

Explaining Kaldor's facts: Effective Economic Models Using Real and Financial Capital in a Lotka-Volterra Framework (Why Money Trickles Up)

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Introduction

- introduces mathematical and simulation models that use basic economic variables
- give straightforward explanations of the distributions of wealth, income, earnings and company sizes
- explains the source of macroeconomic business cycles, including bubble and crash behaviour.
- models give simple formulae for the Bowley ratio; the ratio of returns to labour and capital.
- provide simple effective methods for eliminating poverty without using tax and welfare.

- Ian Wright, Makoto Nirei & Wataru Souma have produced work on similar lines
- the general approach for the macroeconomic models were partly inspired by the work of Steve Keen
- indebted to the work of Levy & Solomon and their GLV models.

Assumptions (1)

- Economics is a statistical mechanical system
- Economics and finance are many bodied systems
- Decisions of agents not coordinated
- Outcomes defined by statistical probability
- This overrides micro interactions
- Can often be modelled by identical (homogeneous) models
- cf Central Limit Theorem
 - Underlying distributions not relevant; can be uniform, normal, triangular, etc
 - Output distributions always:
 - normal if add underlying distributions
 - log-normal if add underlying distributions

Assumptions (2) – Most goods are non-scarce

- Non-scarce:
 - Steel/concrete/plastics/building and industrial materials
 - Priced by cost-plus, Sraffa/post-Keynesian (not marginality)
- Non-scarce long term – sometimes scarce short-term
 - Copper and other specialist metals / minerals
 - Agricultural produce
 - Energy – especially oil
 - Priced long-term cost-plus, short-term market-micro

- Scarce - real
 - Gold, silver
 - Land, urban location
 - Labour
 - Priced market-micro / marginality
- Scarce – artificial
 - Works of art
 - Beanie babies
 - Shares in companies
 - Currency
 - Priced market-micro
 - Not marginality as prices don't formally clear

Assumptions (3) – Classical Economics

“Negentropy theory of value”

- Economic goods have real intrinsic 'value':
 - “Humanly useful free energy”, or just:
 - “Negentropy” - following Schrödinger in biology
- Sources of negentropy – energetic:
 - sun for agriculture
 - food
 - fossil fuels
 - draft animals
 - machines (cf. 'light out factories' in auto industry)
 - human labour

- Sources of negentropy – concentration:
 - oil field
 - lodes of minerals in mines
 - **physical market**
 - supermarket
 - cities
 - **financial market**

- Sources of negentropy – complexity/uniqueness/information:
 - jewellery
 - works of art
 - books
 - computers
 - human ingenuity
 - machines

- Sources of negentropy – Negentropy generators, aka:
“Dissipative Structures”
 - human beings
 - machines
 - computers
 - robots
 - power stations
 - factories
 - cities
 - markets
 - economies

Marx wrong – labour not only source of value
(however was 2/3rds right)

Assumptions (4) – Flow models

- Wealth flows from high negentropy sources in production
 - Fossil fuels, labour, etc
- to low negentropy sinks via consumption
 - Rubbish, waste, refuse, broken toys, crashed cars
 - Waste heat, in factories and houses
- Models are not in thermodynamic equilibrium
 - However can reach a stationary well defined state
 - 'Maximum Entropy Production' 'equilibrium'

- Production – creation of wealth / negentropy
- Consumption – destruction of wealth / negentropy
- Exchange – conservation of wealth / negentropy

- Normal economic exchange
 - Barter is not normal
 - 'half-exchange' is normal
 - one party provides real goods / services
 - other party provides 'monetary token'
 - token is agreed standard store of wealth
 - money, debt instruments, collateral, etc
 - no net gain of wealth (under normal circumstances)
 - 1st party now has tokens
 - goes to find another 'half-exchange'
 - starts process again
- flow system can be modelled as circular flow

- Models don't formally close
- Feedback loops
- Need 'balance' or 'float' variables to accommodate unexpected changes
 - Models similar to those of Godley, Lavoie, Keen

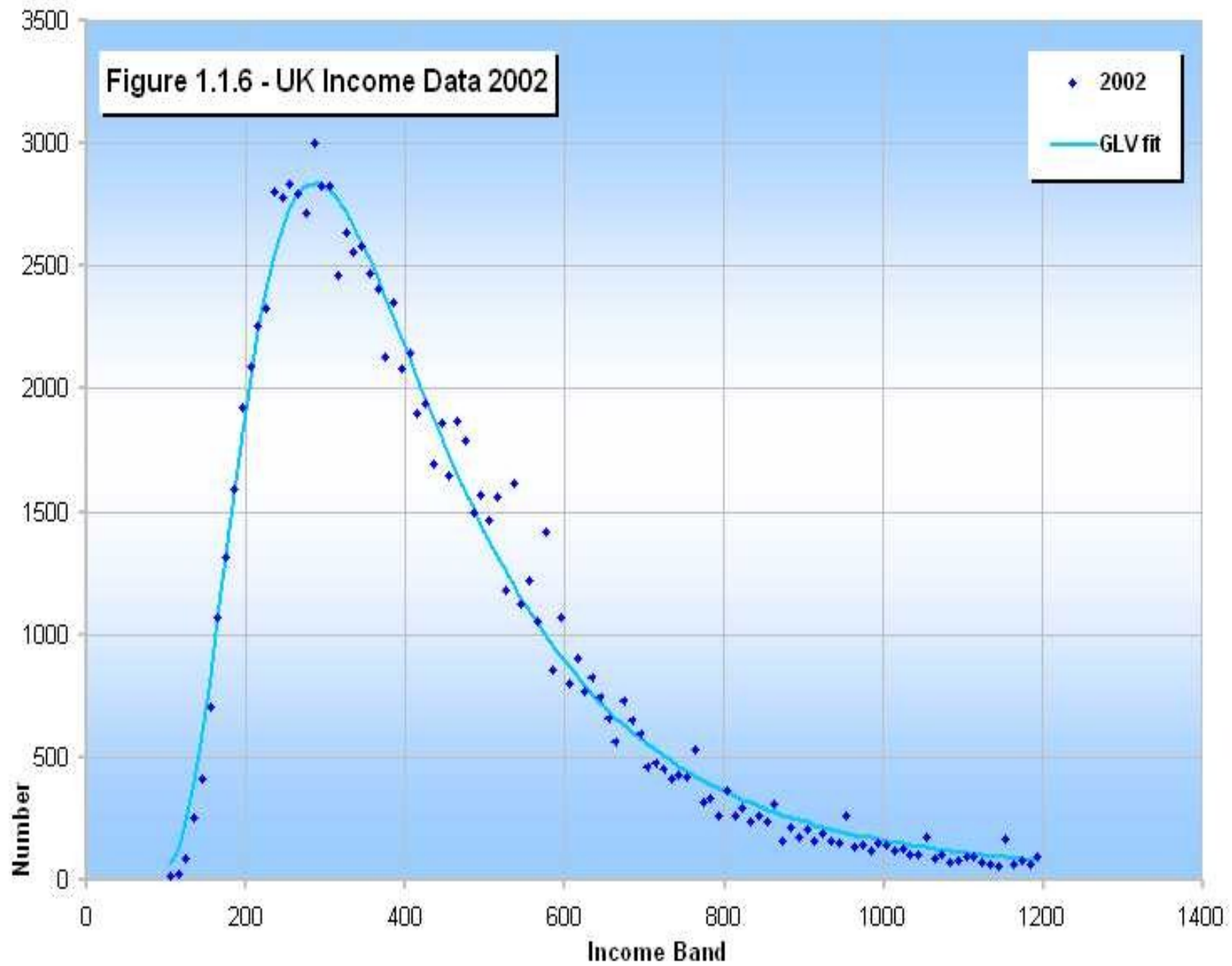
- real capital
 - producer of goods
 - gives source of revenue stream
 - real capital can produce more real capital
 - Sraffian cost plus pricing
- Financial capital
 - ownership of real capital
 - ownership of revenue stream
 - Minskian pricing on revenue stream
 - Standard finance theory

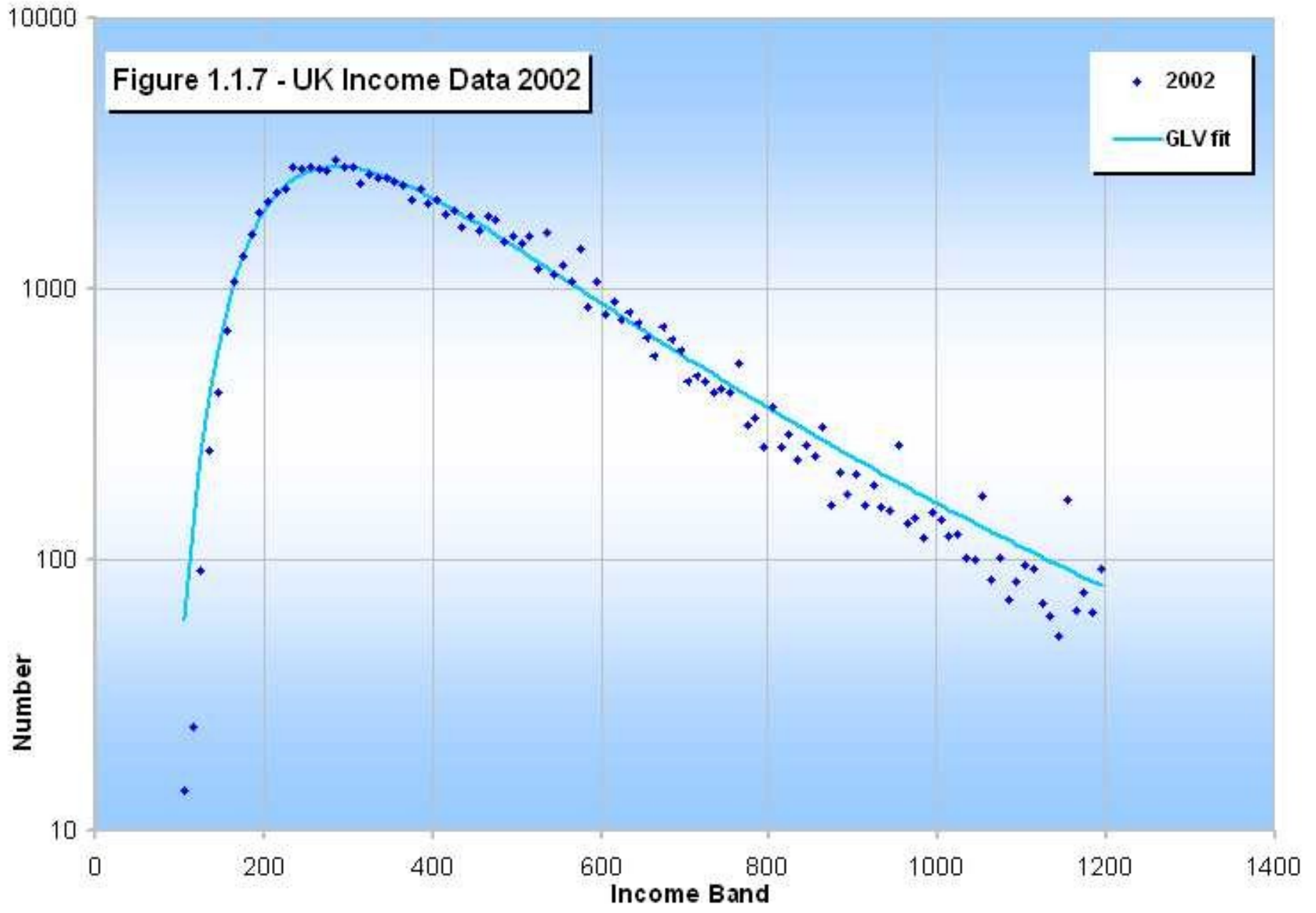
- production of goods and services give source of revenue streams for wages and returns to capital
- mortgage
 - claim on wages stream
- government bonds
 - right to tax revenue streams from wages and capital
- currency
 - claim on governments rights to tax revenues

