is necessary to solve the 0/0 paradox that Marx thought the TP was. But we should not call that a solution.

#### References

Farjoun and Machover (1983). Their book *Laws of Chaos: A Probabilistic Approach to Political Economy*, Verso

Freeman, Alan. 1984. The logic of the transformation problem. In Mandel and Freemand (eds.) 1984, 221-264.

Andrew Kliman: *Reclaiming Marx's "Capital": A Refutation of the Myth of Inconsistency*. Lexington Books 2007

Mandel, E. and Freeman, A. (1984), *Ricardo, Marx and Sraffa*, the Langston Memorial Volume. London: Verso.

Steedman, I., 1977. *Marx after Sraffa*. Verso, London.

Steedman, I., 1981. *The Value Controversy*, Verso

Shaikh, A, 1981 "The Poverty of Algebra" in I. Steedman(ed) *The Value Controversy*. London: Verso

Stiglitz, Joseph E. 1985, *Information and Economic Analysis*, *The Economic Journal*, Vol. 95, Supplement: Conference Papers (1985), pp. 21-41

Stiglitz, Joseph E. 2002, *Information and the Change in the Paradigm of Economics*, *The American Economic Review*, Vol. 92, No. 3, June 2002.

Wright, I., 2007. Prices of production are proportional to real costs, *Open Discussion Papers in Economics*, no. 59. Milton Keynes: The Open University.

Wright, I., 2008. The emergence of the law of value in a dynamic simple commodity economy. *Review of Political Economy* 20 (3), 367–391.

Wright, I., 2009. On nonstandard labour values, Marx's transformation problem and Ricardo's problem of an invariable measure of value. *Boletim de Ciências Económicas LII*.

Wright, I., 2011. A category-mistake in the classical labor theory of value: identification and resolution., [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1963018](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1963018)

Wright, I., 2011b. Classical macrodynamics and the labor theory of value, *Open Discussion Papers in Economics*, no. 76. Milton Keynes: The Open University.