Debt, Financial Fragility and Income Distribution: Minskian Stock-Flow Consistent Economic Models Using Real and Financial Capital

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Introduction

- introduces mathematical and simulation models that use basic economic variables

- give straightforward explanations of the distributions of wealth, income and earnings

- provides simple effective methods for eliminating poverty without using tax and welfare.

- Ian Wright, Makoto Nirei & Wataru Souma have produced work on similar lines

- the general approach for the macroeconomic models were partly inspired by the work of Steve Keen

- indebted to the work of Levy & Solomon and their GLV models.

Assumptions

– Economics is a statistical mechanical system

- Economics and finance are many bodied systems
- Outcomes defined by statistical probability
- Can often be modelled by identical (homogeneous) models

– Classical Economics

- Economic goods have real intrinsic 'value':
    - real capital can produce more real capital
    - Priced by cost-plus, Sraffa/post-Keynesian (not marginality)
    - “Negentropy” - following Schrödinger in biology

– Flow models

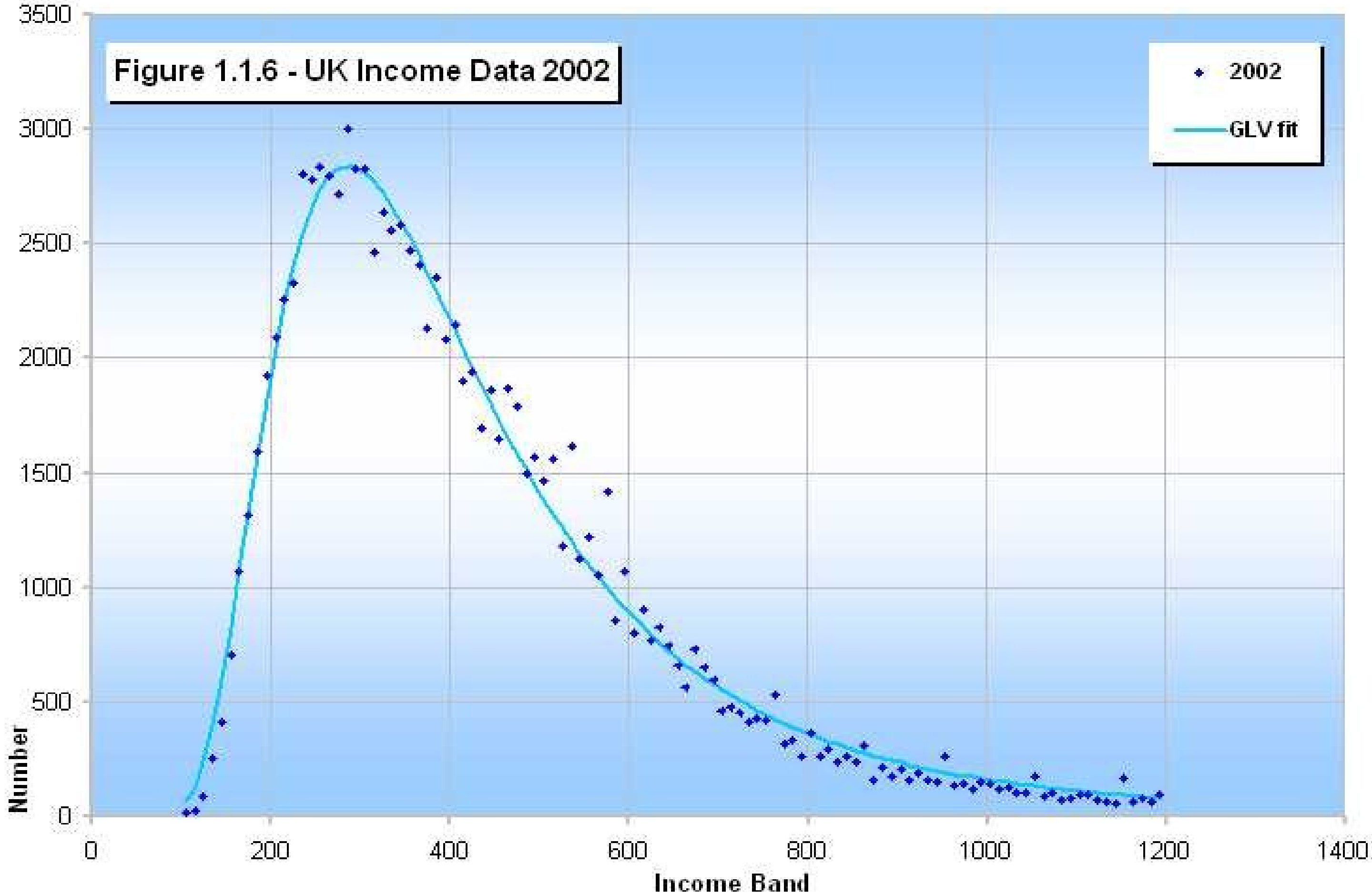
- Production – creation of wealth
- Consumption – destruction of wealth
- Exchange – conservation of wealth

- Stock-flow consistent
    - Models don't formally close
    - Feedback loops
    - Need 'buffer' / 'balance' / 'float' variables to accommodate unexpected changes

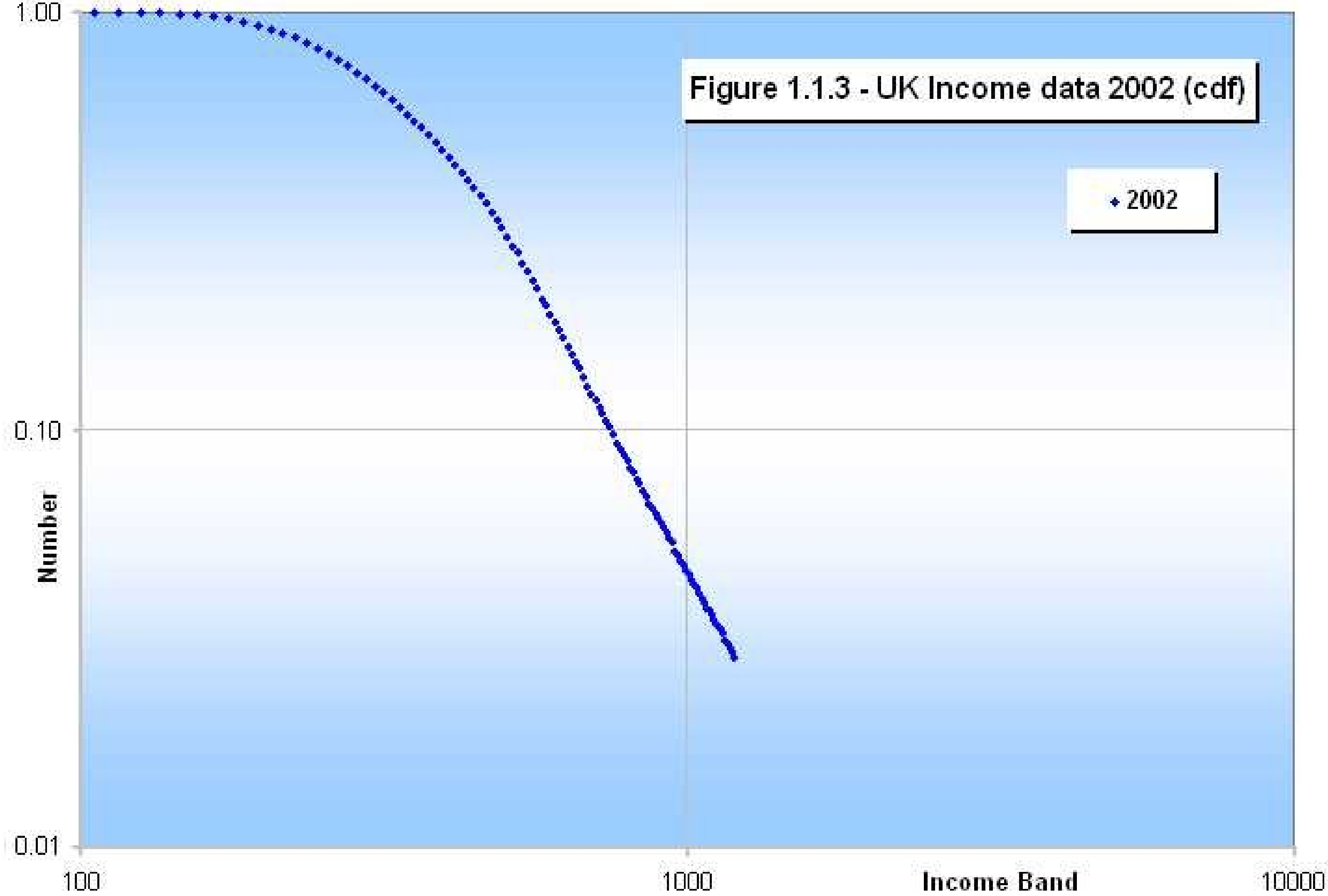
○ Models similar to those of Godley, Lavoie, Keen

1.0 Wealth & Income Data – Empirical Information

- persistent patterns across different economic systems



'log-normal' body offset from zero



Power 'Pareto' tail

- persistent patterns across different economic systems
  - Pareto – 1896
    - Britain, Prussia, Saxony
    - Ireland, Italy, Peru

    - Ancient Egypt
      - "Wealth Distribution in an Ancient Egyptian Society"
        - A. Y. Abul-Magd

- neglected by economists

- fascinates physicists
  - implies deep structure

Different Sources for Income:

- income in 'log-normal' section from wages
- income in Pareto tail from dividends, capital gains, rent, small businesses, etc
    - Clementi & Gallegati, 2005 - “Income Inequality Dynamics”
    - Thomas Hungerford, 2011 - “Changes in the Distribution of Income Among Tax Filers Between 1996 and 2006: The Role of Labor Income, Capital Income, and Tax Policy”
    - Wolff & Zacharias, 2007 - “Class Structure and Economic Inequality”

General Lotka-Volterra Distribution - (GLV)

- can fit power tail
- can fit log-normal body
- has offset from zero
- gives very good fit to data

Figure 1.1.8	Reduced Chi Squared	
	Full Data Set	Reduced Data Set (no power tail)
Gamma/M-B Fit	3.27	1.94
Log Normal Fit	2.12	3.02
GLV Fit	1.21	1.83

1.2.1 Lotka-Volterra systems

- population of prey x (say rabbits)
- population of predators y (say foxes)
- no predators present, natural population growth rate 'a' of rabbits:

$$\frac{dx}{dt} \propto ax \qquad (1.2.1a)$$

- no rabbits to eat, natural death rate 'c' of the foxes:

$$\frac{dy}{dt} \propto -cy \qquad (1.2.1b)$$

- foxes encounter rabbits, rate at which rabbits are killed is proportional to the number of rabbits and the number of foxes:

$$\frac{dx}{dt} \propto -\alpha x y \qquad (1.2.1c)$$

- α is a constant, and the –ve sign, not good for the rabbits.
- However good for the foxes, giving:

$$\frac{dy}{dt} \propto \gamma x y \qquad (1.2.1d)$$

- γ is fixed constant.

- a pair of differential equations:

$$\frac{dx}{dt} = ax - \alpha x y$$

$$= x(a - \alpha y) \qquad (1.2.1e)$$

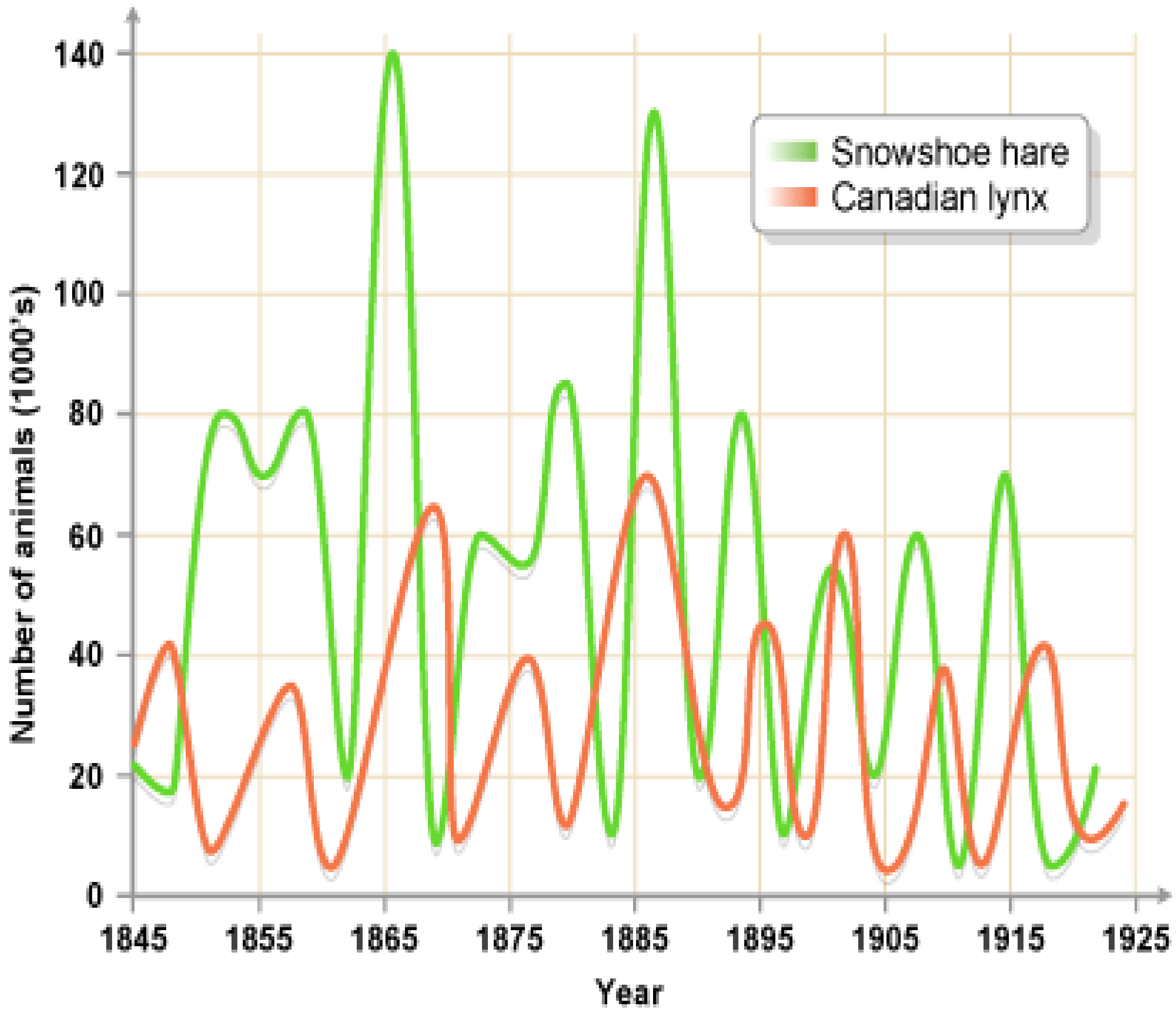
for the rabbits; while for the foxes:

$$\frac{dy}{dt} = \gamma x y - cy$$

$$= y(\gamma x - c)$$

$$= y(-c + \gamma x) \qquad (1.2.1f)$$

- cause and effect in both directions



- Normally unstable system:

1.2.2 General Lotka-Volterra (GLV) systems

- General Lotka-Volterra system (GLV) extends Lotka-Volterra model to multiple predators and prey:

$$\frac{dx_i}{dt} = x_i r_i + \sum_{j=1}^{N} a_{i,j} x_i x_j \qquad (1.2.2a)$$

$$= x_i \left(r_i + \sum_{j=1}^{N} a_{i,j} x_j \right) \qquad (1.2.2b)$$

- dx_i/dt is rate of change for the i-th species, out of a total of N species.

$$\frac{dx_i}{dt} = x_i\left(r_i + \sum_{j=1}^{N} a_{i,j}\,x_j\right) \qquad (1.2.2\,b)$$

- first term natural growth (or death) rate, r_i, for the species with population x_i. Rate r_i is equivalent to 'a' or '-c' in equations (1.2.1e/f).
- second term is sum of interactions with all j other species. Here $a_{i,j}$ is interaction rate between species i and j.
- $a_{i,j}$ is negative if j is a predator, positive if i is a predator. $a_{i,j}$ is equivalent to α or γ of equations (1.2.1e/f).
- Equations (1.2.2a/b) are generalisations of equations (1.2.1e/f) for many interacting species.
- potentially N! separate differential equations are needed to describe the whole system.