1. Wealth & Income Models

1.0 Wealth & Income Data – Empirical Information

- Power 'Pareto' tail
- 'log-normal' body
- offset from zero
- persistent patterns across societies with different economic systems



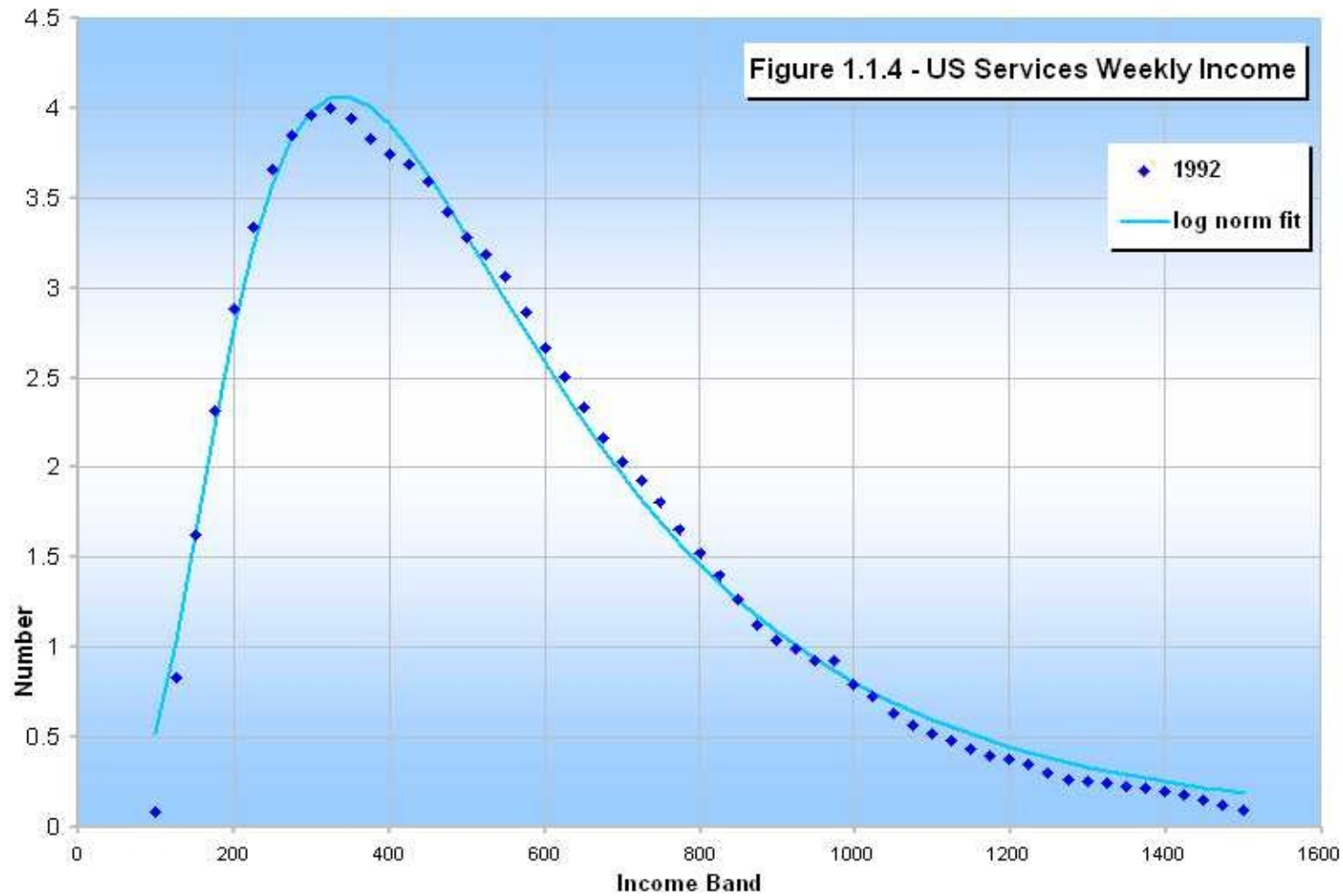


General Lotka-Volterra - (GLV)

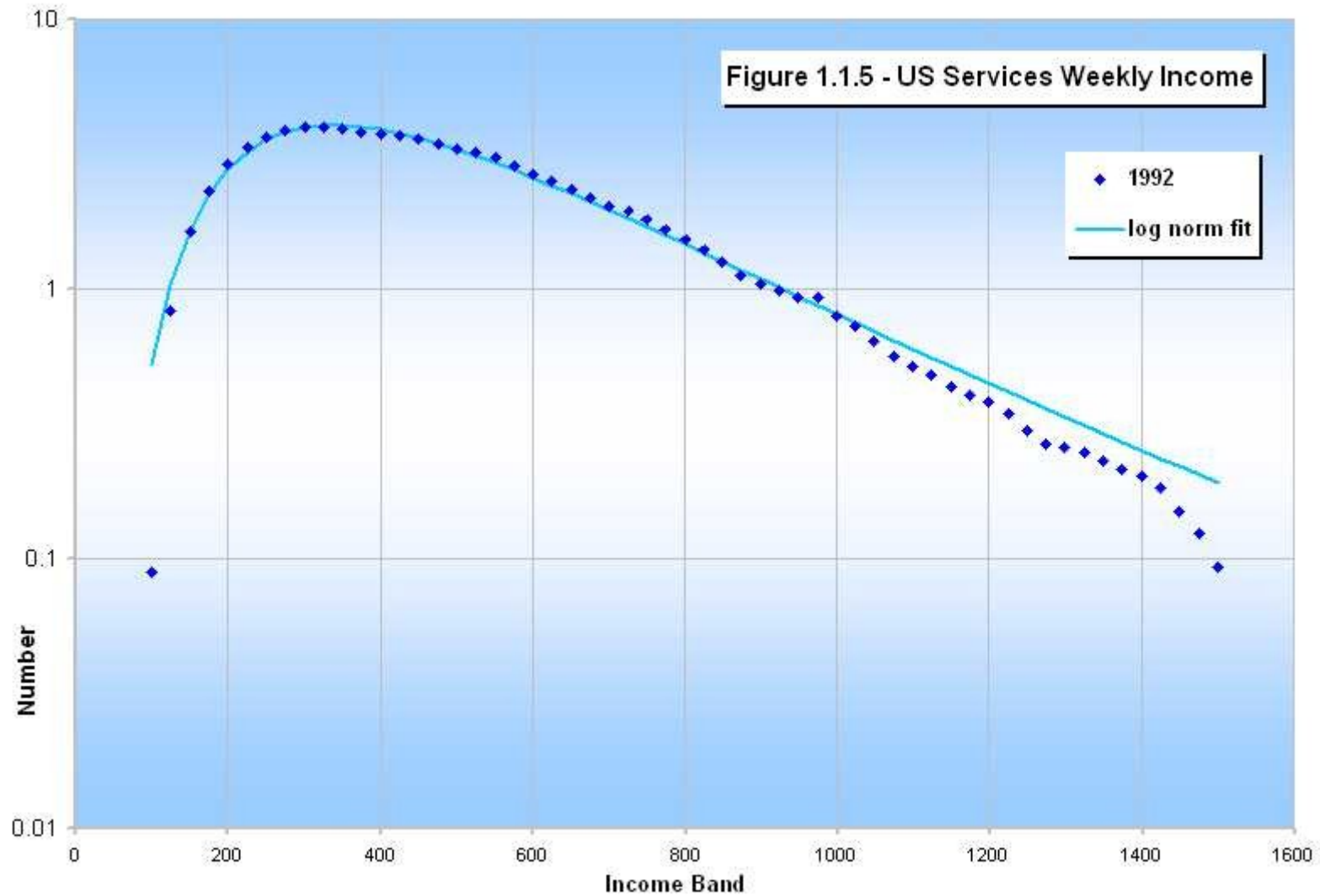
- can fit power tail
- can fit log-normal body
- has offset from zero
- gives very good fit to data

Figure 1.1.8	Reduced Chi Squared	
	Full Data Set	Reduced Data Set (no power tail)
Gamma/M-B Fit	3.27	1.94
Log Normal Fit	2.12	3.02
GLV Fit	1.21	1.83