- Simplified by Solomon & Levy [Solomon 2000]
- difference equation for city population sizes.

$$w_{i,t+1} = w_{i,t} + rw_{i,t} + a_t \bar{w}_t - c_t \bar{w}_t w_{i,t}$$

$$w_{i,t+1} = \lambda w_{i,t} + a_t \bar{w}_t - c_t \bar{w}_t w_{i,t} \qquad (1.2.2c)$$

- uses w bar as average population
- λ is the natural growth rate of the population w of city i,
- a_t is the arrival rate of population from other cities, multiplied by the average population $\bar{w}$ of all the cities.
- c_t gives the rate of population leaving each city

$$w_{i,t+1} = \lambda w_{i,t} + a_t \bar{w}_t - c_t \bar{w}_t w_{i,t} \qquad (1.2.2c)$$

- λ, a and c are universal rates, apply to all agents.
- λ and a 'positive autocatalytic' (positive feedback), increase the population w of each city.
- negative value of c decreases the population of each city.
- Without the negative feedback term, the populations of the cities can increase to infinity
- Without the positive growth of λ in the first term, the second and third terms will cause all of the population to end up in a single city.
- Normally one or more variables are assumed to be stochastic.

- Gives a stable probability distribution for city populations:

$$P(w) \propto (e^{-(\alpha-1)/w})/(w^{(1+\alpha)}) \qquad (1.2.2d)$$

$$P(w) = K(e^{-(\alpha-1)/(w/L)})/((w/L)^{(1+\alpha)}) \qquad (1.2.2e)$$

- Lotka-Volterra – feedback from x to y,

  and also feedback from y to x.
- GLV – feedback from x_i to all the other $x_{j.}$

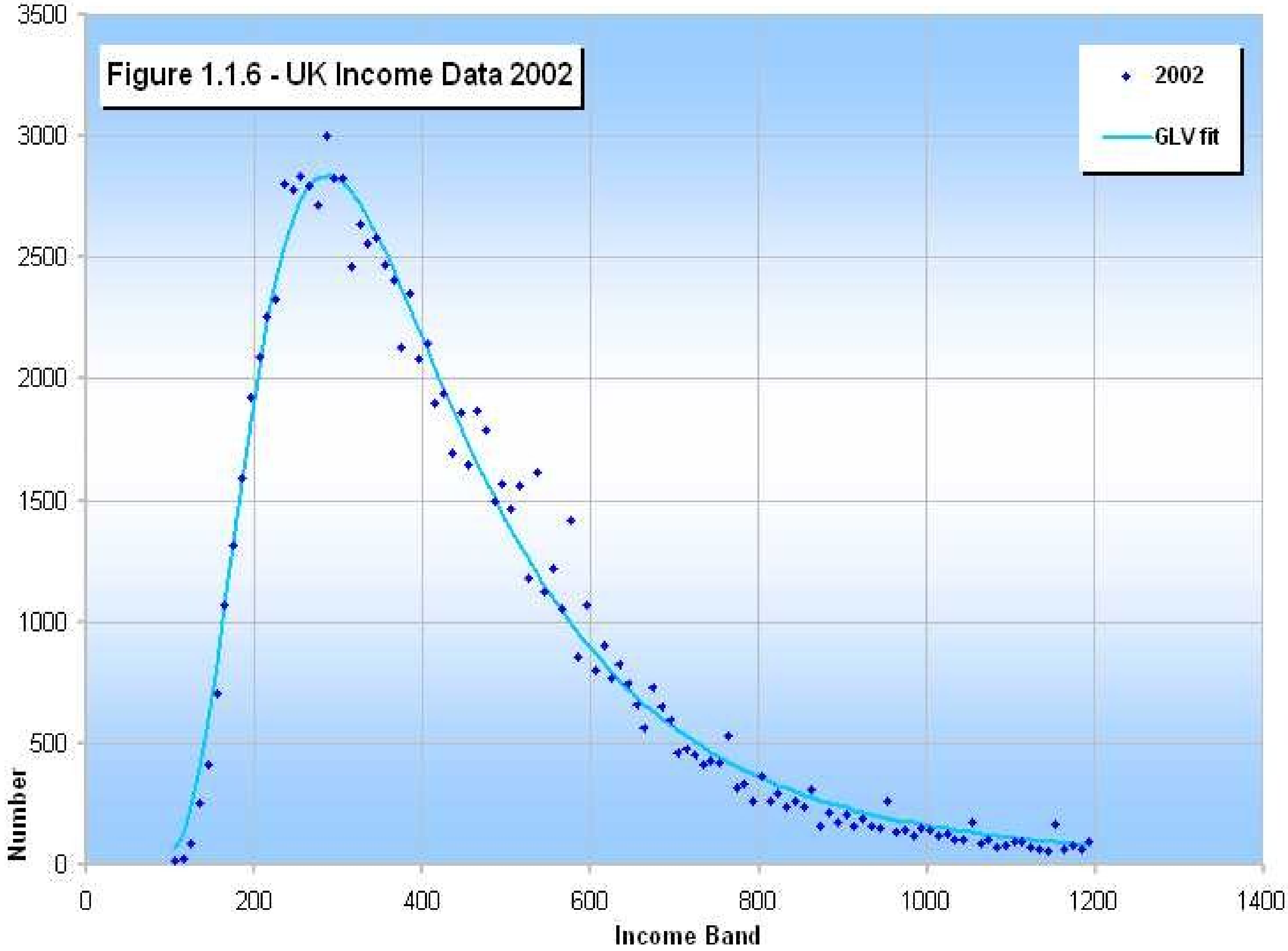
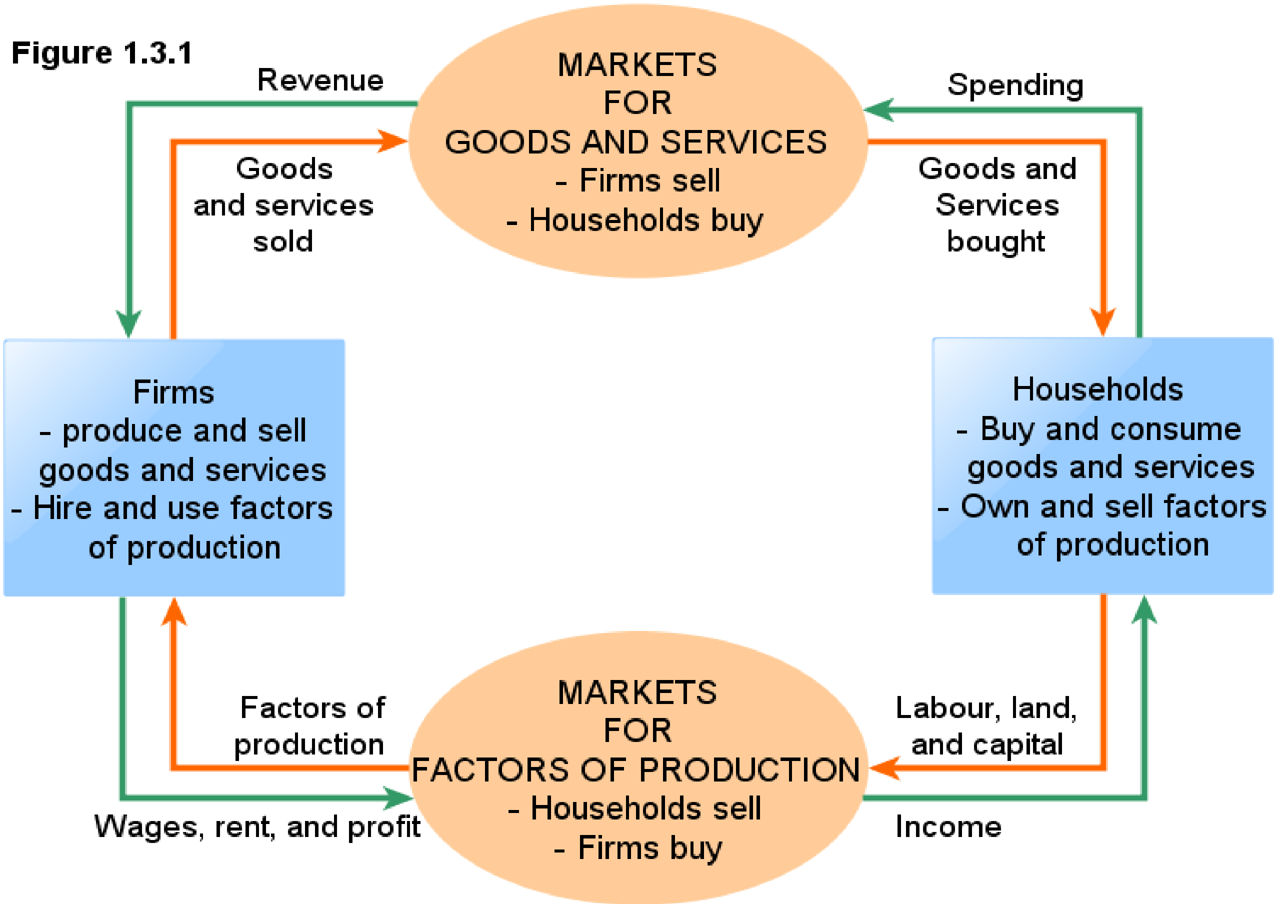


Figure 1.1.6 - UK Income Data 2002
2002
GLV fit
Number
Income Band
0
500
1000
1500
2000
2500
3000
3500
0
200
400
600
800
1000
1200
1400

1.3 Wealth & Income Models - Modelling

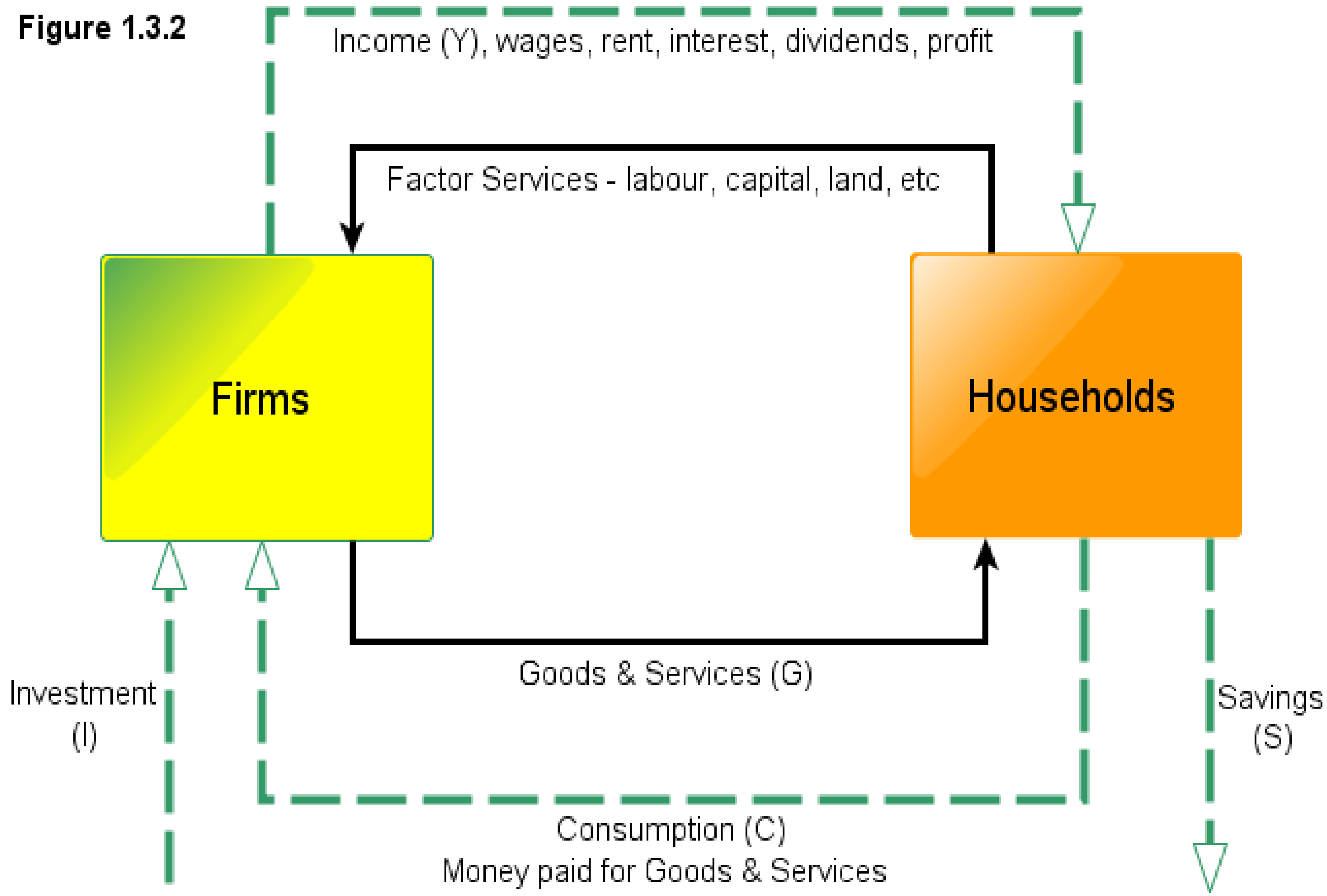
- traditional economic model:

Figure 1.3.1



• Typical 'circular flow'

Figure 1.3.2



- Incorrect – shows flow of capital and land from households to firms

- Householders don't sell blast furnaces to companies

- Investment & saving not main source of capital

- *“Most corporations, in fact, do not finance their investment expenditure by borrowing from banks.”* [Miles & Scott 2002, 14.2. Corbett J, Jenkinson T, 1997. How is investment financed?]

TABLE 14.1 The Financing of Investment: Flow-of-funds Estimated (%) (1970–1994)

	Germany	Japan	UK	USA
Internal finance	78.4	69.9	95.6	94.0
Bank finance	12.0	30.1	15.0	12.8
Bond finance	−1.0	3.4	3.8	15.3
New equity	−0.02	3.4	−5.3	−6.1
Other	10.6	−6.8	−9.1	−16.0

Note: Internal finance comprises retained earnings and depreciation. The other category includes trade credit and capital transfers. The figures represent weighted averages where the weights for each country are the level of real fixed investment in each year in that country.

Source: Corbett and Jenkinson, "How Is Investment Financed?" *The Manchester School* (1996) vol. LXV, pp. 69–94.

• Value >1 - most of the time

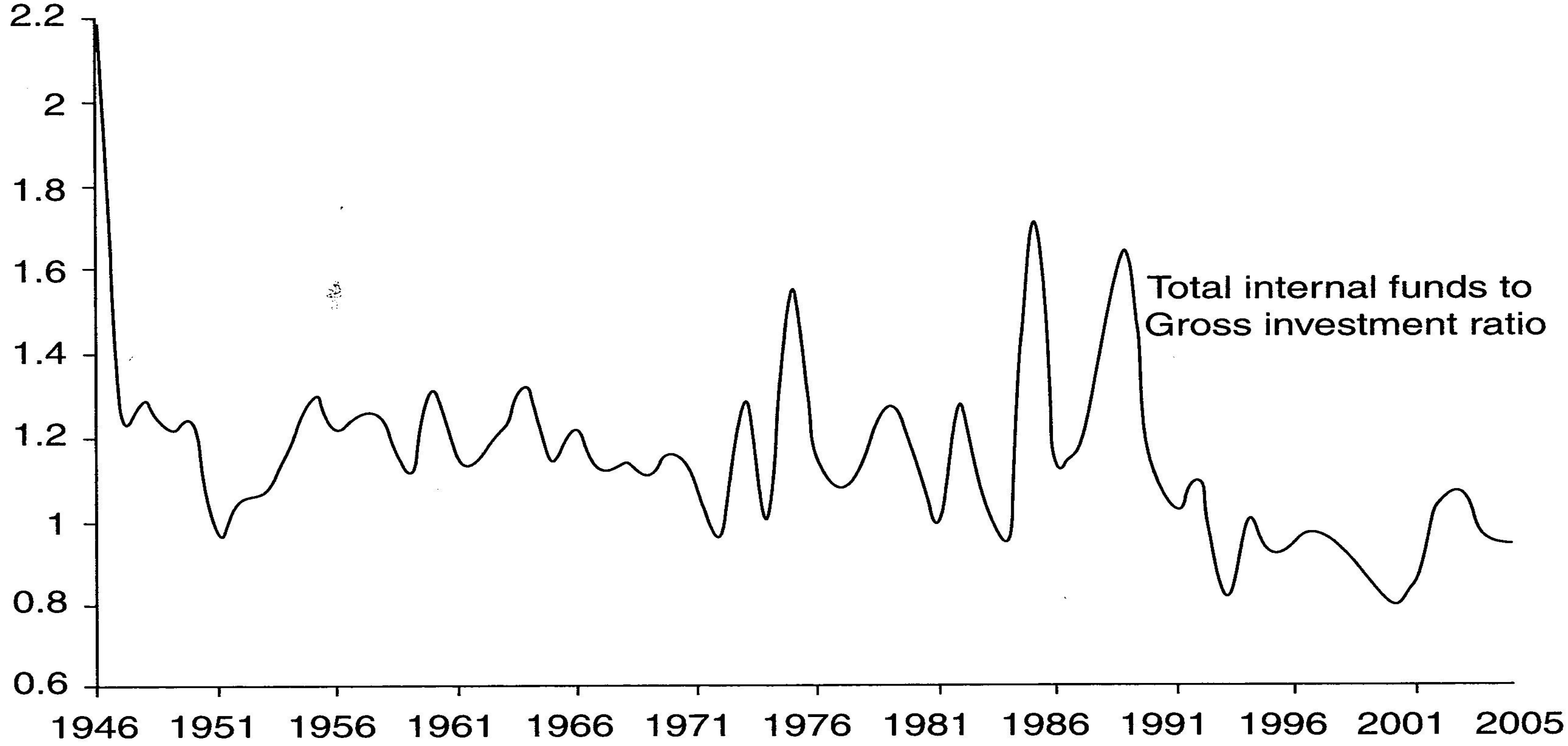
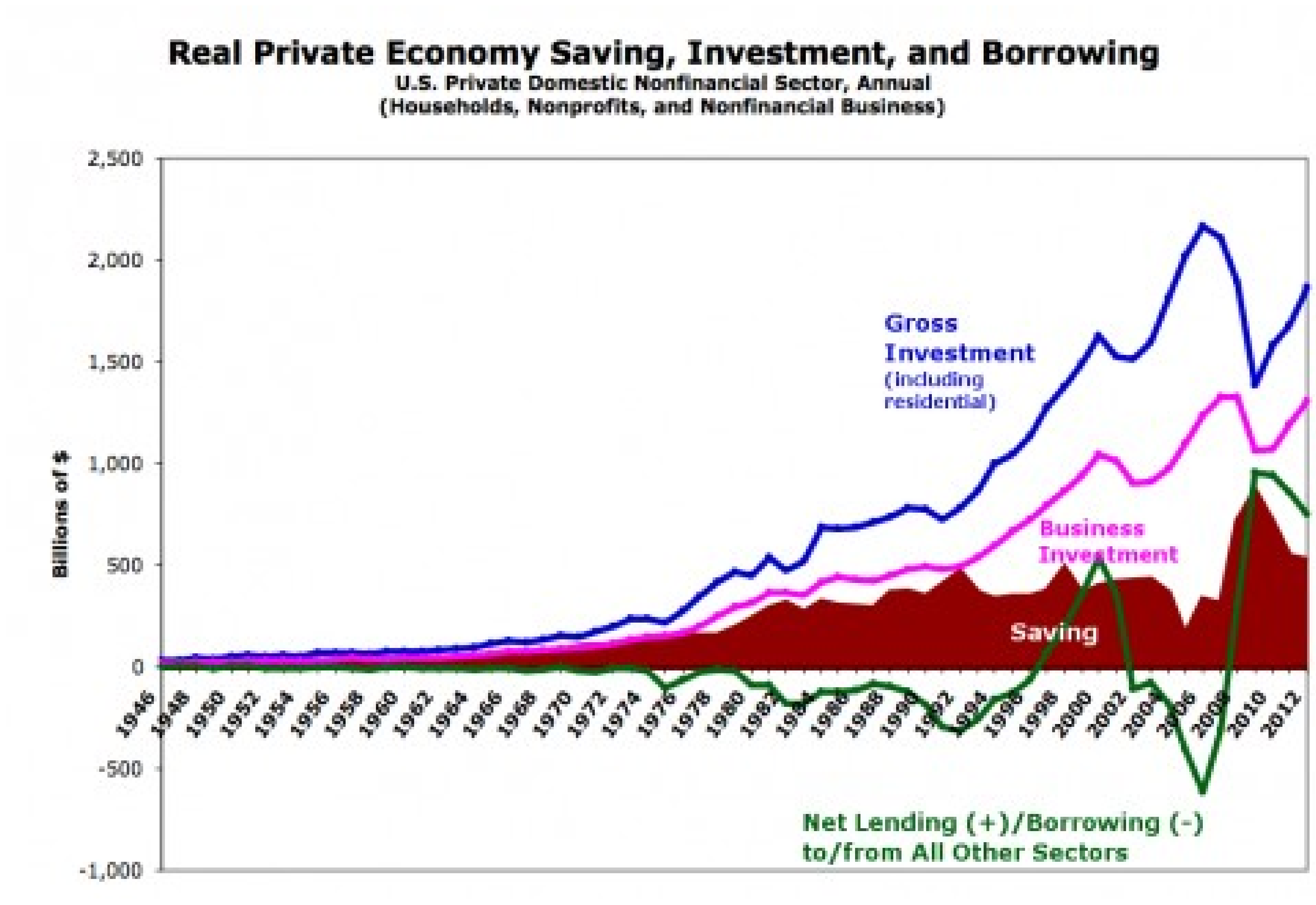


Figure 2.1 Total internal funds (including IVA) to gross investment ratio, USA, 1946–2005.

Source: Z.1 statistics of the Federal Reserve, www.federalreserve.gov/releases/z1, table F102, Non-farm non-financial corporate business. The curve plots the ratio of lines 9 and 10.

○ companies investing and saving at the same time

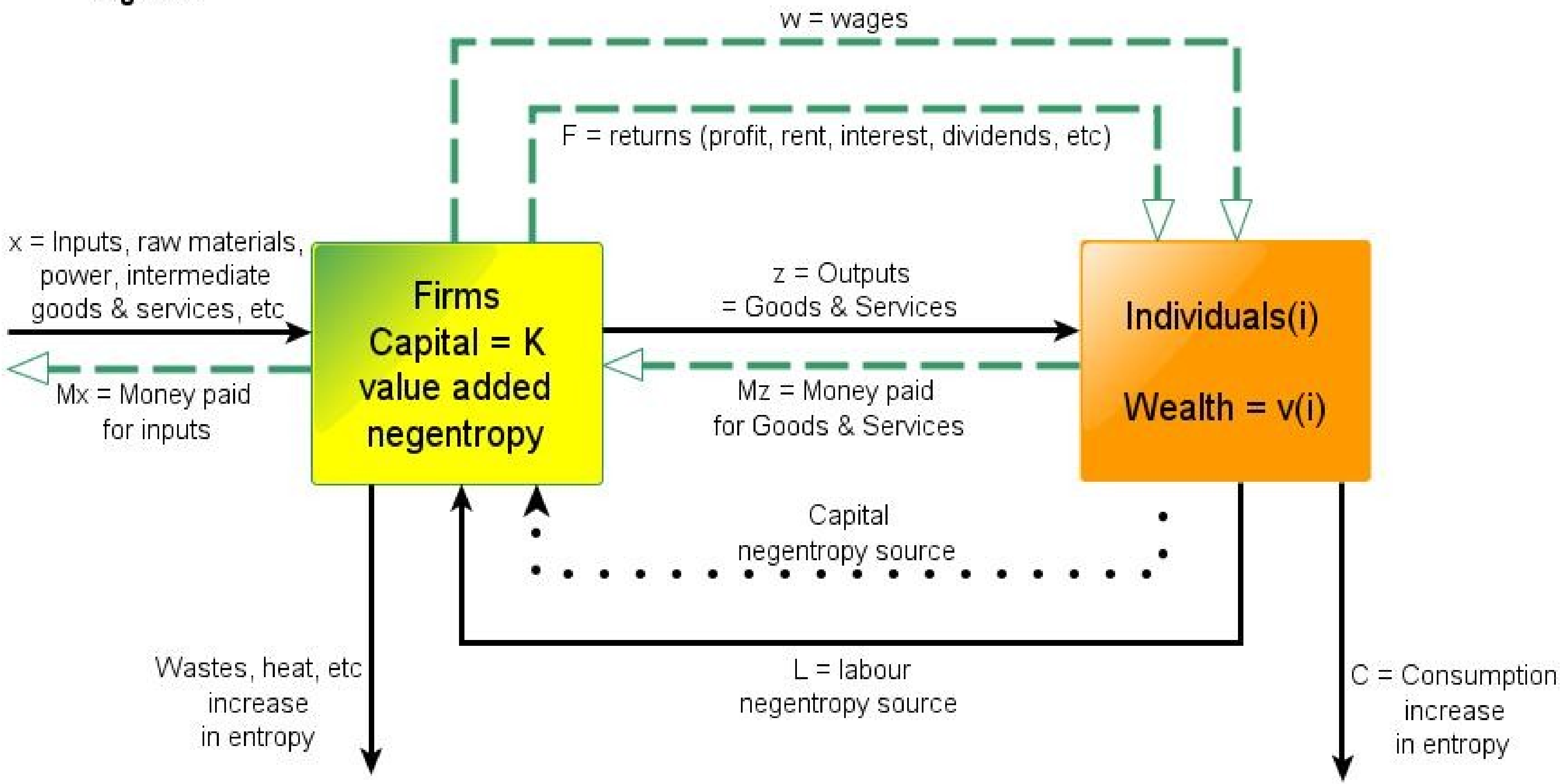

Real Private Economy Saving, Investment, and Borrowing
U.S. Private Domestic Nonfinancial Sector, Annual
(Households, Nonprofits, and Nonfinancial Business)
2,500
2,000
1,500
1,000
500
0
-500
-1,000
Billions of $
Gross Investment (including residential)
Business Investment
Saving
Net Lending (+)/Borrowing (-) to/from All Other Sectors
1946
1948
1950
1952
1954
1956
1958
1960
1962
1964
1966
1968
1970
1972
1974
1976
1978
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010
2012

- Models in this paper assume capital is invested using internal finance

- Investment and saving ignored as secondary loop
  - returned to at end of paper

- Gives base model:

Figure 3



w = wages
F = returns (profit, rent, interest, dividends, etc)
x = Inputs, raw materials, power, intermediate goods & services, etc
Firms
Capital = K
value added
negentropy
z = Outputs
= Goods & Services
Individuals(i)
Wealth = v(i)
Mx = Money paid for inputs
Mz = Money paid for Goods & Services
Capital
negentropy source
Wastes, heat, etc
increase
in entropy
L = labour
negentropy source
C = Consumption
increase
in entropy

- K – 'real' capital; machines, land, buildings, etc
- V – 'financial' capital; shares, bonds, loans, etc
- black lines flows of real goods
- green lines flows of money / financial instruments
- dotted line indicates ownership of K by V
- dotted line is not a flow
- Consumption shown as real flow
- Mz – economist's consumption
- Returns Y split into wages W and returns F

Simple Model:

- economy is isolated; no imports or exports.
- no government sector, no taxation, welfare payments, government spending, etc.
- no unemployment; all individuals are employed
- Labour and capital are complementary inputs and are not interchangeable.
- The role of money is ignored.
- no debt
- fixed level of capital
- capital and consumption goods interchangeable
  - Sraffian 'corn' model
- waste streams included so that the 2nd law of thermodynamics is not violated.

- Individual wealth – financial capital – v – is the float variable
- assume total wealth is constant:

$$\text{total}\,V \;=\; \text{total}\,K \qquad\qquad (1.3c) \qquad \text{or:}$$

$$\sum v_i \;=\; V \;=\; K \;=\; \text{constant} \qquad\qquad (1.3d)$$

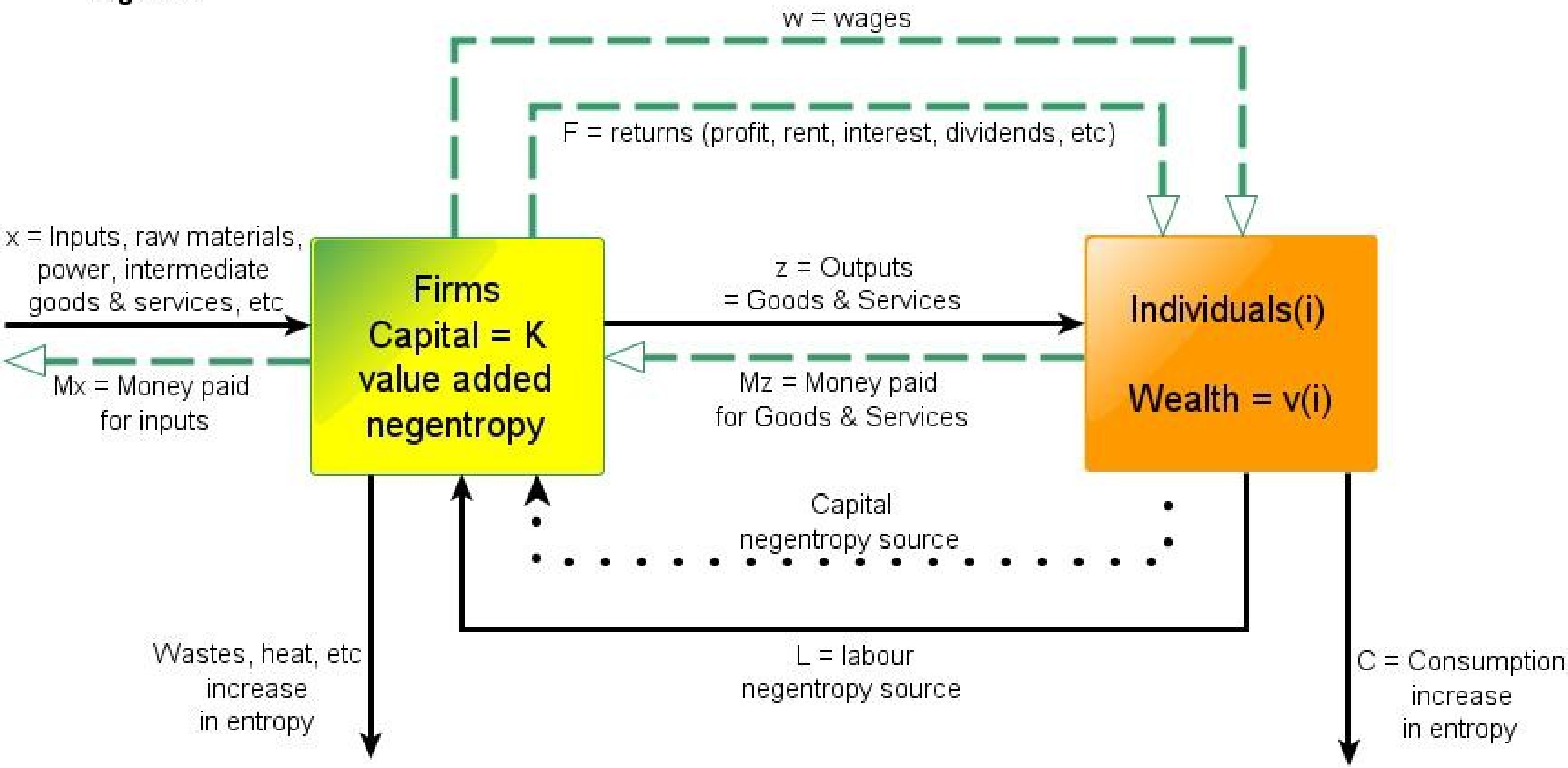
- at steady state equilibrium.

$$\text{total} \;\; C \;=\; \text{total} \;\; Y \;=\; \text{total} \;\; Mz$$

- but C and Y may be different for individuals

$$\sum v_{i,t} \;=\; \sum v_{i,t+1} \;=\; V \;=\; K \;=\; \text{constant} \qquad\qquad (1.3e)$$

Figure 3



$$v_{i,t+1} = v_{i,t} + z_{i,t} - Mz_{i,t} + w_{i,t} + F_{i,t} - C_{i,t} - labour_{i,t} - capital_{i,t} \qquad (1.3f)$$

- For single individual in the box on the right hand side:

$$v_{i,t+1} = v_{i,t} + z_{i,t} - Mz_{i,t} + w_{i,t} + F_{i,t} - C_{i,t} - labour_{i,t} - capital_{i,t} \qquad (1.3f)$$

- $z_{i,t}$, $C_{i,t}$, labour and capital are real units, others are financial.