US earnings data



- offset from zero



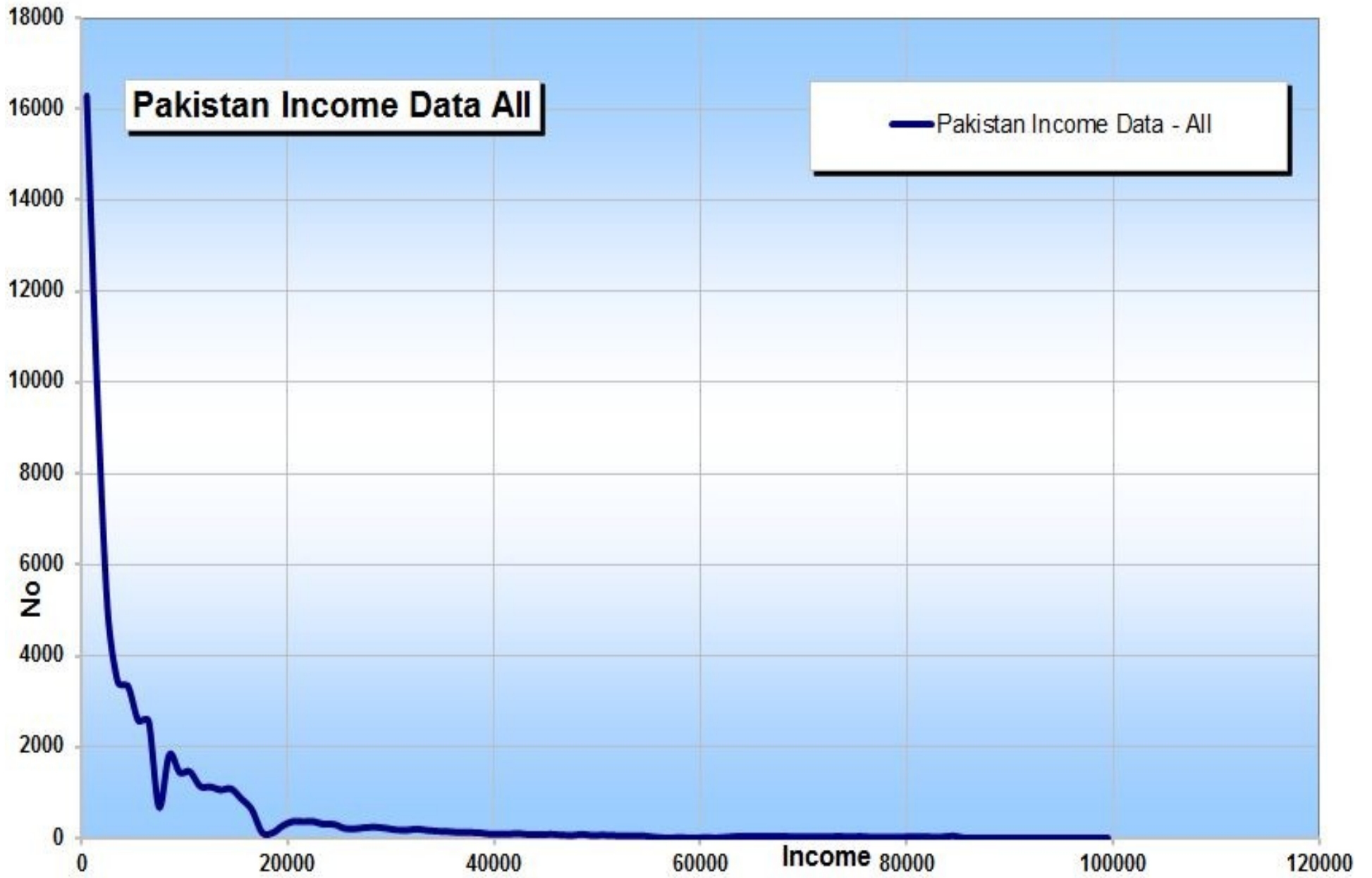
- No power law / 'Pareto' tails

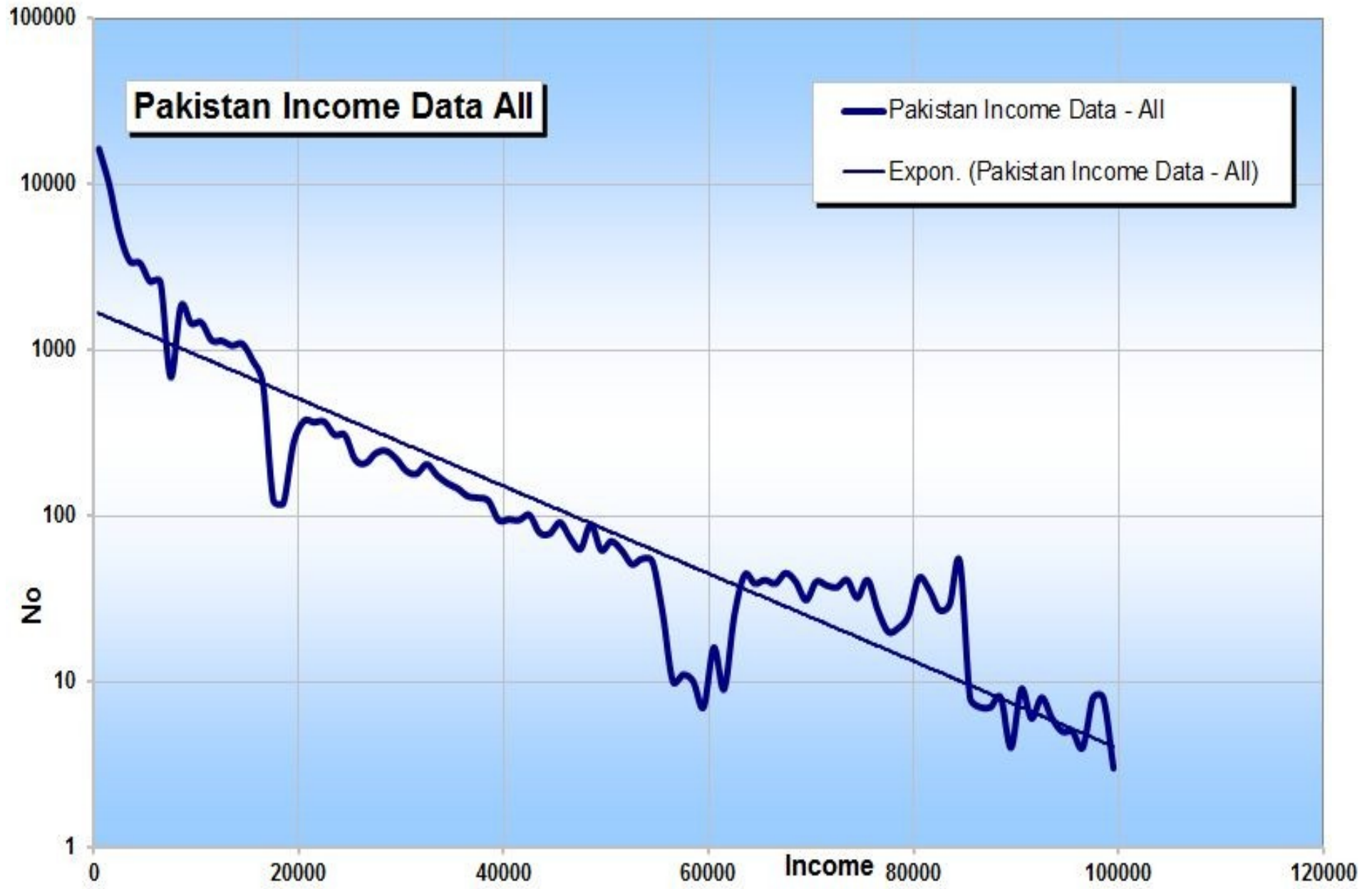
- 'log-normal' like body
 - actually fit very well by gamma / simple Maxwell-Boltzmann