- Looking only at the financial flows:

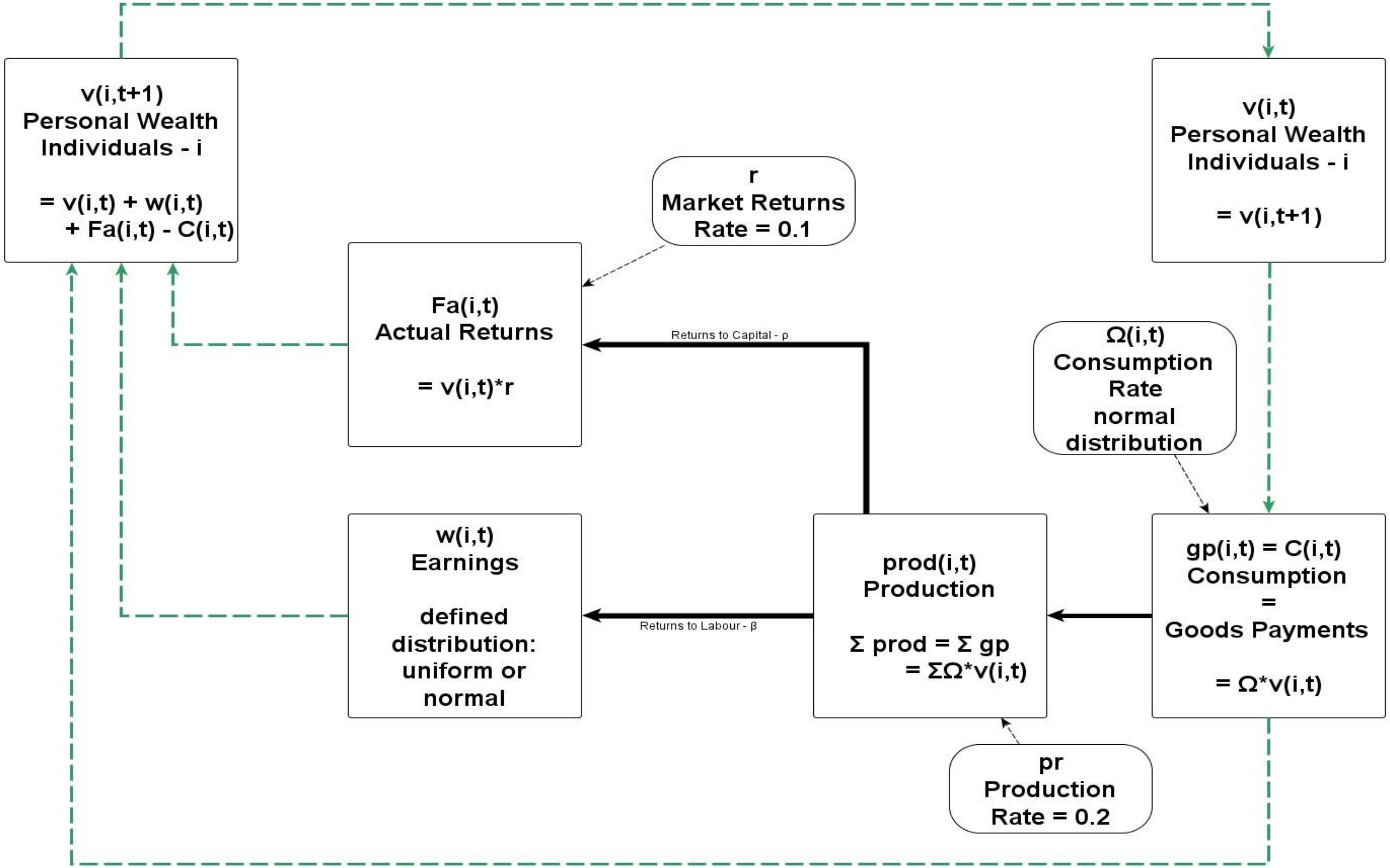
$$v_{i,t+1} = v_{i,t} - Mz_{i,t} + w_{i,t} + F_{i,t} \qquad (1.3g)$$

- Use $C_{i,t}$ in place of $-Mz_{i,t}$. $C_{i,t}$ is now a monetary unit; reverts to standard economics usage gives:

$$v_{i,t+1} = v_{i,t} + w_{i,t} + F_{i,t} - C_{i,t} \qquad (1.3h)$$

- In a single iteration, the paper wealth w of an individual i
  - increases by the wages earned w
  - increases by the profits received F.
  - reduces by the amount spent on consumption C.

Figure 1.3.9
Wealth & Income Model
Iteration Diagram



- w – is either uniform distribution or normal distribution, defined in model – exogenous

- profit is proportional to wealth, given by market profit rate r:

$$F_{i,t} = v_{i,t} r \qquad (1.3j) \qquad \text{for each of the i agents.}$$

- Consumption also proportional to wealth, given by consumption rate Ω :

$$C_{i,t} = v_{i,t} \Omega \qquad (1.3n)$$

Substitute into (1.3h) gives the difference equation for each agent:

$$v_{i,t+1} = v_{i,t} + w_{i,t} + v_{i,t}r - v_{i,t}\Omega \qquad (1.3o)$$

- Equation (1.3o) is base equation for a single agent in all income models.
- v is the only variable.
- wealth is Godleian float / buffer variable

- w, r and Ω are all constants; though can be stochastic around long-term constant value

- In income models:

$$Y = \sum w_i + \sum F_i = \text{constant}, \quad \text{always} \qquad (1.3p) \quad \text{and}$$

$$\sum w_i = \sum F_i = \frac{Y}{2} \qquad \text{usually} \qquad (1.3q)$$

- Accords with 'Bowley's Law' returns to labour typically between 0.75 and 0.5 of total returns.

Some definitions:

$$\text{Income rate} \qquad \Gamma = \frac{\sum Y}{\sum v} \qquad (1.3s)$$

$$\text{Profit rate} \qquad r = \frac{\sum F}{\sum v} \qquad (1.3r)$$

$$\text{Labour Share / Bowley ratio} \quad \beta = \frac{\sum w}{\sum Y} \qquad (1.3t)$$

$$\text{Profit Share / Profit ratio} \quad \rho = \frac{\sum F}{\sum Y} \qquad (1.3u)$$

- by definition:

$$\beta + \rho = 1 \qquad (1.3v)$$

$$\text{Profit ratio} \quad \rho = \frac{r}{\Gamma} \qquad (1.3w)$$

important subtlety:

- textbook economics; C = Y by definition
- in these models consumption becomes equal to income automatically by adjusting wealth
- final consumption term gives automatic feedback and stability

Formula for iterations:

$$v_{i,t+1} = v_{i,t} + w_{i,t} + v_{i,t} r - v_{i,t} \Omega \qquad (1.3o)$$

	One Household		Firms		
	Real Capital	Current Account	Current Account	Inventories	Installed Capital
Net Worth (v) time=t	0	$+v_t$	0	0	ΣK_t
Labour wages	$-NEP_W$ = work	+w (from distribution)	-w	$+K.pr_W$	
Capital Profits	$-NEP_F$	$+F^a$ = v.r	$-F^a$ = -v.r	$+K.pr_F$	
Consumption	+z	-C = -Mz = -V.Ω	+C = V.Ω	-z	
Change in real Capital				+Δin	
Net Worth time=t+1	0	$+v_{t+1}$ $=v_{t+1}+w+F^a-C$	0	ΣΔin = 0	ΣK_{t+1} $=\Sigma K_t$

NEP – negentropy. Ω is stochastic, or set from distribution

1.4 Wealth & Income Modelling - Results

1.4.1 Model 1A Identical Waged Income, Stochastic on Consumption

- Earnings – uniform distribution
- all agents have identical productive ability

- consumption stochastic from normal distribution
- consumption constant and identical over long run

- **All agents absolutely identical**
- perfect fit to GLV for wealth (as expected)

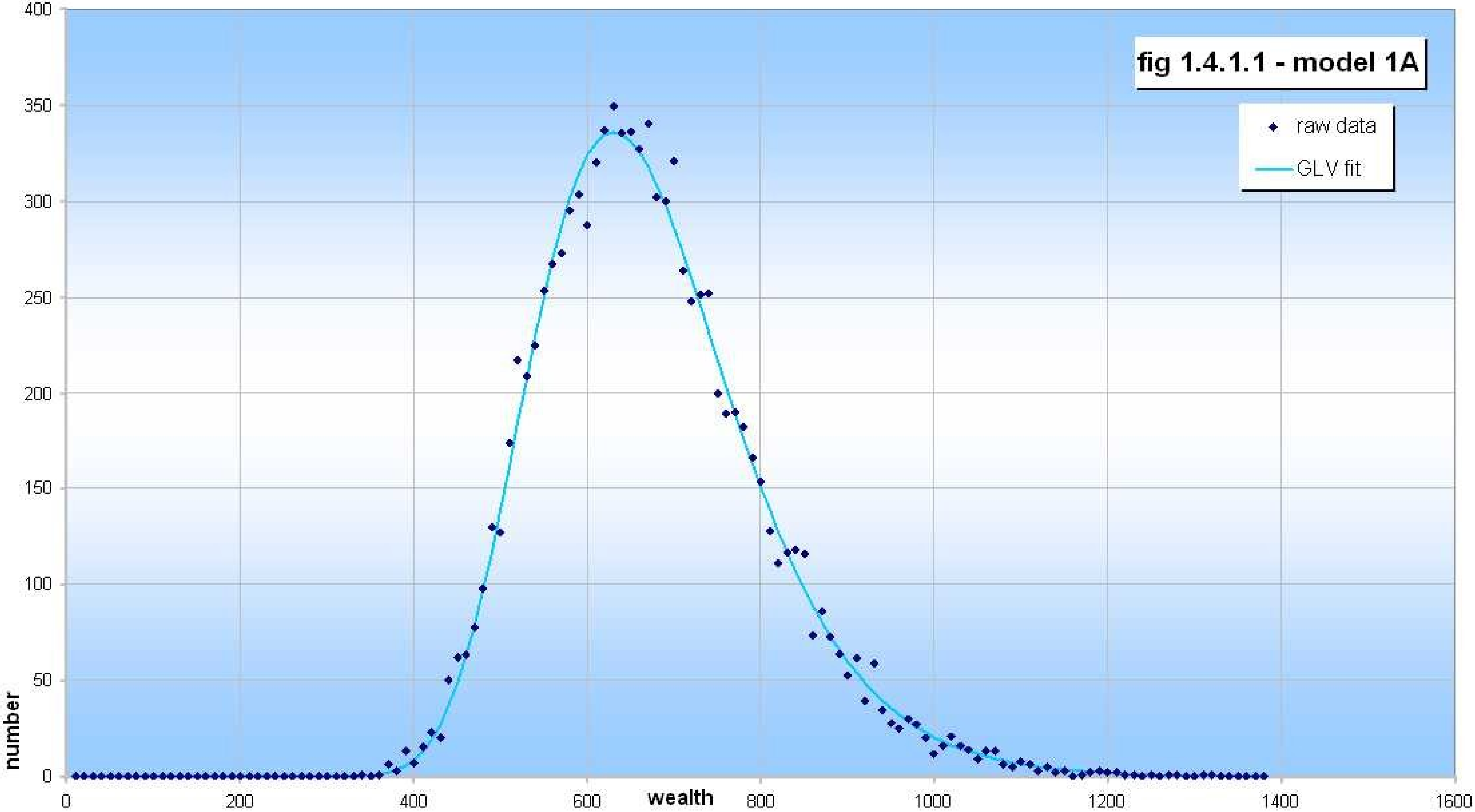
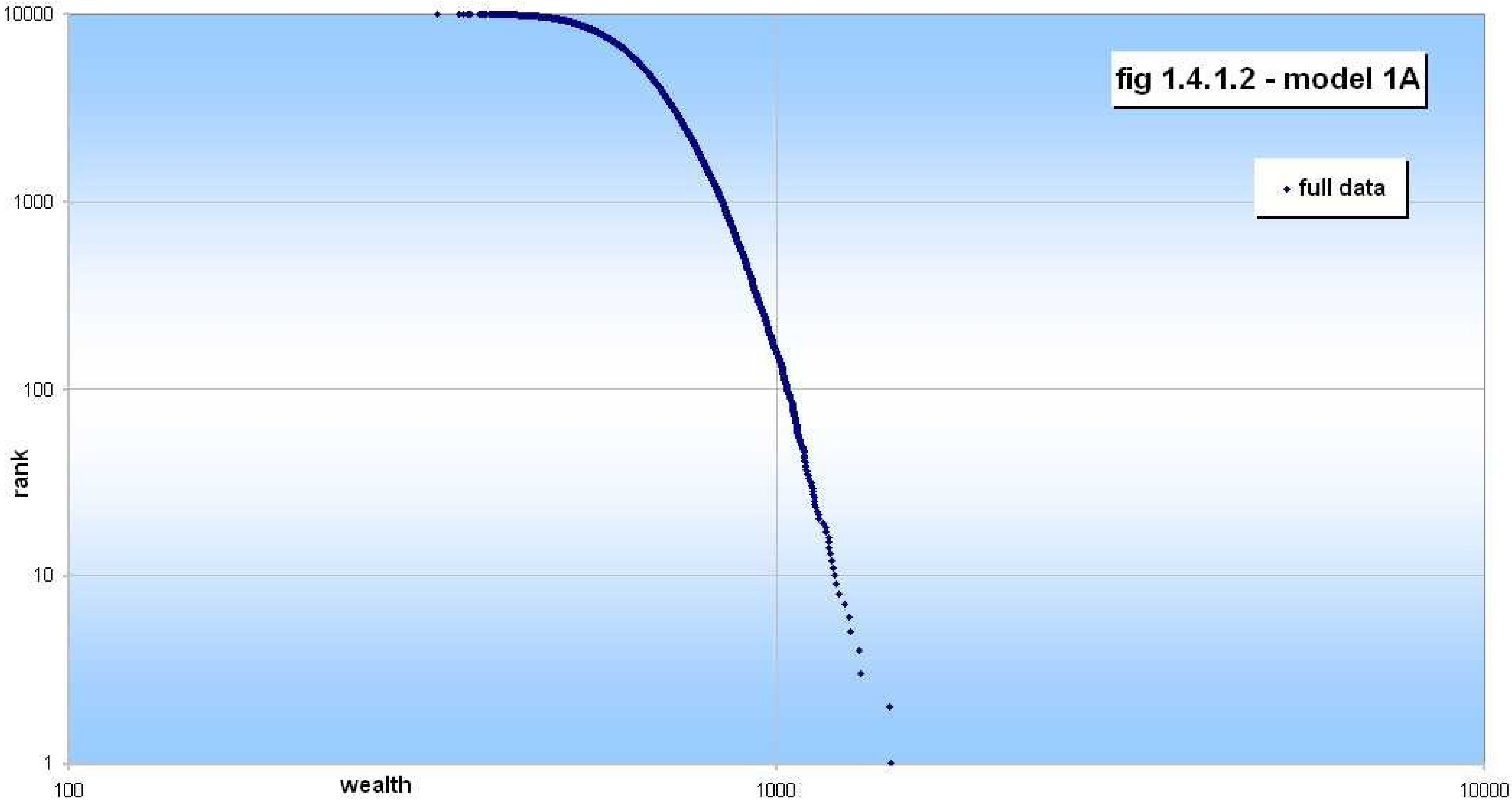


fig 1.4.1.1 - model 1A
raw data
GLV fit
number
wealth
400
350
300
250
200
150
100
50
0
0
200
400
600
800
1000
1200
1400
1600



• also gives power tail:

- **highly unequal wealth distribution produced from identical agents**

- Wealth distribution is a simple result of statistical mechanics; of entropy.

- The fundamental driver forming this distribution of wealth is not related to ability or utility in any way whatsoever.

- Distribution on returns instead of consumption produces similar results
- Income not analysed as agents move up and down in the distribution very rapidly

1.4.2 Model 1B Distribution on Waged Income, Identical Consumption, Non-stochastic

- Earnings – normal distribution at start of run (not stochastic)
- Consumption rate – uniform distribution
- dull model – output distribution is identical to input distribution
- **distribution of consumption / savings rates is key to wealth condensation effects**

1.4.4 Model 1D Distribution on Consumption and Waged Income, Non-stochastic

- Earnings – normal distribution at start of run (not stochastic)
- Consumption rate – normal distribution at start of run (not stochastic)
- Produces GLV for wealth distribution
- Produces apparent GLV distribution for income

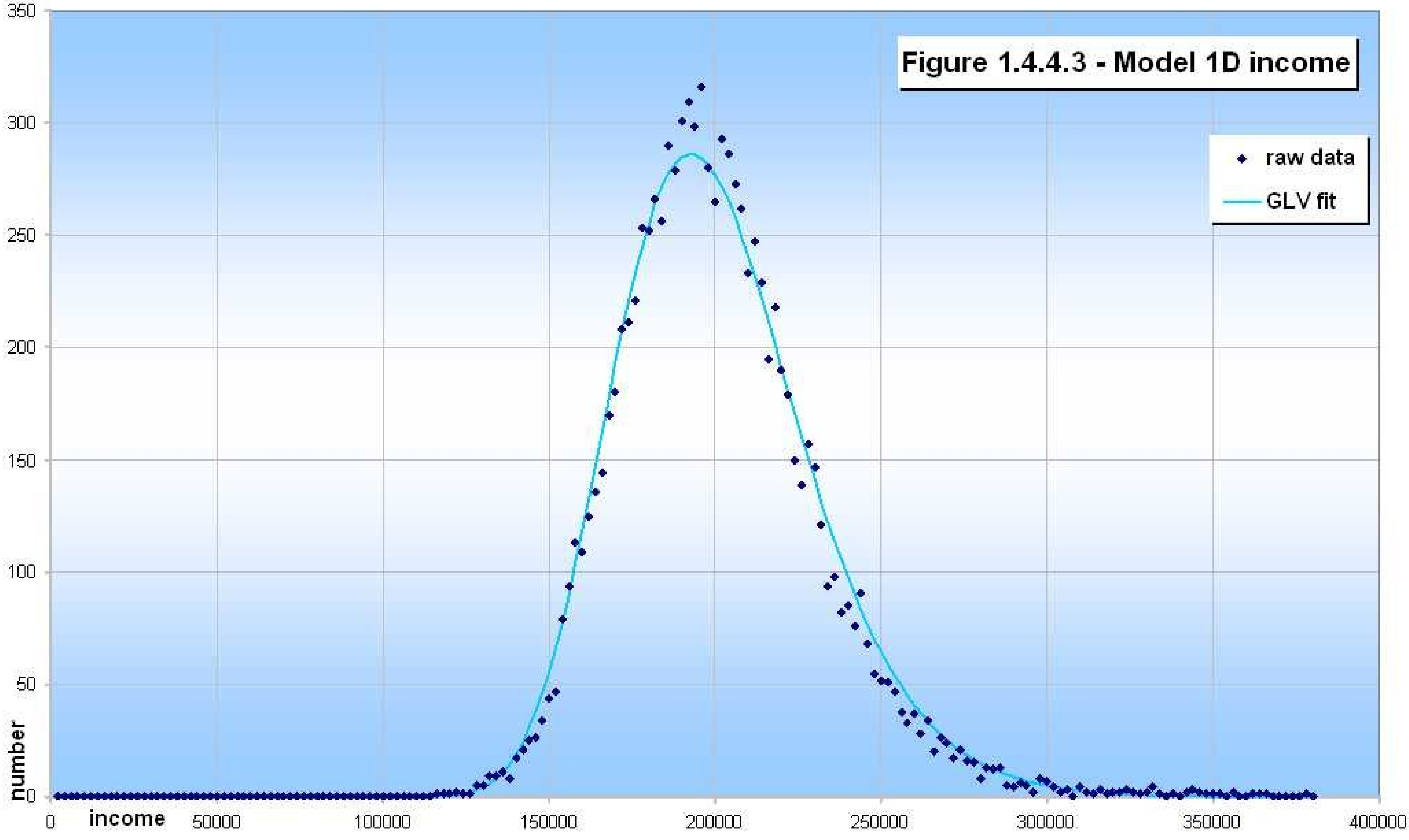


Figure 1.4.4.3 - Model 1D income
raw data
GLV fit
number
income
0
50
100
150
200
250
300
350
0
50000
100000
150000
200000
250000
300000
350000
400000

- Produces apparent GLV distribution for income
  - actually a combination of two underlying distributions:
    - GLV distribution of income from wealth – which is proportional to wealth (via r)
    - and normal distribution of earnings income – defined in model
  - result looks like GLV

1.5 Wealth & Income Modelling - Discussion

- Output distributions for wealth and income are much more unequal than input earnings / consumption distributions.

- Wealth condensation model – caused by statistical mechanics

- System involving capital: changes normal distributions into power tail distributions

- natural split between wealth owning class and working/middle class dependent on earnings

- rather than 'predator-prey' model better to think as grazing model – sheep graze grass, humans 'graze' wool from sheep.

- ownership of capital allows 'grazing'
- geometric 'pyramid of grazing' – Pareto distribution

- Rupert Murdoch grazes on many people due to ownership of many newspapers
- Apex grazer is (was) Bill Gates, can graze on Murdoch as Murdoch companies use Windows software

- The more capital you have got, the more grazing you get to do.

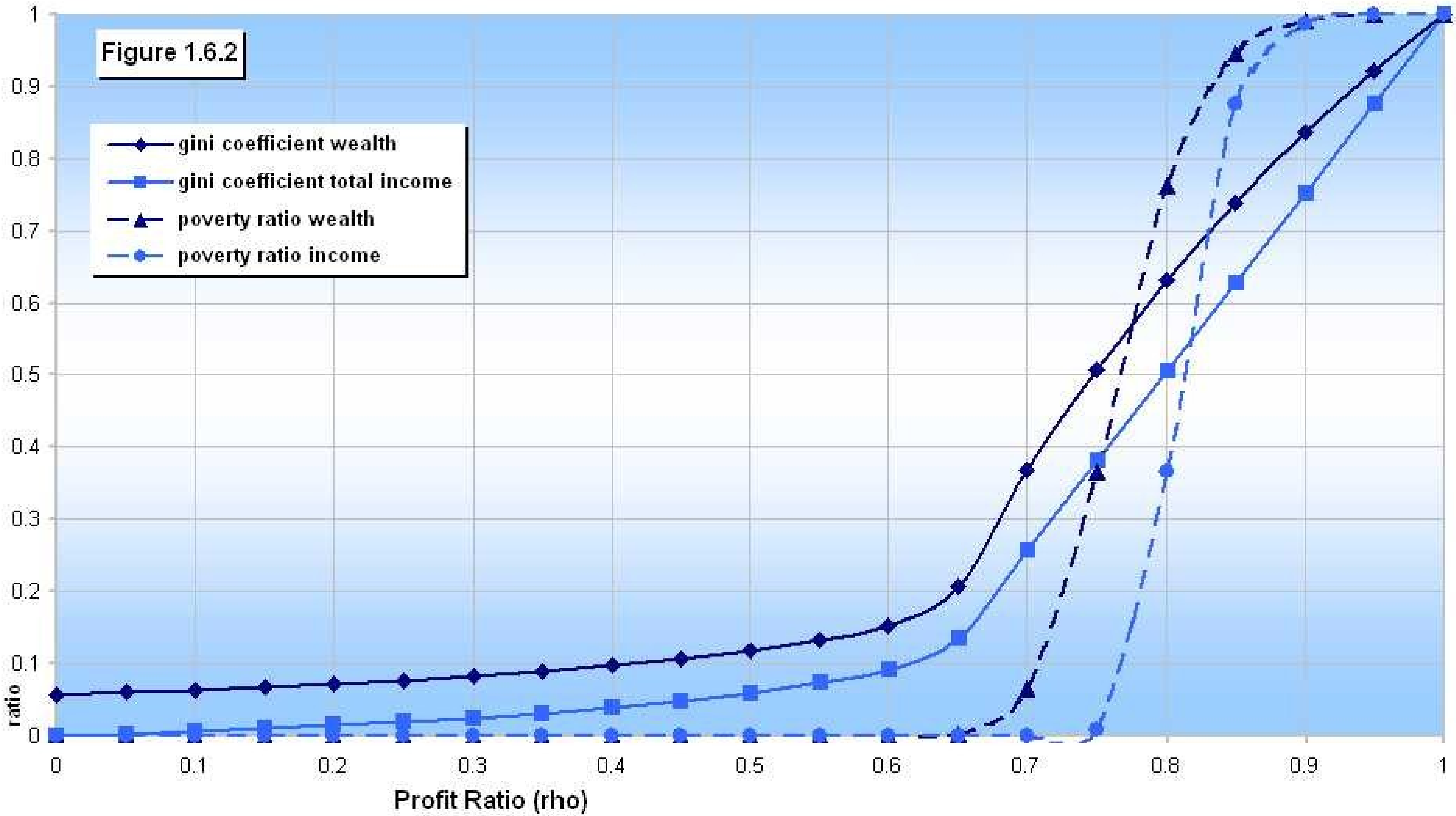
Don’t need any of the following:

- Different initial endowments
- Savings rates that change with wealth
- Different earning potentials
- Economic growth
- Expectations (rational or otherwise)
- Behaviouralism
- Marginality
- Utility functions
- Production functions

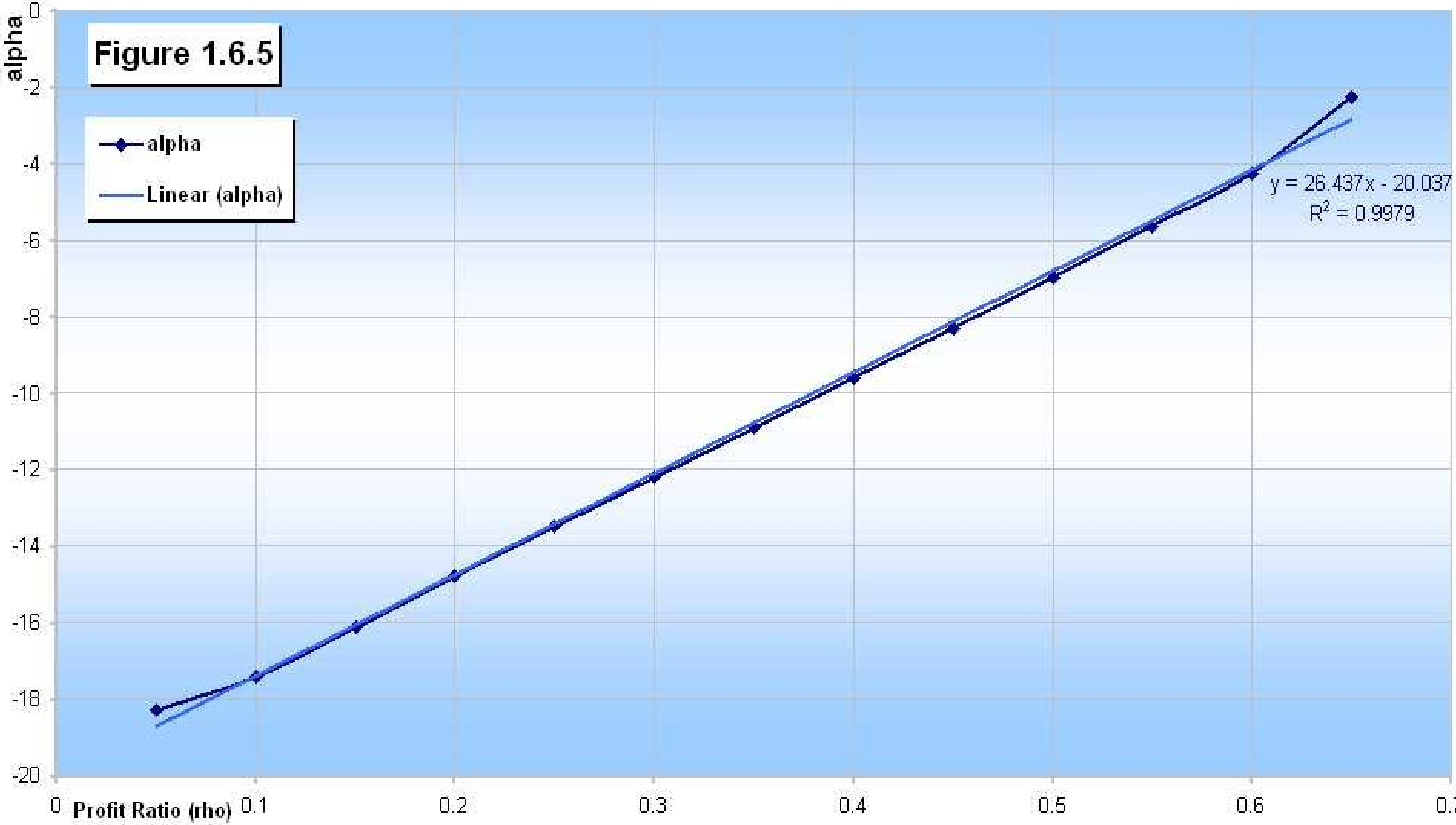
Very simple model

- Stock of capital
  - works with any capital
    - Industry
    - Land – cf mediaeval Europe, ancient Egypt, etc
- Income from capital
- Income wages / earnings
- Agents are identical
  - other econophysics models need differing saving propensity, etc to produce power tail
- Ian Wright – similar outcomes
  - 'pure' ABM – 'rule' based at agents
  - not explained mathematically

1.6 Enter Sir Bowley - Labour and Capital



- when $\rho = 0$, $\beta = 1$, Gini index is zero
- when $\rho = 1$, $\beta = 0$, all earnings are returned as capital
  - the individual with the highest saving propensity, becomes the owner of all the wealth
  - Gini index goes to 1.



- power tail exponent for wealth varies linearly with profit share/ Bowley ratio

- following formulae extracted empirically from data
  - (not proved analytically):

$$\alpha = \frac{1.36(1 - \rho)}{\mathrm{var}^{1.15}} \qquad (1.6\mathrm{d})$$

$$\alpha = \frac{1.36\beta}{\mathrm{var}^{1.15}} \qquad (1.6\,\mathrm{e})$$

v is the variance of the normal distribution of consumption rates

direct link from macro earnings share to inequality (magnified) supported by data – Daudey & Garcia-Peñalosa, 2005

- increase in profit ratio / decrease in Bowley ratio has two effects on income distribution

    - simple change in income shares – bad

    - change slope of power tail – very, very, bad indeed

- Second effect much more important than first

Supported by maths?

- Power law appears to be result of two growth rates cf. [Newman 2005], which gives a general formula for α as:

$$\alpha = 1 - a/b \qquad (1.6f)$$

- In wealth model:

$$\text{Profit ratio} \qquad \rho = \frac{\text{direct returns to capital}}{\text{total income from capital}}$$

$$\text{Profit ratio} \qquad \rho = \frac{r}{\Gamma} \qquad (1.3w)$$

ρ is the growth rate that capitalists get on capital, r, divided by Γ, the growth rate that everybody (capitalists and workers) gets on capital. (Consumption balances growths.)

1.7 Modifying Wealth and Income Distributions

1.7.2 Compulsory Saving

- If any agent’s current wealth was less than 90% of the average wealth, that agent was obliged to decrease their consumption rate by 20 percent.