$$y(x) \propto x.e^{-x}$$

- even better fit (output from genetic algorithm)
 - inverse Gaussian

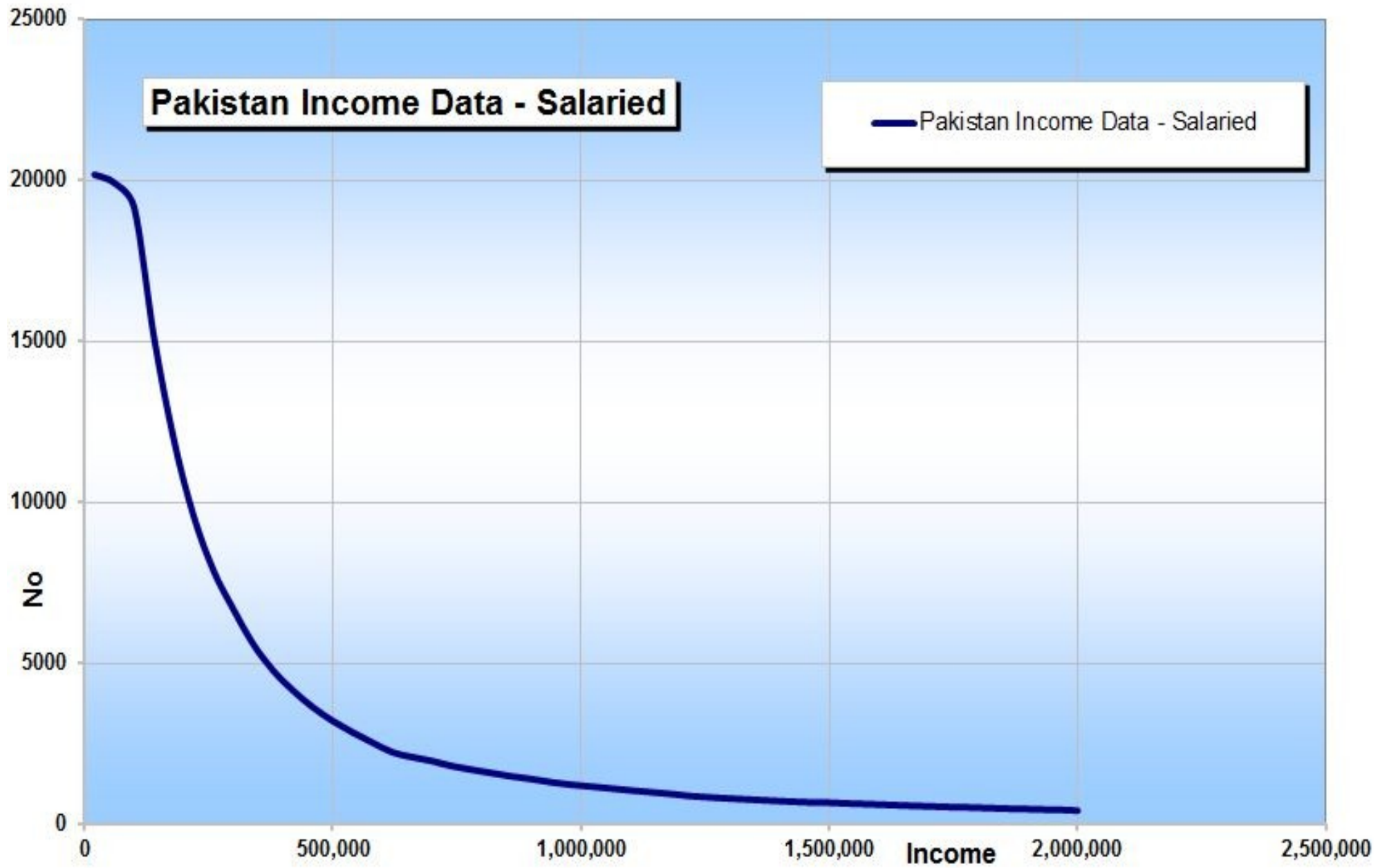
$$y(x) \propto x.e^{-(ax+b/x)}$$





Pakistan income data - all

- NOT offset from zero
- 'exponential' like body



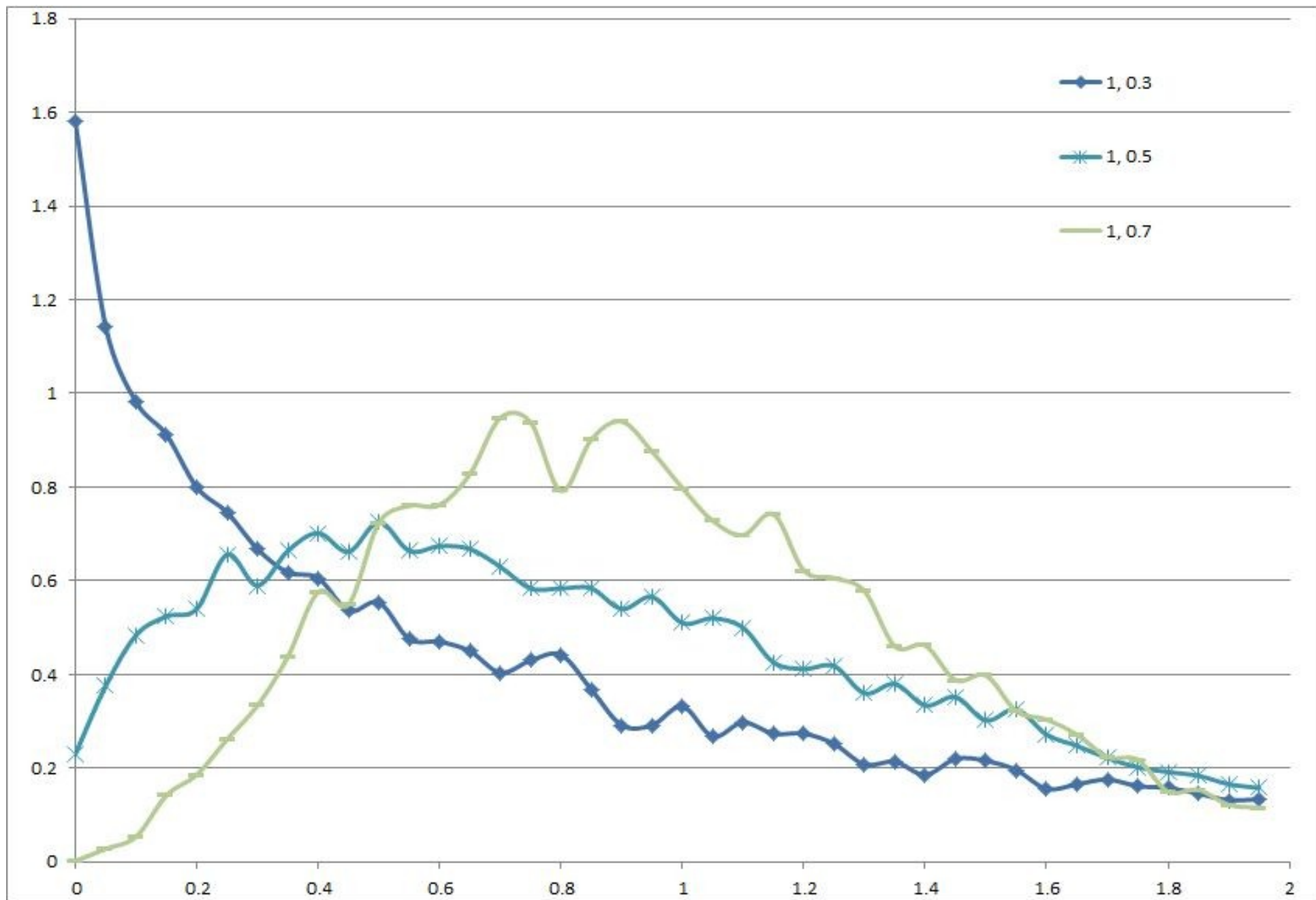
Pakistan income data – Salaried

- hybrid? - exponential / gamma ?

1.1 Service Economy Model

- real capital – self owned
- homesteader / subsistence economy
 - land owned communally, allotted 'fairly'
 - owners produce most of their own basic foods
 - also produce surpluses / specialist goods & services
- Could just pair up and exchange surpluses in barter
 - inefficient, restricts possible exchanges
 - use nominal store of value, with fixed supply, cowrie shells, gold coins, etc 'money'
 - now have 'half-exchanges'
 - allows analysis as a continuous cycle

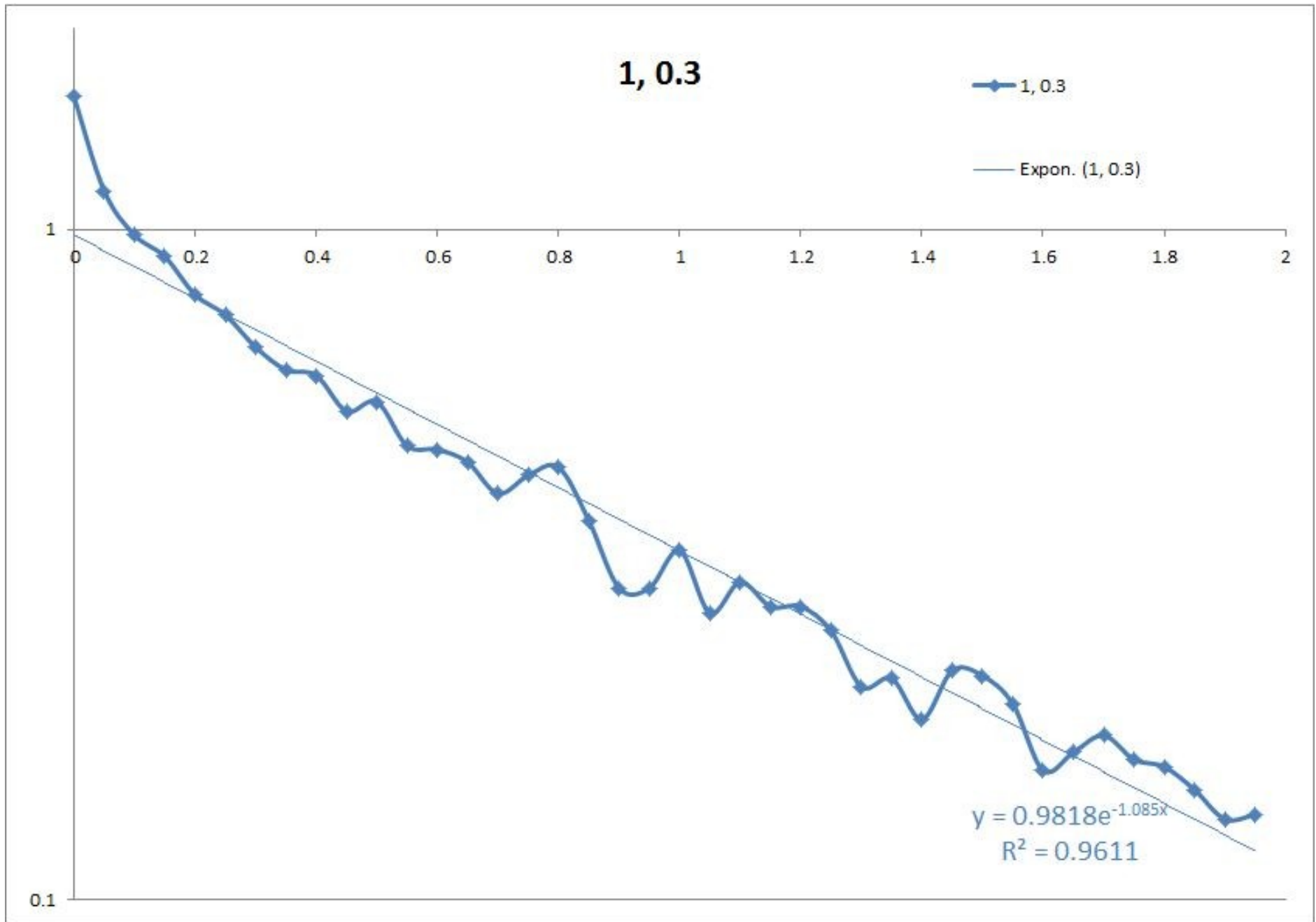
- one individual could use all their coins to buy surplus from just one other individual – spend all on one good
 - more likely to split purchases and buy from a number of different individuals
- N individuals total
- each individual starts with wealth W (measured in coins)
- spends (consumes) portion Ω on goods / services from others
- saves portion $(1-\Omega)$ for next cycle (for security)
- splits spending equally between m others.
 - In model – m other individuals picked at random
 - each receives $1/(m.\Omega)$
- all individuals identical – homogenous agents



$m = 1$, consumption Ω large, saving rate small

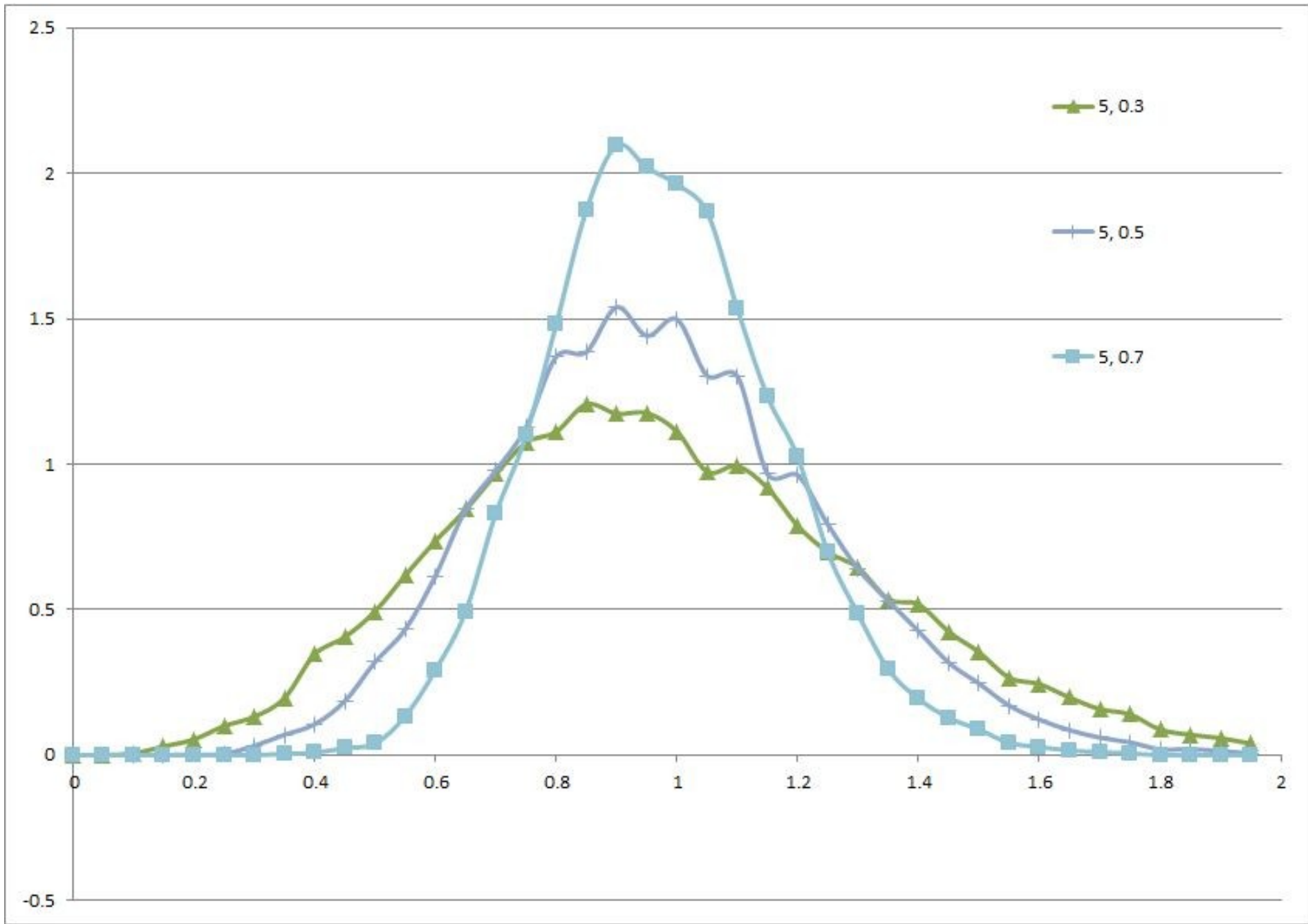
near exponential

Undeveloped subsistence economy, low levels of saving, low levels of trading partners



$m = 1$, consumption $\Omega < 0.5$, saving large

- gives close to $x \cdot \exp(-x)$ – no offset from zero
- possibly analagous to pair exchange model
- reduces to simple one dimensional Maxwell-Boltzmann