- Moving ownership of a portion of capital into the hands of the poor

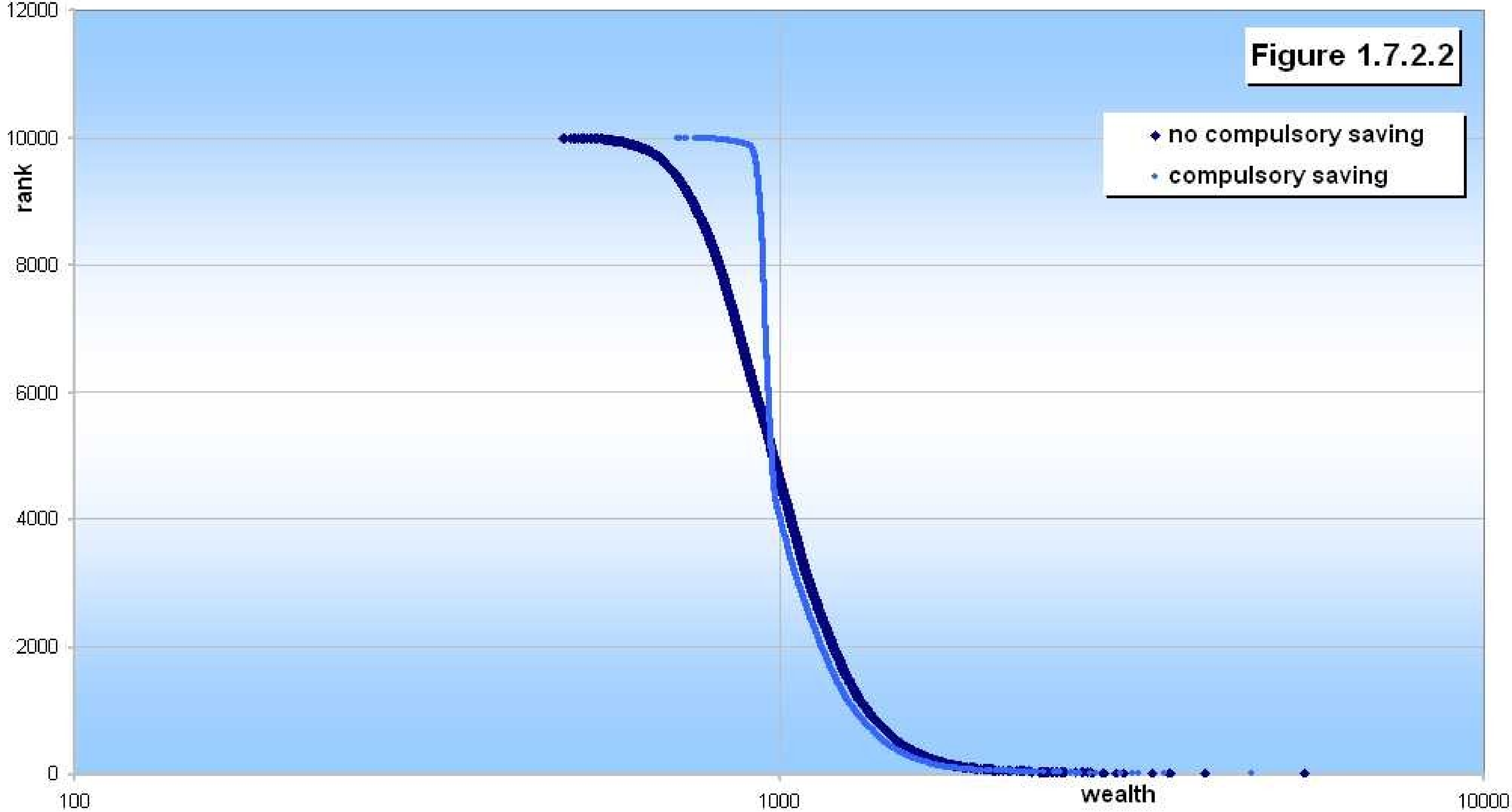


Figure 1.7.2.2
no compulsory saving
compulsory saving
rank
wealth
12000
10000
8000
6000
4000
2000
0
100
1000
10000

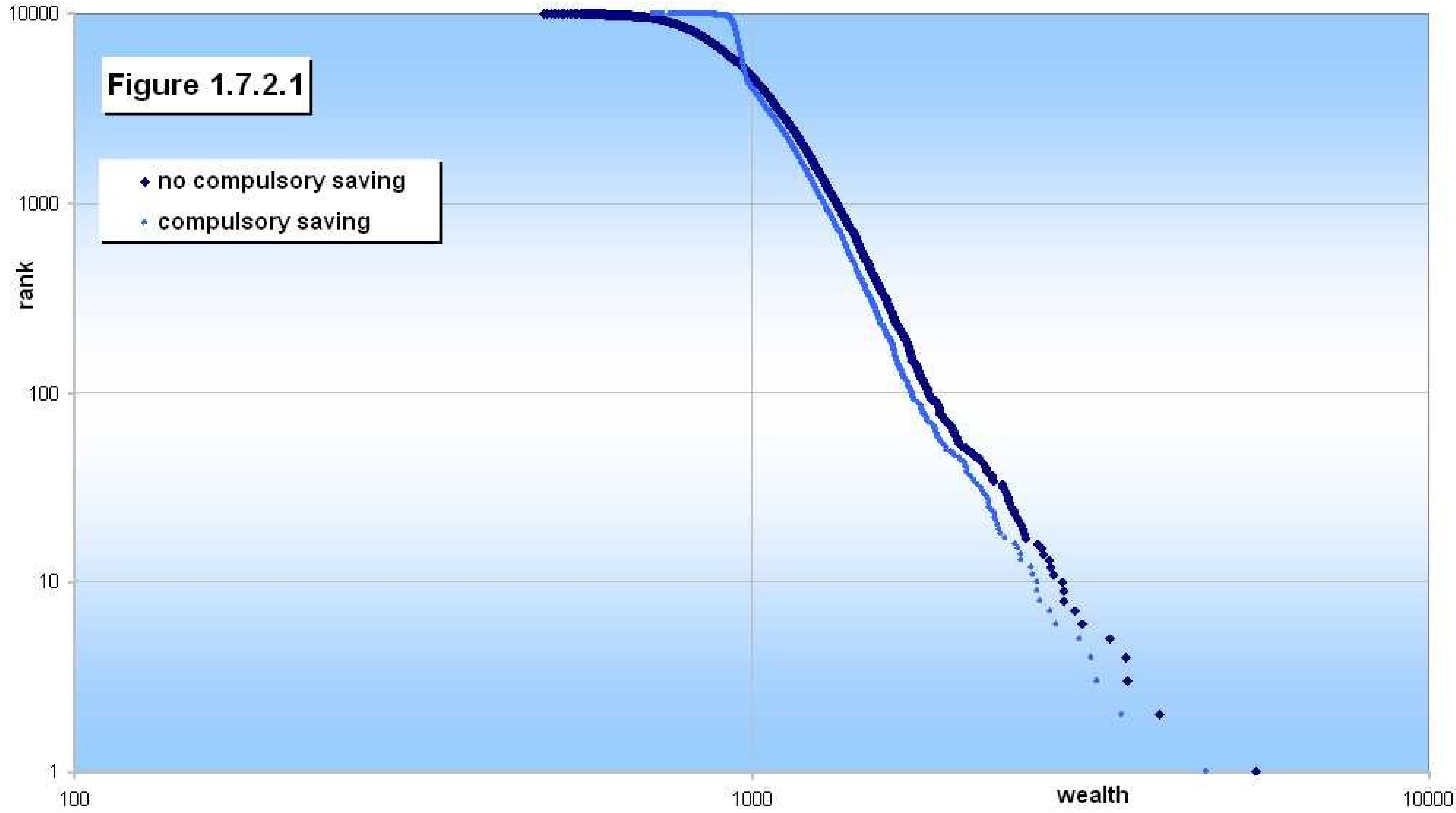


Figure 1.7.2.1
no compulsory saving
compulsory saving
rank
wealth
10000
1000
100
10
1
100
1000
10000

- Poverty largely eliminated
- still have power tail for the most talented
- rich are not taxed
- poor are compelled to save.

- In practise use system like Chilean / Singapore / Australian compulsory pensions
  - but can receive payments at all ages
- give extra assistance for low earners

2. Companies Models

- ABM model of companies

- **real capital**
  - producer of goods
  - gives source of revenue stream
  - Sraffian models

- **Financial capital**
  - ownership of real capital
  - ownership of revenue stream
  - Minskian pricing on revenue stream
  - Standard finance theory

- Allow V to differ from K
- Share prices can be different to company fundamental values
- shareholders are myopic – shares valued on previous dividends
  - as financial pricing:

$$\text{Present Value} = \frac{\text{Dividend}_1}{r}$$

r is the relevant market interest/profit rate; Dividend_1 is the latest dividend payment, and capital growth is ignored

[Brealey et al 2008, chapter 5].

Figure 2.2.1

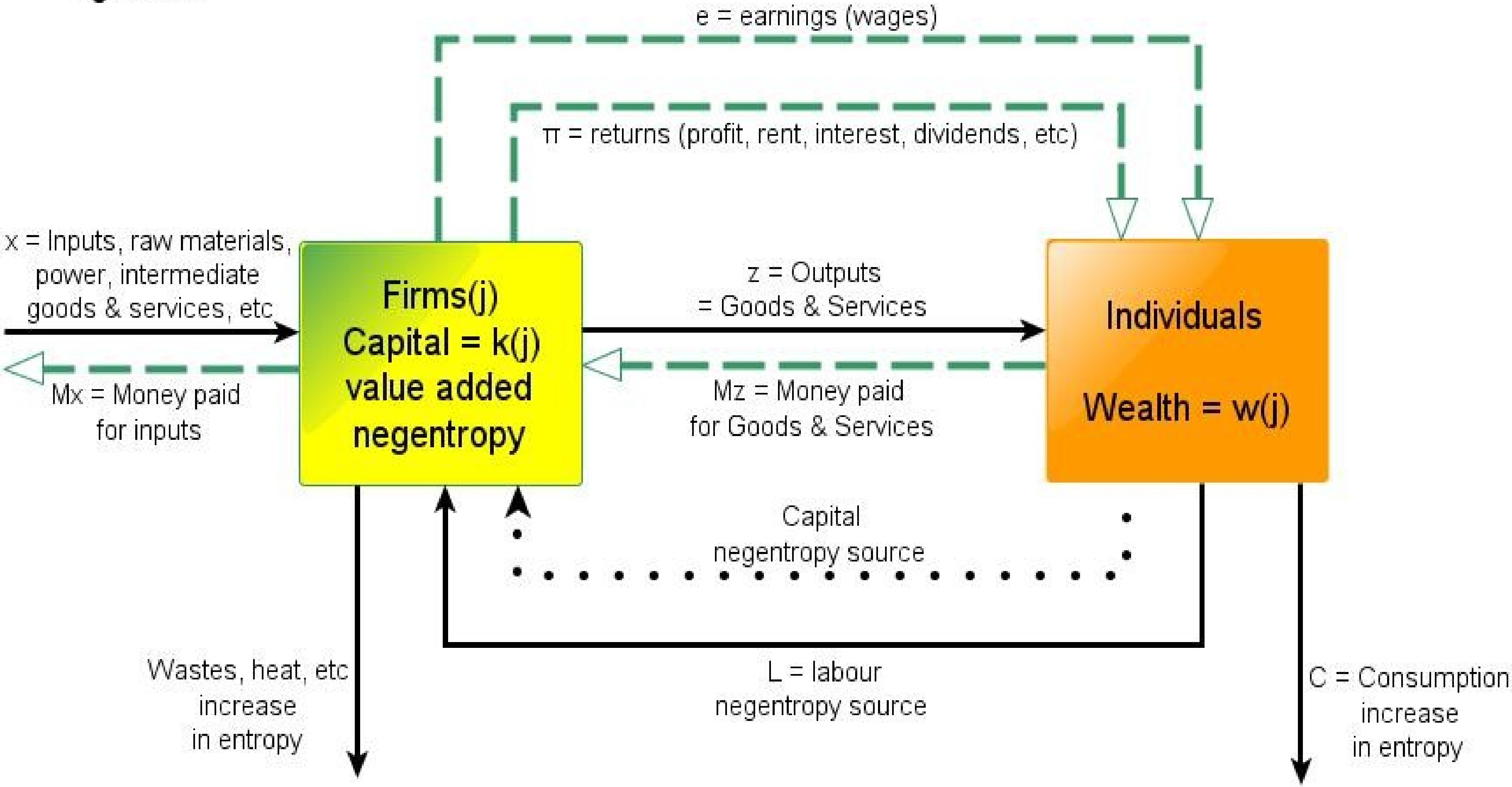
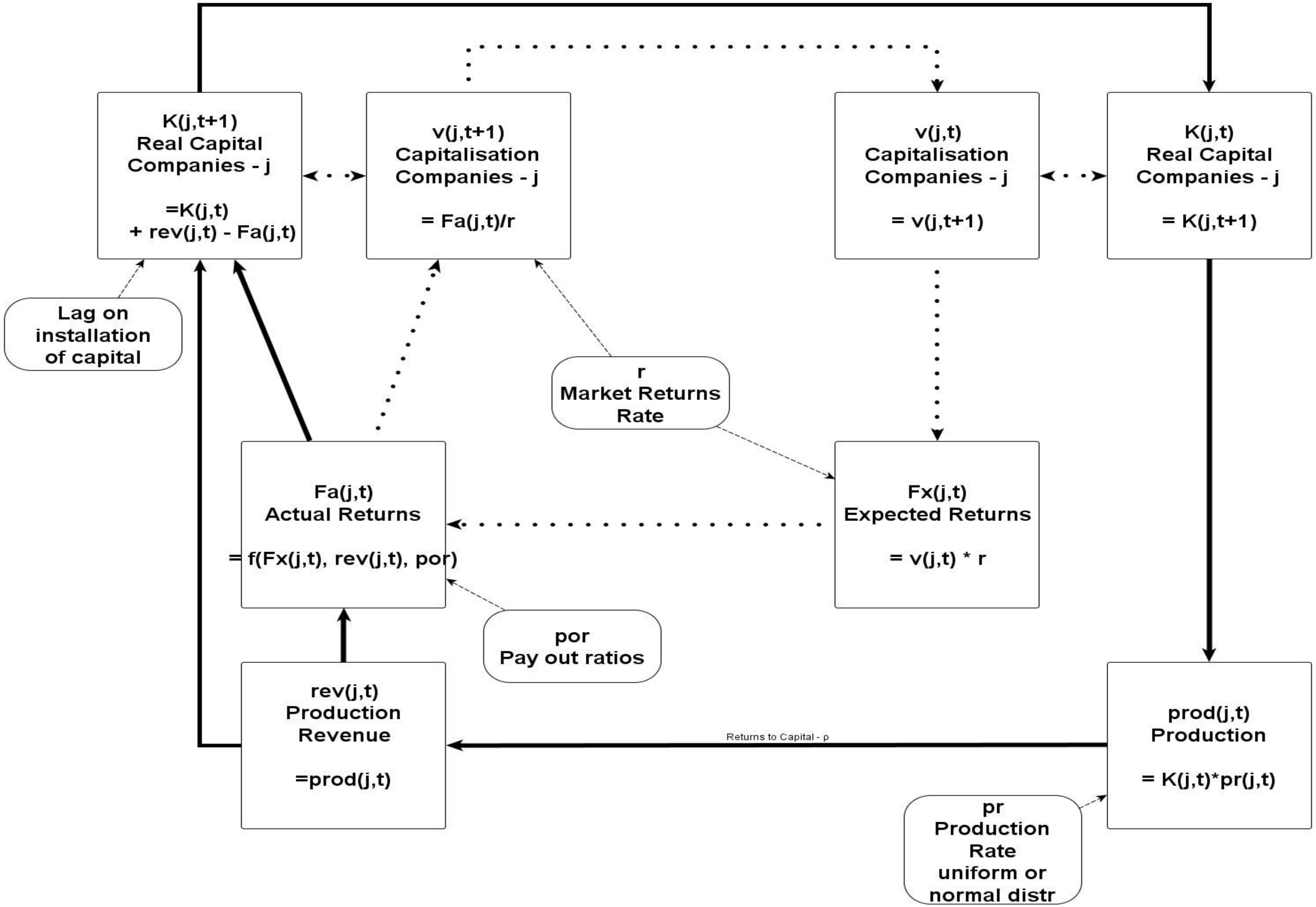

e = earnings (wages)
π = returns (profit, rent, interest, dividends, etc)
x = Inputs, raw materials, power, intermediate goods & services, etc
Firms(j)
Capital = k(j)
value added
negentropy
z = Outputs
= Goods & Services
Individuals
Wealth = w(j)
Mx = Money paid for inputs
Mz = Money paid for Goods & Services
Capital negentropy source
Wastes, heat, etc increase in entropy
L = labour negentropy source
C = Consumption increase in entropy

Figure 2.2.2
Companies Model
Iteration Diagram



Two cycles for capital:

- 'Sraffian', 'real' capital cycle – black heavy arrows
  - production of commodities by means of commodities
  - 'real' goods with intrinsic value
- 'Minskian', 'financial' capital cycle – in dotted arrows
  - valuation by revenue stream
- So:
  - V(j,t) is a function of K(j,t), and
  - K(j,t) is a function of V(j,t)
- Gives a (General) Lotka-Volterra system with two different types of stock
  - Real capital K
  - Capitalisation V

- Labour and earnings ignored
- Production rate; pr - defined distribution – uniform or normal
    - Leontief coefficient
- market expected returns on capital; r – constant
- 'Capital hoarding' via 'payout ratios' – actual returns reduced to keep capital in company

Formula for iterations:

$$K_{j,t+1} = K_{j,t} + K_{j,t}\,\mathrm{prodrate}_{j,t} - f\left([W_{j,t}r],[K_{j,t}],\mathrm{por}\right)$$

$$W_{j,t+1} = \frac{1}{r}\, f\left([W_{j,t}r],[K_{j,t}],\mathrm{por}\right)$$

NEP – negentropy, pr_{Π} – is stochastic; change in prod -> sell/buy real capital to/from other firms