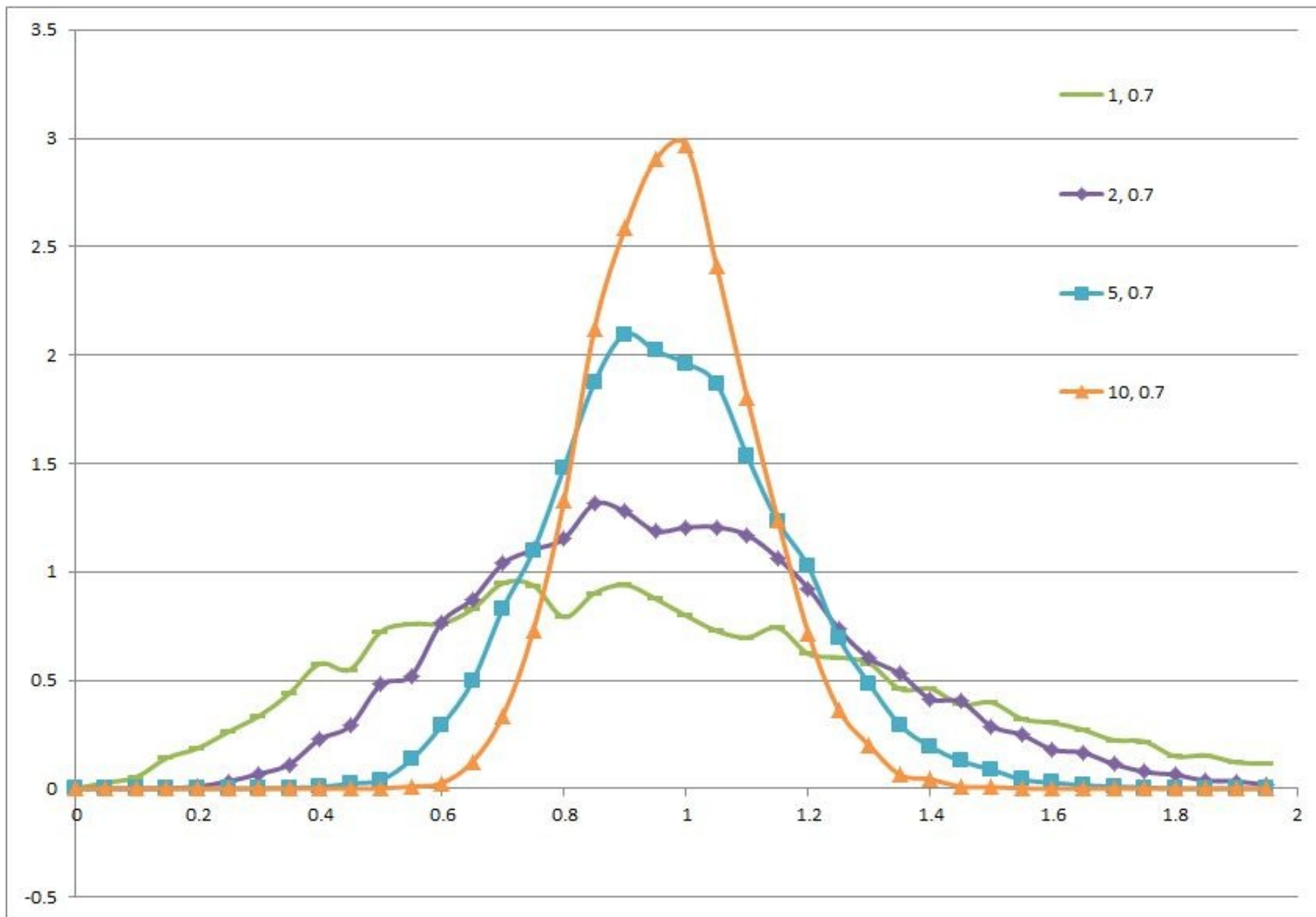
$m > 2$

appears that $x \cdot \exp(-(ax + b/x))$

- gives very good fit to data from model
- matches US data and truncated UK data
 - still investigating analytically

Note that:

- increasing savings rate decreases inequality
- increasing m decreases inequality



More m , more natural distribution, more equality, ultimately as m tends to N , all agents have same wealth, infinitely narrow spike.

as Ω decreases, saving increases, more equal distribution

important as decreasing saving; for example by allowing increased debt, (or zero deposit mortgages) results in more inequality.

Achieve greater equality by:

- Restricting personal debt
- Compulsory saving

Distributive statistical mechanical model

key is $m > 1$

tends to distribute income more fairly

- Most jobs naturally distributive, need people employed roughly in proportion to goods received:
 - manufactures, haircuts, meals in restaurants, education, etc
- Not true of some jobs, 1 person can supply services to many:
 - Film stars, pop stars, sportsmen, authors, etc
 - financial investors
 - These can probably gain much more than the above model suggests

Thanks to Ray Rogers for assistance with the work on this model.

- Have looked at source of income from labour
- Now look at income from capital

1.2.1 Lotka-Volterra systems

- population of prey x (say rabbits)
- population of predators y (say foxes)
- no predators present, natural population growth rate 'a' of rabbits:

$$\frac{dx}{dt} \propto ax \quad (1.2.1 a)$$

- no rabbits to eat, natural death rate 'c' of the foxes:

$$\frac{dy}{dt} \propto -cy \quad (1.2.1 b)$$

- foxes encounter rabbits, rate at which rabbits are killed is proportional to the number of rabbits and the number of foxes:

$$\frac{dx}{dt} \propto -\alpha x y \quad (1.2.1 c)$$

- α is a constant, and the $-ve$ sign, not good for the rabbits.
- However good for the foxes, giving:

$$\frac{dy}{dt} \propto \gamma x y \quad (1.2.1 d)$$

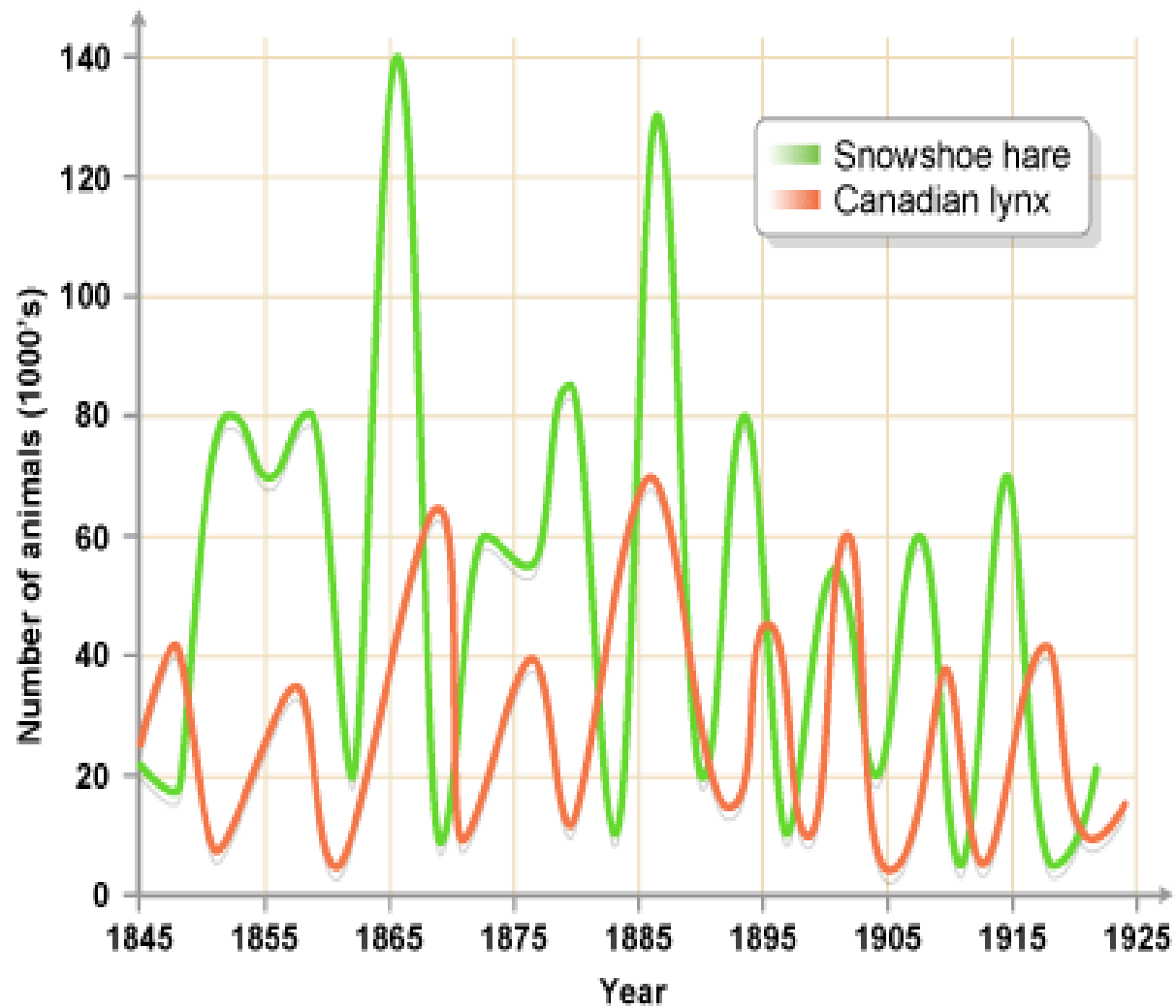
- γ is fixed constant.

- gives a pair of differential equations:

$$\begin{aligned}\frac{dx}{dt} &= ax - \alpha xy \\ &= x(a - \alpha y)\end{aligned}\quad (1.2.1 e)$$

for the rabbits; while for the foxes:

$$\begin{aligned}\frac{dy}{dt} &= \gamma xy - cy \\ &= y(\gamma x - c) \\ &= y(-c + \gamma x)\end{aligned}\quad (1.2.1 f)$$



- Normally unstable system:

1.2.2 General Lotka-Volterra (GLV) systems

- General Lotka-Volterra system (GLV) extends Lotka-Volterra model to multiple predators and prey:

$$\frac{dx_i}{dt} = x_i r_i + \sum_{j=1}^N a_{i,j} x_i x_j \quad (1.2.2 a)$$

$$= x_i \left(r_i + \sum_{j=1}^N a_{i,j} x_j \right) \quad (1.2.2 b)$$

- dx_i/dt is rate of change for the i -th species, out of a total of N species.

- first term natural growth (or death) rate, r_i , for the species with population x_i . Rate r_i is equivalent to the growth rate 'a' in equation (1.2.1e) or the death rate '-c' in equation (1.2.1f).
- second term gives the sum of all the interactions with the j number of other species. $a_{i,j}$ is the interaction rate defining the relationship between species i and j .
- $a_{i,j}$ is negative if species j is a predator, positive if species i is a predator. $a_{i,j}$ is equivalent to the α of equation (1.2.1e) or the γ of equation (1.2.1f).
- Equations (1.2.2a) and (1.2.2b) are generalisations of equations (1.2.1e) and (1.2.1f) for many interacting species.
- potentially $N!$ separate differential equations are needed to describe the whole system.