* line cancels to zero – effectively ignore labour. Shares, Q don't 'add-up'

	Households			Firm - A				Firms - others			
	Real Capital	Current Account	Shares	Current Account	Invent-ories	Real Capital	Capital-isation	Current Account	Invent-ories	Real Capital	Capital-isation
Net Worth time=t	0	0	$+Q_t$	0	0	$K_{A,t}$	$-Q_t$	0	0	$K_{oth,t}$	na
Labour* earnings	$-NEP_W$ = work	+w		$-K_A.pr_W$	$+K_A.pr_W$						
Capital Profits	$-NEP_F$	$+F^{actual}$ $=f(Q,K,pr_n,por)$		$-F^{actual}$ $=f(Q,K,pr_n,por)$	$+K_A.pr_F$						
Consumption* (from wages)	$+z_W$	$-C_W$		$+C_W$	$-z_W$						
Consumption (from profits)	$+z_F$	$-C_F$		$+C_F$	$-z_F$						
Business to Business (B2B)				$+C_{B2B}$	$-z_{B2B}$			$-C_{B2B}$	$+z_{B2B}$		
Change in real Capital					$+\Delta in_A$	$-\Delta K_A$			$-\Delta in_{other}$	$+\Delta K_{other}$	
Net Worth time=t+1	0#	0	$+Q_{t+1}$ $+(F^a/r)$	0	0	$K_{A,t+1}$ $=K_{A,t} -\Delta K$	$-Q_{t+1}$ $-(F^a/r)$	0	0	$K_{oth,t+1}$ $=K_{oth,t} +\Delta K$	na

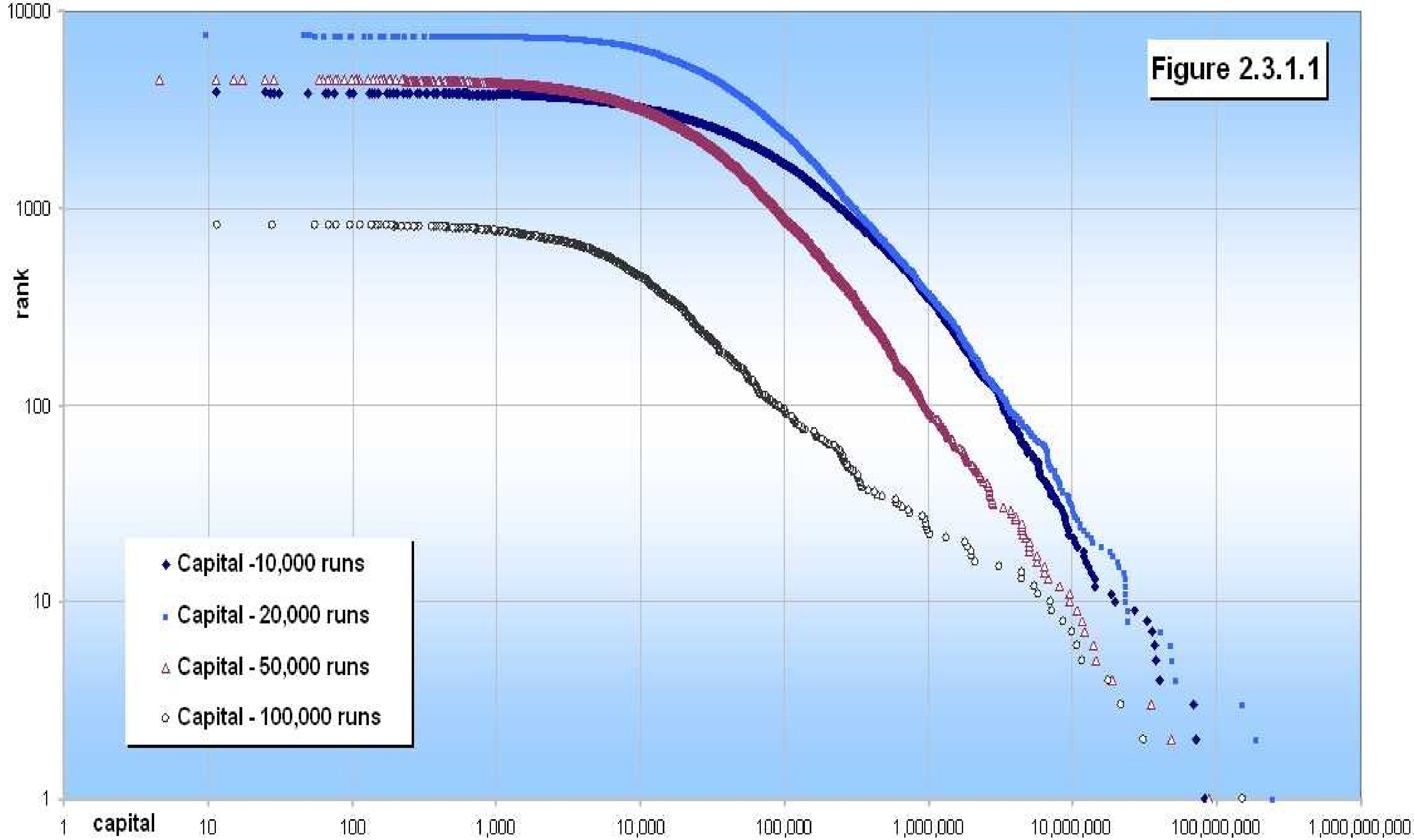


Figure 2.3.1.1
rank
capital
10000
1000
100
10
1
1
10
100
1,000
10,000
100,000
1,000,000
10,000,000
100,000,000
1,000,000,000
Capital -10,000 runs
Capital - 20,000 runs
Capital - 50,000 runs
Capital - 100,000 runs

- Production rate stochastic, same for each company over long run

- **Companies are identical**

- **produces power tail distribution from identical companies**

- power tail correct value; approx = 1

- but problems with model
  Ian Wright better

4. Minsky goes Austrian à la Goodwin – Macroeconomic Models

- Macroeconomic – not ABM (done in Excel)
- Consumption is a fixed proportion of consumers' paper wealth, as income models
- Companies have real capital which can produce a fixed proportion of output, and needs a proportional supply of labour, as all models above.
- price of paper wealth assets is defined by the preceding revenue stream; as in the companies model above.
- The price of labour is non-linear according to supply. That is real wage rates go up when there is a shortage of labour, and go down when there is a surplus of labour.

- Consumers can receive more income than they spend in consumption, alternatively can spend less.
  - So have a cash/credit-balance H for excess income
    - assumed held in non invested cash / credit account
    - allows imbalances generated to be accommodated

  - total wealth V is the sum of the capitalisation Q and cash-balance H, so:

$$C = V\Omega \qquad (4.2a) \qquad \text{or:}$$

$$C = (Q + H)\Omega \qquad (4.2b)$$

- H is Godleian buffer / float variable

- if H +ve, consumers:
  - have spare cash, or
  - or give credit to companies

- if H -ve, consumers:
  - have debts, or
  - receive credit from companies

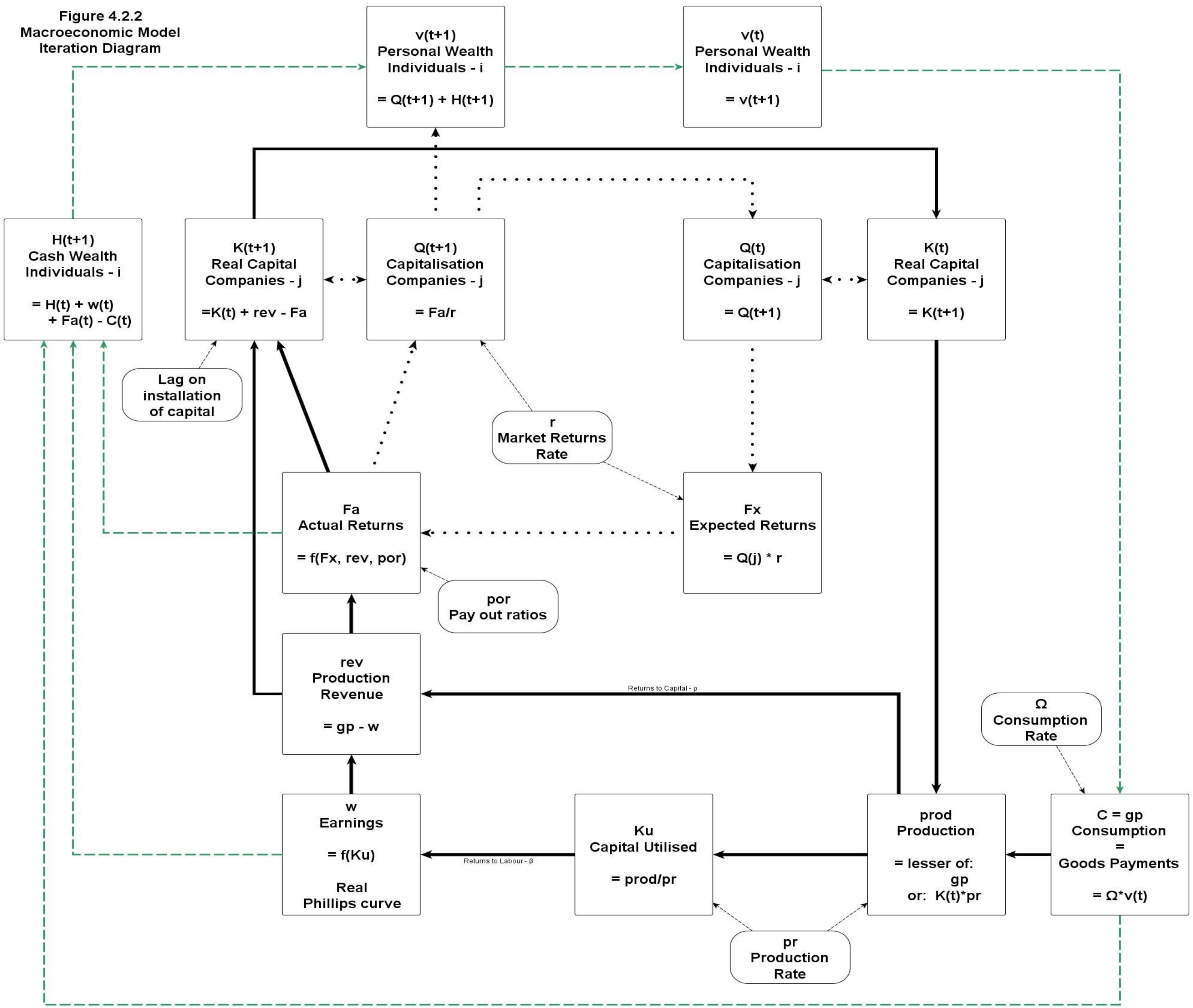
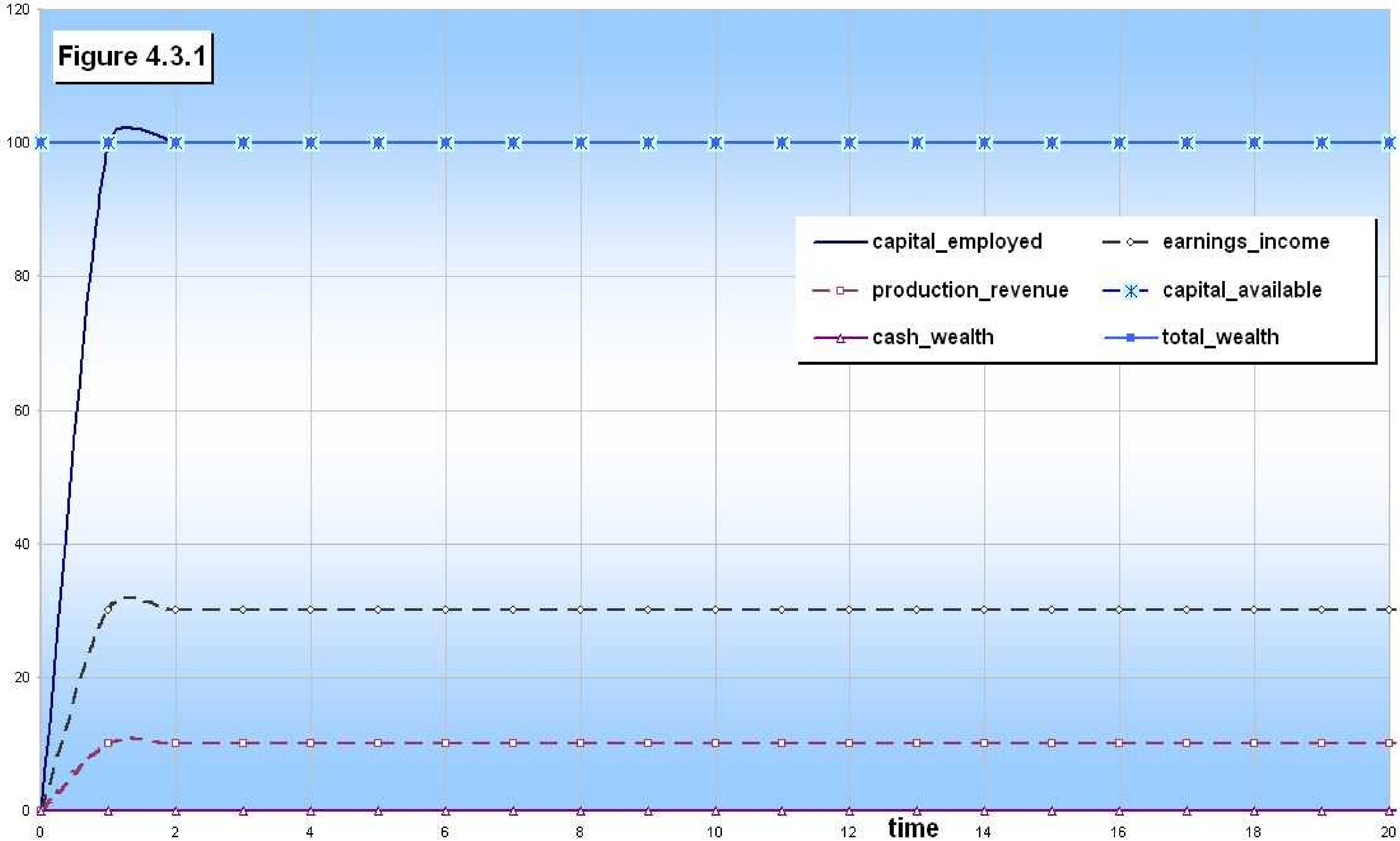


Figure 4.2.2
Macroeconomic Model
Iteration Diagram

	Households			Firms			
	Real Capital	Current Account	Shares	Current Account	Inventories	Real Capital	Capitalisation
Net Worth time=t	0	$+H_t$	$+Q_t$	0	0	K_t	$-Q_t$
Labour earnings	$-NEP_e$ = work	+w		-w $=f(K,K^2)$	$+K.pr_e$		
Capital Profits	$-NEP_F$	$+F^a$ $=f(Q,K,K^2,pr_F, por)$		$-F^a$ $=f(Q,K,K^2,pr_F, por)$	$+K.pr_\Pi$		
Consumption	+z	-C = -Mz		+C =v.Ω	-z		
Change in real Capital					$+\Delta IN$	$-\Delta K$	
Net Worth time=t+1	0	$+H_{t+1}$ $=H_{t+1}+w+F^a-C$	$+Q_{t+1}$ $+(F^a/r)$	0	0	K_{t+1} $=K_t-\Delta K$	$-Q_{t+1}$ $-(F^a/r)$

- Model can show stable or complex or explosive behaviour:



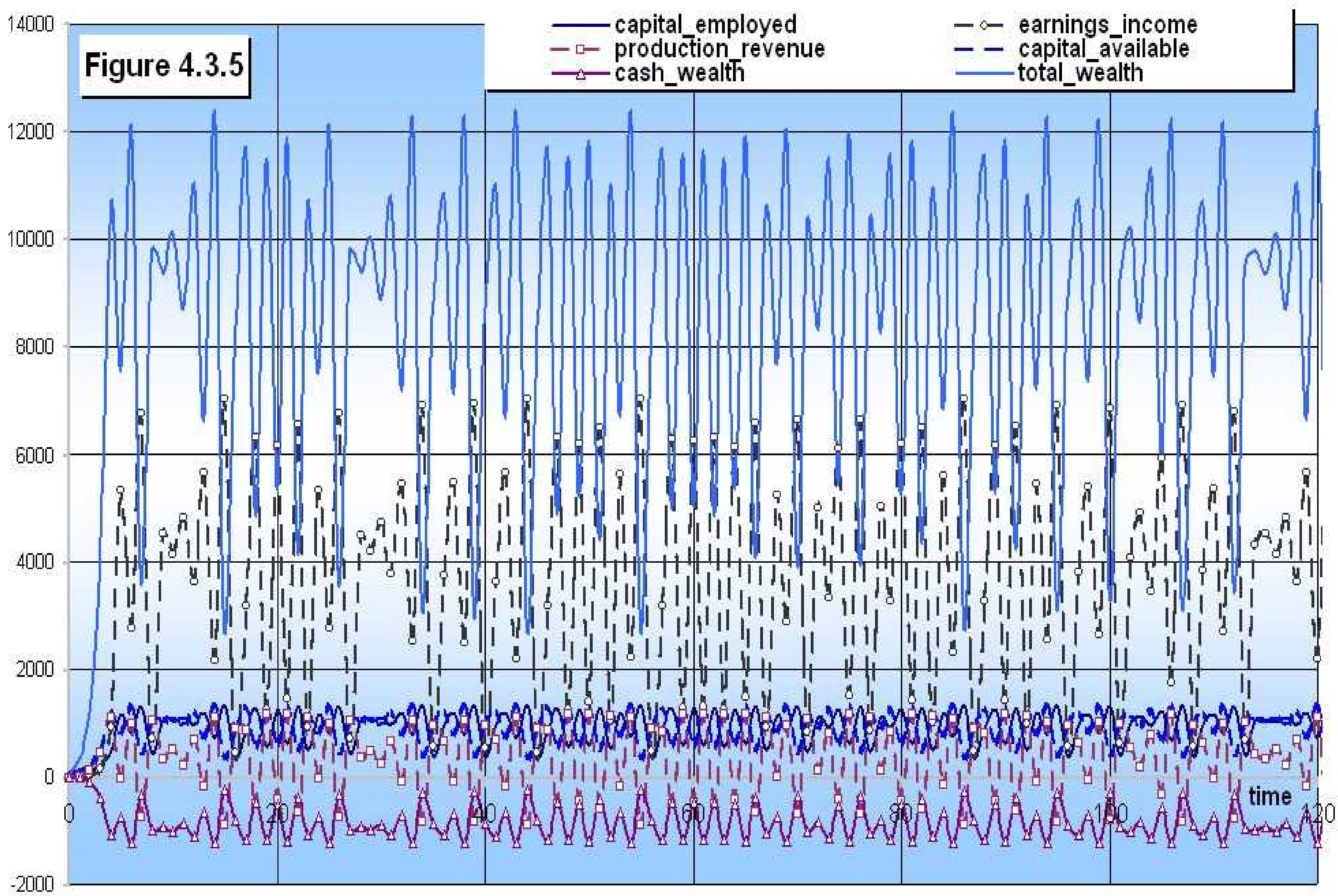


Figure 4.3.5
capital_employed
production_revenue
cash_wealth
earnings_income
capital_available
total_wealth
14000
12000
10000
8000
6000
4000
2000
0
-2000
0
20
40
60
80
100
120
time

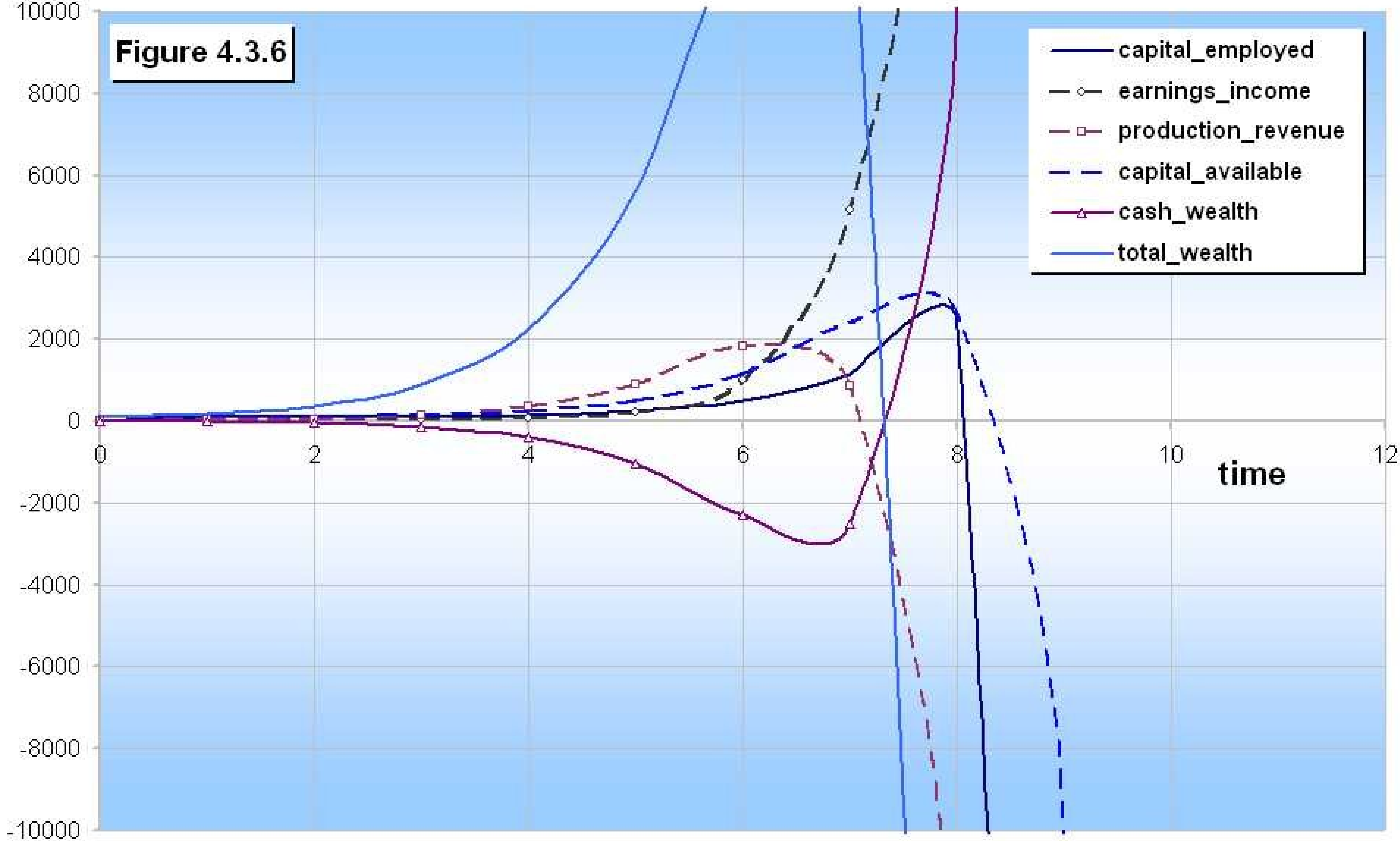


Figure 4.3.6
capital_employed
earnings_income
production_revenue
capital_available
cash_wealth
total_wealth
time
10000
8000
6000
4000
2000
0
-2000
-4000
-6000
-8000
-10000
0
2
4
6
8
10
12

- even with simple behaviour long term equilibrium can be very different according to initial conditions

  - Dynamic systems can have many different equilibiria

- **Basic instability due to pricing of paper assets on past / predicted future cash flow**
  - **So V can be different to K**

4.5 A Present for Philip Mirowski? – A Bowley-Polonius Macroeconomic Model

Bowley ratios from the models:

Figure 4.3.7	β
Model 4A	0.75 (exactly)
Model 4B	0.92
Model 4C	0.78
Model 4D	0.85

Not far off reality

- The source of the value of the Bowley ratio in the model was investigated empirically

- while holding the cash balance at zero; the following formula was 'discovered' from the model:

$$\begin{aligned} \beta &= \text{Bowley ratio} \\ &= 1 - \frac{r}{\Omega} \qquad (4.5k) \end{aligned}$$

- this can be derived trivially:

$Y = w + F \quad \Rightarrow$ by definition, so: $\quad w = Y - F$

$\frac{w}{Y} = 1 - \frac{F}{Y} \quad \Rightarrow$ so: $\quad \beta = 1 - \frac{F}{Y}$ but:

Consumption = Income $\quad \Rightarrow \quad C = Y$ so:

$\beta = 1 - \frac{F}{C}$ or:

$\beta = 1 - \frac{F/V}{C/V}$ but: $\quad F/V = r$ and $C/V = \Omega$

So: $\quad \beta = 1 - \frac{r}{\Omega}$

$$\beta = 1 - \frac{r}{\Omega}$$

- **Do not need any models to produce this equation!**
- The proportion of returns to labour is determined macroeconomically by consumption / savings rates.
- Not determined by production functions
- the consumption rate Ω defines Γ; the ratio of total income to capital.
- r is smaller than Ω - gives Bowley ratio between 0.5 and 1.0 – matches real values
- r and Ω are exogenous
  - Consumption rates known exogenous variables
    - life cycle theory – save for pensions

- Real returns known to be very stable over long term

- though reasons not clear

- If r and omega fixed (and also zero debt) then Bowley ratio fixed by eqn:

$$\beta = 1 - \frac{r}{\Omega}$$

"I mean the stability of the proportion of national dividend accruing to labour, irrespective apparently of the level of output as a whole and of the phase of the trade cycle. This is one of the most surprising, yet best-established, facts in the whole range of economic statistics………Indeed…the result remains a bit of a miracle." [Keynes 1939]

"…no hypothesis as regards the forces determining distributive shares could be intellectually satisfying unless it succeeds in accounting for the relative stability of these shares in the advanced capitalist economies over the last 100 years or so, despite the phenomenal changes in the techniques of production, in the accumulation of capital relative to labour and in real income per head." [Kaldor 1956]

challenge to economists – is the maths flawed?

$Y = w + F \Rightarrow$ by definition, so: $w = Y - F$

$\frac{w}{Y} = 1 - \frac{F}{Y} \Rightarrow$ so: $\beta = 1 - \frac{F}{Y}$ but:

Consumption = Income $\Rightarrow$ $C = Y$ so:

$\beta = 1 - \frac{F}{C}$ or:

$\beta = 1 - \frac{F/V}{C/V}$ but: $F/V = r$ and $C/V = \Omega$

So: $\beta = 1 - \frac{r}{\Omega}$

4.6 Unconstrained Bowley Macroeconomic Models

- If the cash-balance balance is allowed to change from zero, then Bowley ratio given by:

$$\rho = \frac{rQ}{\Omega(Q + H)}$$

$$\beta = \frac{\Omega + \Omega(H/Q) - r}{\Omega + \Omega(H/Q)}$$

$$= \frac{1 + (H/Q) - (r/\Omega)}{1 + (H/Q)} \qquad (4.6a)$$

- also trivial to derive from basic algebra

- If the cash balance is positive and increasing; Bowley ratio heads closer to unity, good for workers, bad for capitalists.

- if H is negative (a debt) and the size of the debt is increased, then the size of both the numerator and denominator reduce, however the value of the numerator reduces more rapidly than the size of the denominator, and the Bowley ratio slowly decreases.

- (At least at first.)

- If debt is allowed to continue increasing, then the Bowley ratio drops rapidly to zero, and then shortly afterwards heads off to negative infinity.

- In the model it isn't possible to reach these points; as the Bowley ratio heads to zero the model becomes unstable, and explosive

  - the economy blows up in an bubble of excess real capital and even more excess debt.

    - *This may sound familiar.*

Note also:

- value of H has a direct effect on Bowley ratio β in eq 4.6(a)

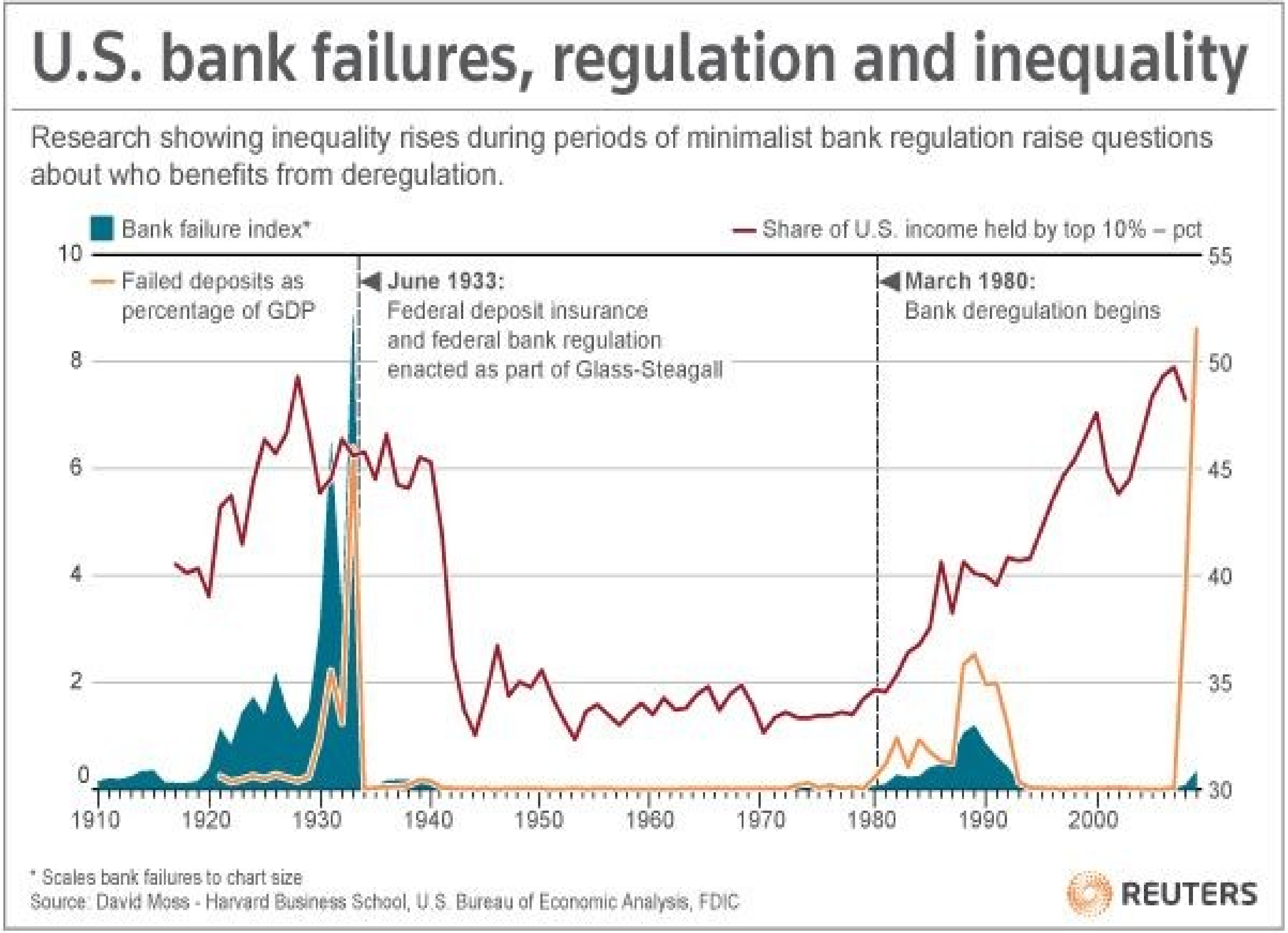
$$\beta = \frac{\Omega + \Omega(H/Q) - r}{\Omega + \Omega(H/Q)}$$
$$= \frac{1 + (H/Q) - (r/\Omega)}{1 + (H/Q)} \qquad (4.6a)$$

- β has a direct effect on alpha, the exponent of the power tail in the wealth distribution in eq 1.6(e).

$$\alpha = \frac{1.36(1 - \rho)}{var^{1.15}} \qquad (1.6d)$$

$$\alpha = \frac{1.36\beta}{var^{1.15}} \qquad (1.6e)$$

- So, levels of debt have a direct effect on wealth inequality



Reuters graphic/Stephen Culp 02/12/11

- More realistically, do something with the cash H, change to savings S
- Companies take in savings S and supply bonds to public
  - pay r on bonds
  - pay R on share capital
  - R > r

Then:

$$\rho = \frac{RQ + rS}{\Omega\left(Q + S\right)}$$

- Same conclusions arise regarding debt as above.
- Investment and Saving is a secondary loop

Conclusions – Modelling

- Simple models explain
  - wealth / income distributions
  - company size distributions
  - macroeconomic cycles
  - ratio of returns to capital and labour

- Biologists have the right models

Further reading:

- econodynamics.org