- Simplified by Solomon & Levy [Solomon 2000]
- difference equation for city population sizes.

$$w_{i,t+1} = w_{i,t} + r w_{i,t} + a_t \bar{w}_t - c_t \bar{w}_t w_{i,t}$$

$$w_{i,t+1} = \lambda w_{i,t} + a_t \bar{w}_t - c_t \bar{w}_t w_{i,t} \quad (1.2.2c)$$

- uses \bar{w} as average population
- λ is the natural growth rate of the population w of city i ,
- a_t is the arrival rate of population from other cities, multiplied by the average population \bar{w} of all the cities.
- c_t gives the rate of population leaving each city

- λ , a and c are universal rates, applicable to all members of the system.
- λ and a 'positive autocatalytic' (positive feedback), increase the population w of each city.
- negative value of c decreases the population of each city.
- Without the negative feedback term, the populations of the cities can increase to infinity
- Without the positive autocatalytic growth of λ in the first term, the second and third terms will cause all of the population to end up in a single city.
- Normally one or more variables are assumed to be stochastic.

- Gives a stable probability distribution for city populations:

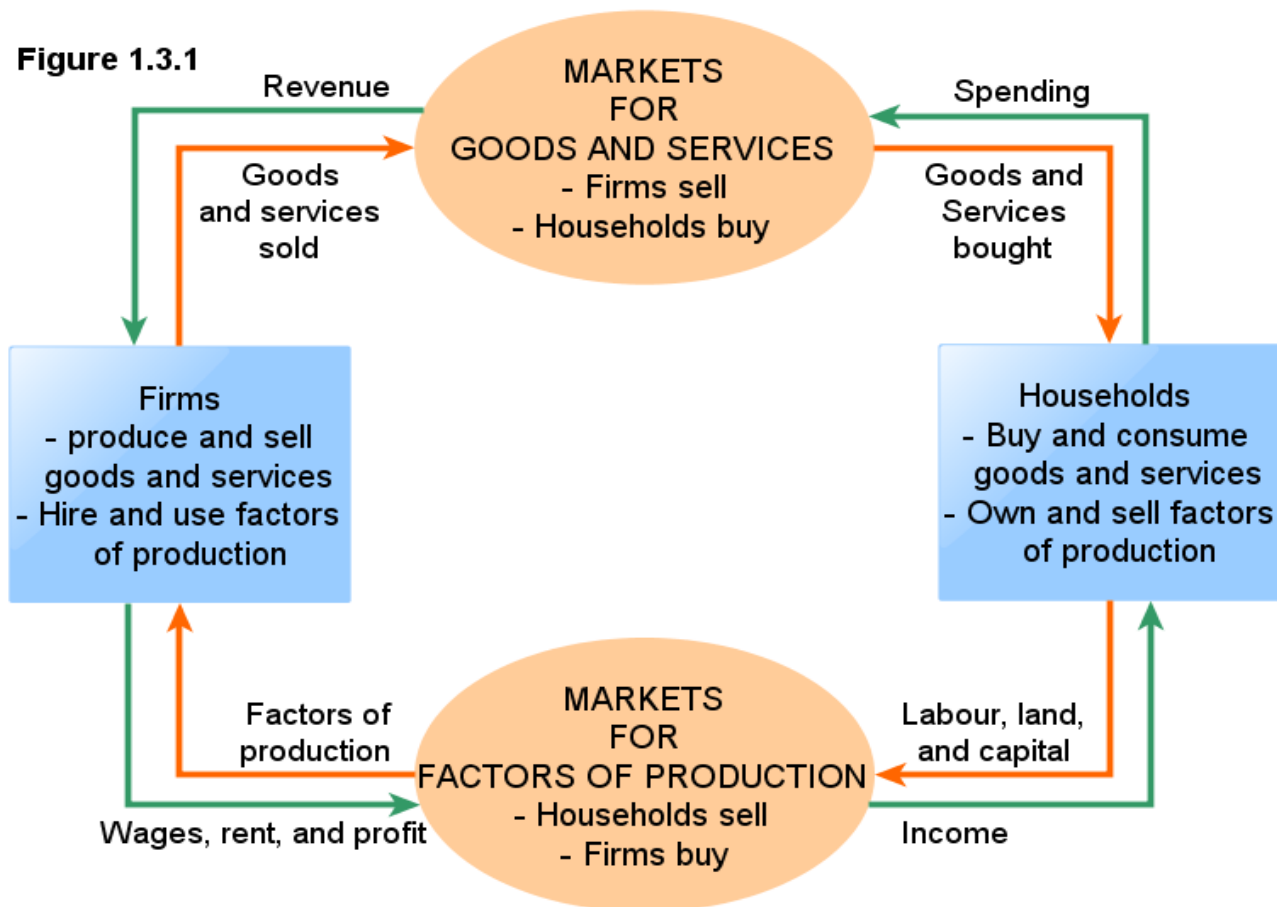
$$P(w) \propto (e^{-(\alpha-1)/w})/(w^{(1+\alpha)}) \quad (1.2.2 d)$$

$$P(w) = K (e^{-(\alpha-1)/(w/L)})/((w/L)^{(1+\alpha)}) \quad (1.2.2 e)$$

- Lotka-Volterra – feedback from x to y ,
and also feedback from y to x .
- GLV – feedback from x_i to all the other x_j .

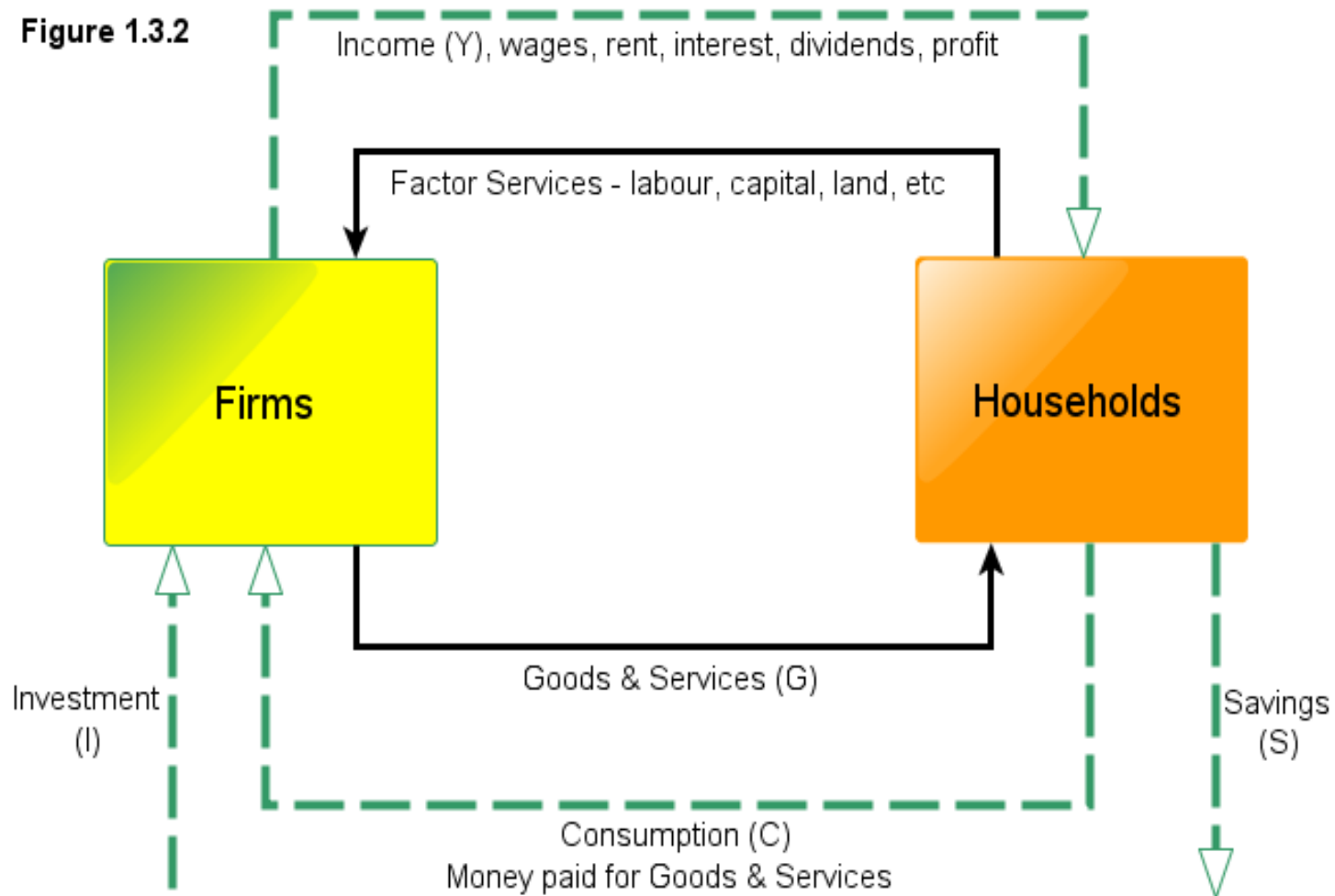
1.3 Wealth & Income Models - Modelling

- traditional economic model:



- Typical 'circular flow'

Figure 1.3.2



- Incorrect – shows flow of capital and land from households to firms
- Householders don't sell blast furnaces to companies
- Investment & saving not main source of capital
- *"Most corporations, in fact, do not finance their investment expenditure by borrowing from banks."* [Miles & Scott 2002, 14.2]

TABLE 14.1 The Financing of Investment:
Flow-of-funds Estimated (%) (1970–1994)

	Germany	Japan	UK	USA
Internal finance	78.4	69.9	95.6	94.0
Bank finance	12.0	30.1	15.0	12.8
Bond finance	−1.0	3.4	3.8	15.3
New equity	−0.02	3.4	−5.3	−6.1
Other	10.6	−6.8	−9.1	−16.0

Note: Internal finance comprises retained earnings and depreciation. The other category includes trade credit and capital transfers. The figures represent weighted averages where the weights for each country are the level of real fixed investment in each year in that country.

Source: Corbett and Jenkinson, “How Is Investment Financed?” *The Manchester School* (1996) vol. LXV, pp. 69–94.

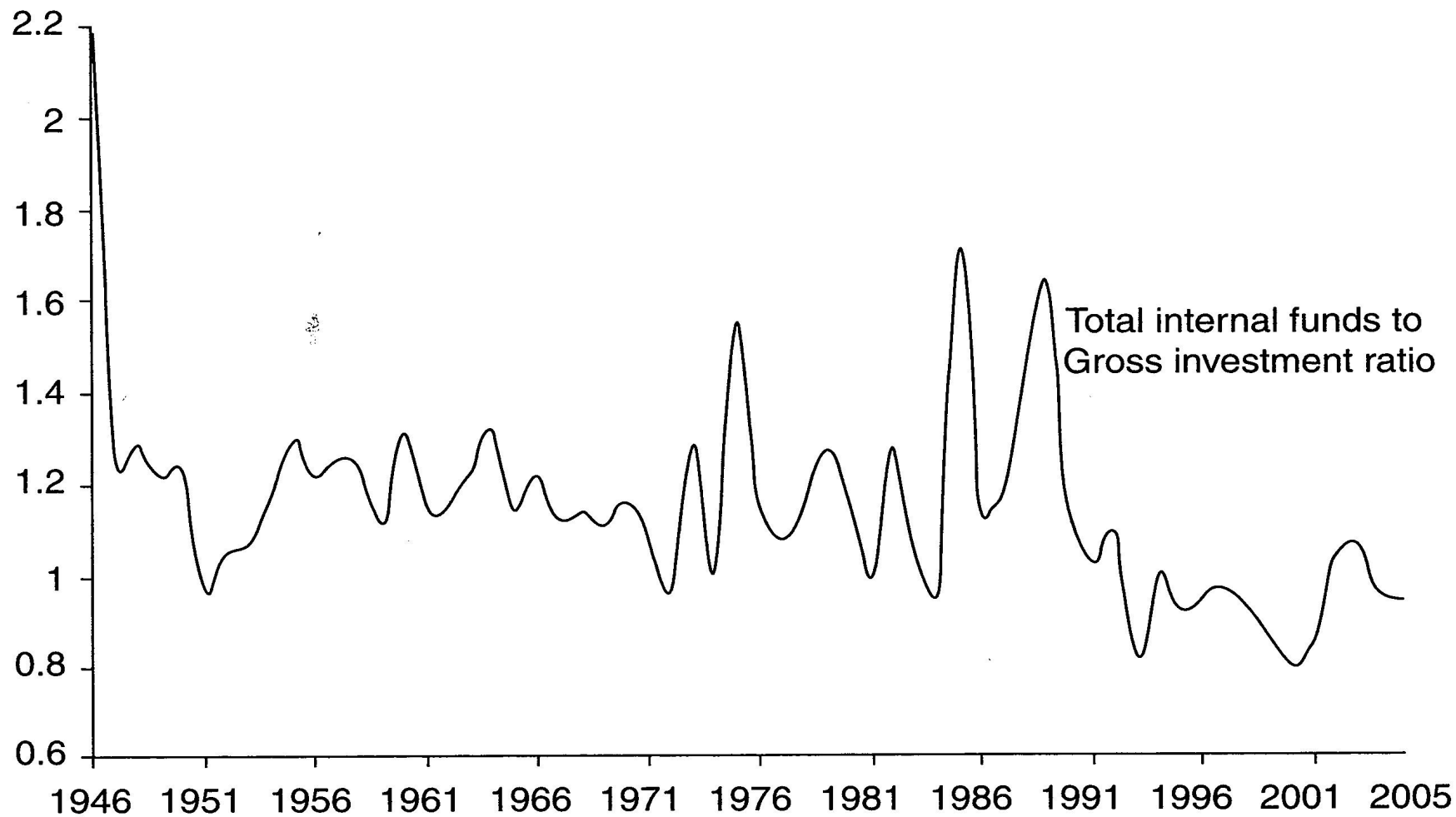
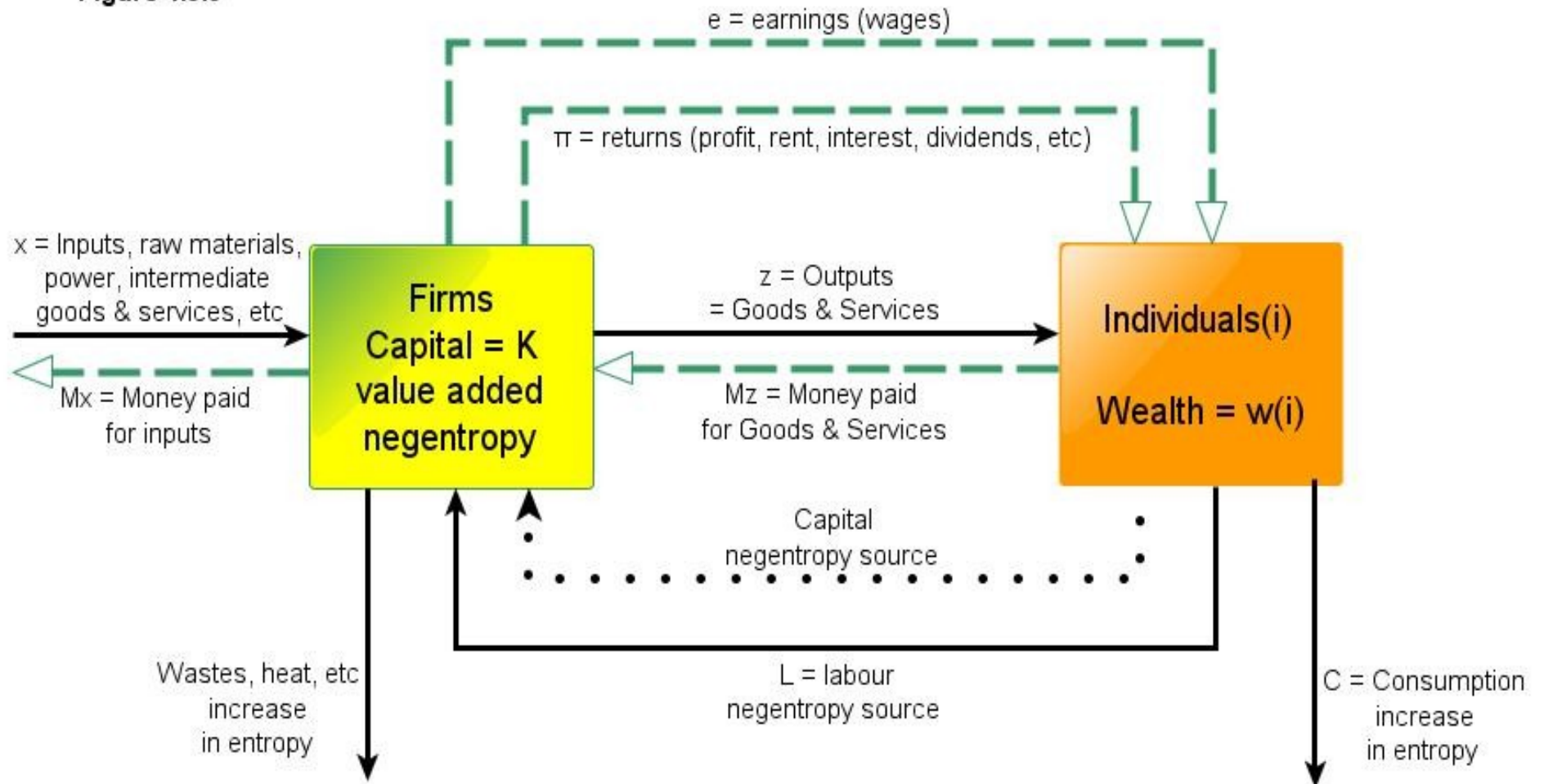


Figure 2.1 Total internal funds (including IVA) to gross investment ratio, USA, 1946–2005.

Source: Z.1 statistics of the Federal Reserve, www.federalreserve.gov/releases/z1, table F102, Non-farm non-financial corporate business. The curve plots the ratio of lines 9 and 10.

- Value >1 - most of the time
 - companies investing and saving at the same time
- Models in this paper assume capital is invested using internal finance
- Investment and saving ignored as secondary loop
 - returned to at end of paper
- Gives base model:

Figure 1.3.5



- K – 'real' capital; machines, land, buildings, etc
- W – 'financial' capital; shares, bonds, loans, etc
- black lines flows of real goods
- green lines flows of money / financial instruments
- dotted line indicates ownership of K by W
- dotted line is not a flow
- Consumption shown as real flow
- Mz – economist's consumption
- Returns Y split into earnings e and returns π

- in wealth & income models, assume total wealth is constant:

$$\text{total } W = \text{total } K \quad (1.3c) \quad \text{or:}$$

$$\sum w_i = W = K = \text{constant} \quad (1.3d)$$

Simple Model:

- economy is isolated; no imports or exports.
- no government sector, no taxation, welfare payments, government spending, etc.
- no unemployment; all individuals are employed
- Labour and capital are complementary inputs and are not interchangeable.
- The role of money is ignored.
- no debt
- fixed level of capital
- capital and consumption goods interchangeable
 - Sraffian 'corn' model
- waste streams included so that the 2nd law of thermodynamics is not violated.

- at steady state equilibrium.

$$\text{total } C = \text{total } Y = \text{total } Mz$$

- but C and Y may be different for individuals

$$\sum w_{i,t} = \sum w_{i,t+1} = W = K = \text{constant} \quad (1.3 e)$$

- For single individual:

$$w_{i,t+1} = w_{i,t} + y_{i,t} - Mz_{i,t} + e_{i,t} + \pi_{i,t} - C_{i,t} - \text{labour}_{i,t} - \text{capital}_{i,t} \quad (1.3f)$$

- $y_{i,t}$, $C_{i,t}$, labour and capital are real units, others are financial.
- Looking only at the financial flows:

$$w_{i,t+1} = w_{i,t} - Mz_{i,t} + e_{i,t} + \pi_{i,t} \quad (1.3g)$$

- Use $C_{i,t}$ in place of $-Mz_{i,t}$. $C_{i,t}$ is now a monetary unit; reverts to standard economics usage gives:

$$W_{i,t+1} = W_{i,t} + e_{i,t} + \pi_{i,t} - C_{i,t} \quad (1.3h)$$

- In a single iteration, the paper wealth w of an individual i
 - increases by the wages earned e
 - increases by the profits received π .
 - reduces by the amount spent on consumption C .

- e – is either uniform distribution or normal distribution, defined in model – exogenous
 - note – not using gamma distribution from earnings model above
 - not had time to incorporate
- profit is proportional to wealth, given by market profit rate r :

$$\pi_{i,t} = W_{i,t} r \quad (1.3j) \quad \text{for each of the } i \text{ agents.}$$

- Consumption also proportional to wealth, given by consumption rate Ω :

$$C_{i,t} = w_{i,t} \Omega \quad (1.3n)$$

Substitute into (1.3h) gives the difference equation for each agent:

$$w_{i,t+1} = w_{i,t} + e_{i,t} + w_{i,t}r - w_{i,t}\Omega \quad (1.3o)$$

- Equation (1.3o) is base equation for a single agent in all income models.

- w is the only variable.
- e , r and Ω are all constants; though can be stochastic around long-term constant value

- In income models:

$$Y = \sum e_i + \sum \pi_i = \text{constant}, \quad \text{always} \quad (1.3p) \quad \text{and}$$

$$\sum e_i = \sum \pi_i = \frac{Y}{2} \quad \text{usually} \quad (1.3q)$$

- Accords with 'Bowley's Law' returns to labour typically between 0.75 and 0.5 of total returns.

Some definitions:

$$\text{Profit rate} \quad r = \frac{\sum \pi}{\sum w} \quad (1.3r)$$

$$\text{Income rate} \quad \Gamma = \frac{\sum Y}{\sum w} \quad (1.3s)$$

$$\text{Bowley ratio} \quad \beta = \frac{\sum e}{\sum Y} \quad (1.3t)$$

$$\text{Profit ratio} \quad \rho = \frac{\sum \pi}{\sum Y} \quad (1.3u)$$

- by definition:

$$\beta + \rho = 1 \quad (1.3v)$$

Profit ratio $\rho = \frac{r}{\Gamma} \quad (1.3w)$

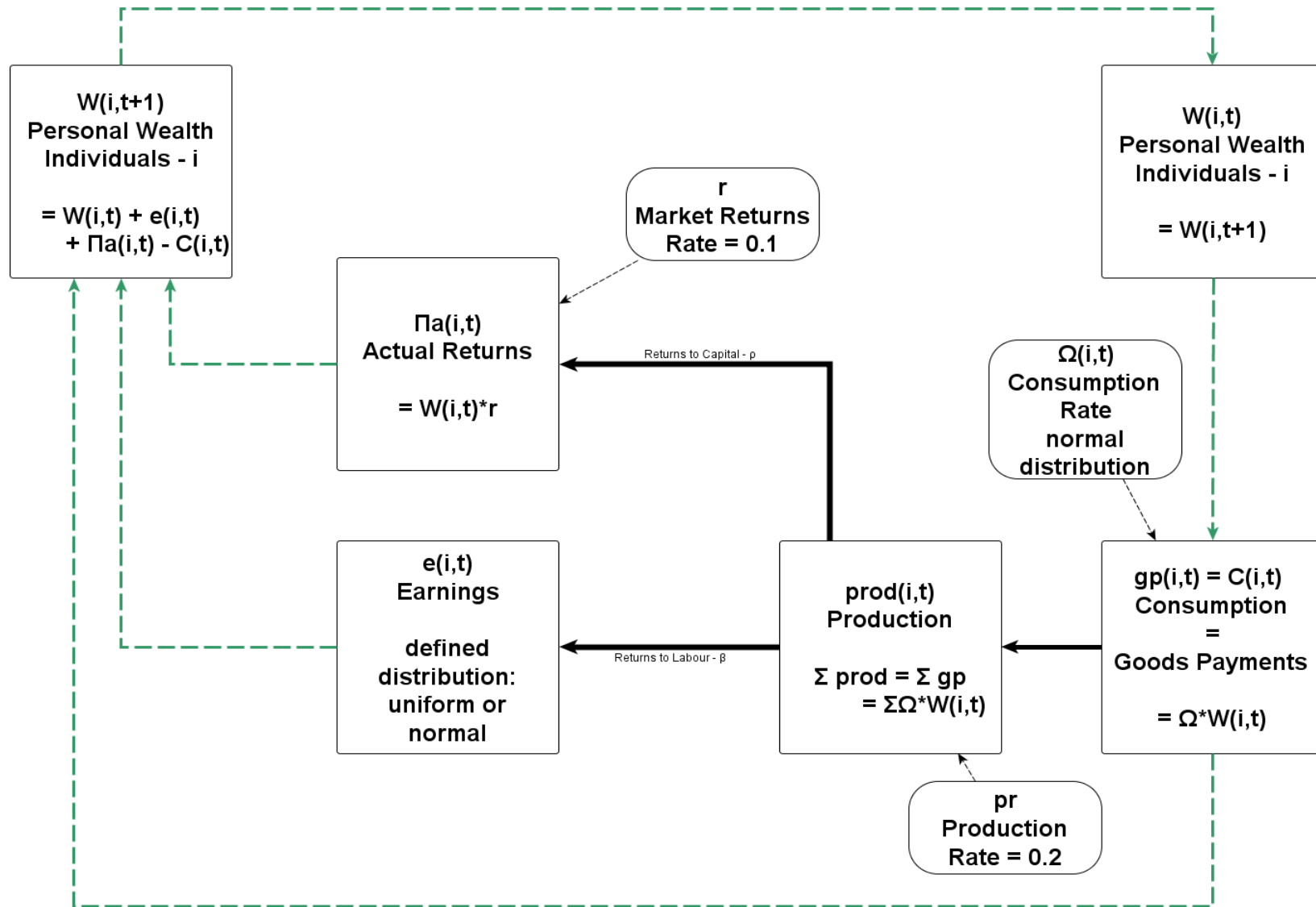
important subtlety:

- textbook economics; $C = Y$ by definition
- in these models consumption becomes equal to income automatically by adjusting wealth
- final consumption term gives automatic feedback

Formula for iterations:

$$w_{i,t+1} = w_{i,t} + e_{i,t} + w_{i,t}r - w_{i,t}\Omega \quad (1.3o)$$

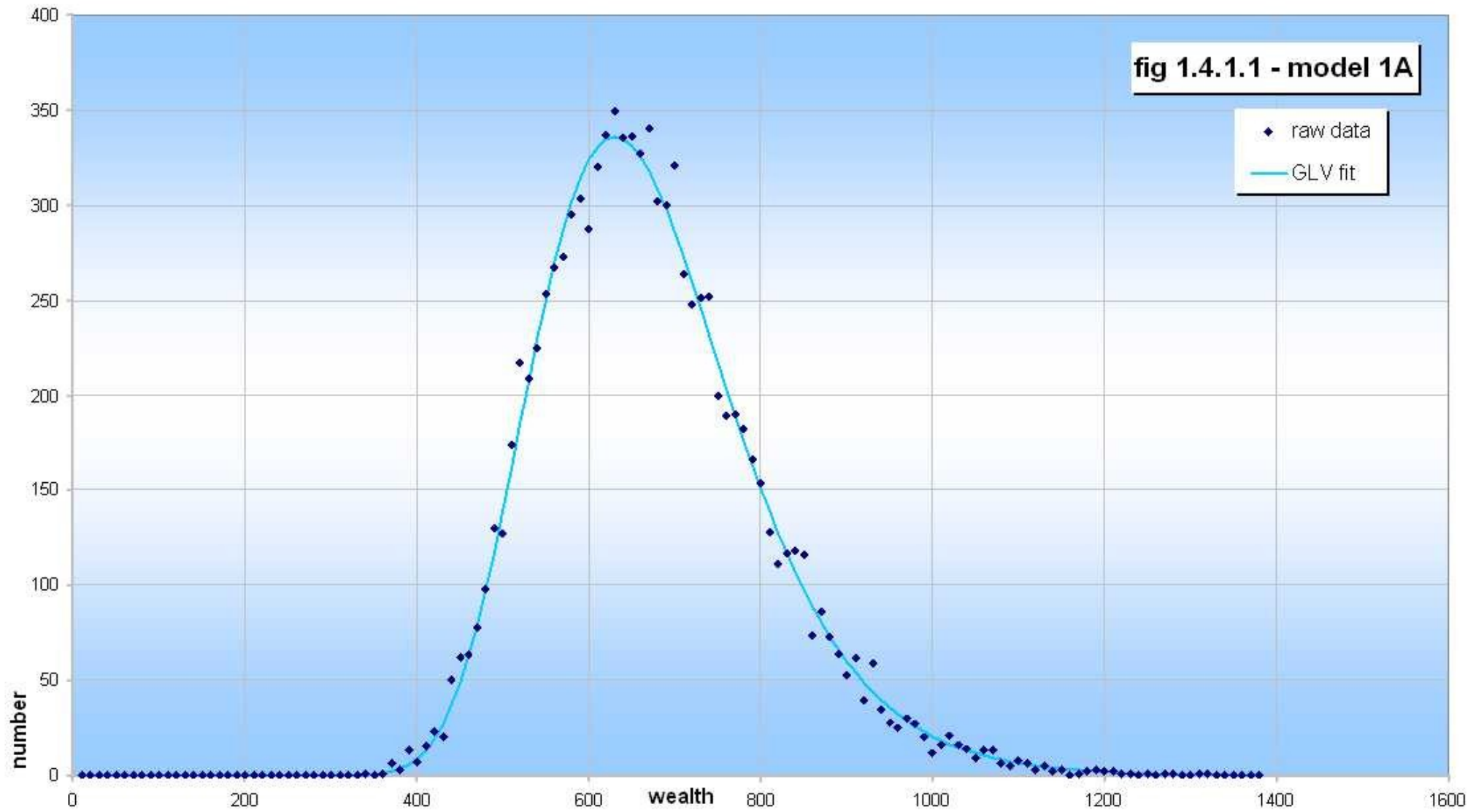
Figure 1.3.9
Wealth & Income Model
Iteration Diagram

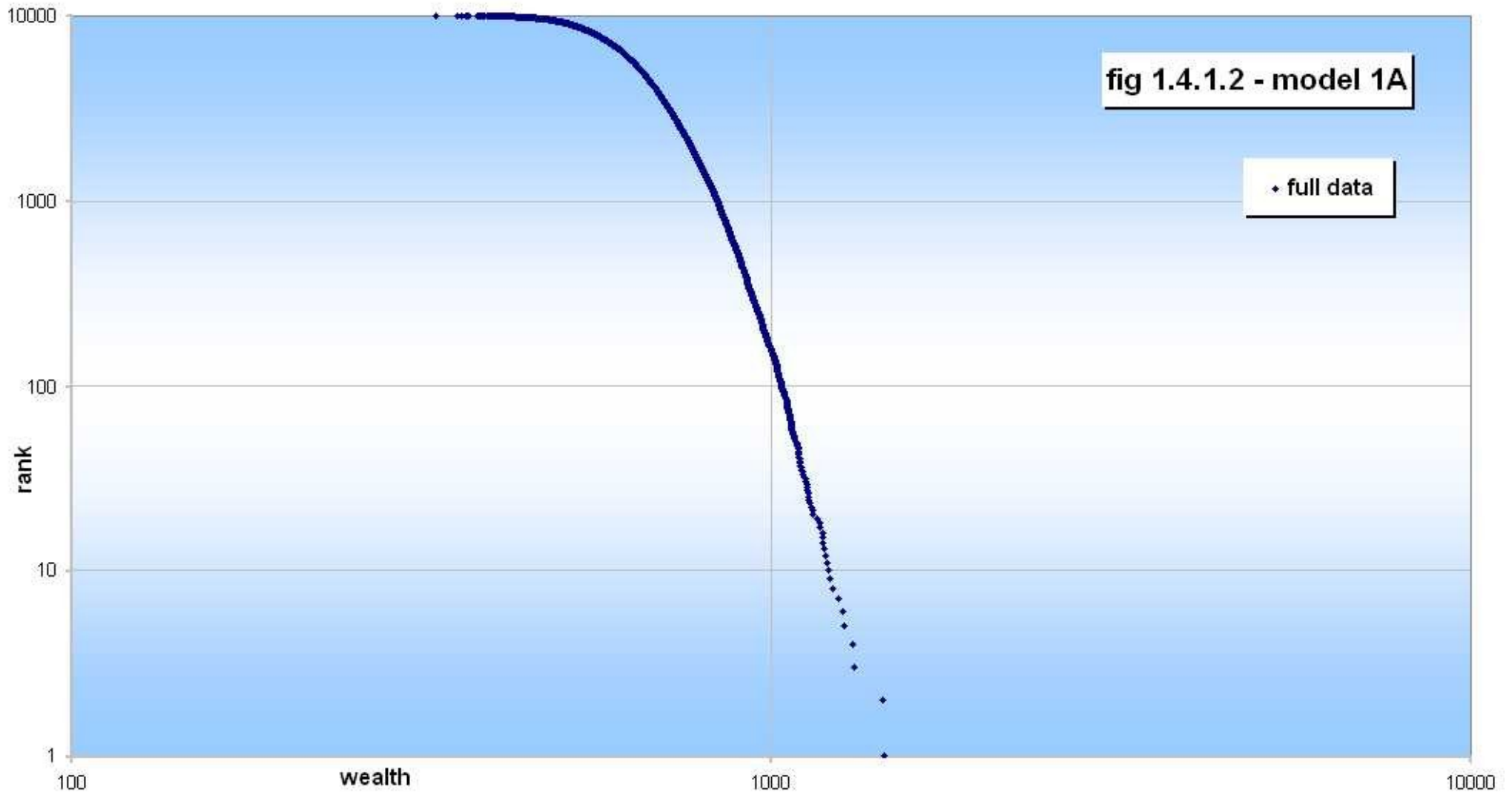


1.4 Wealth & Income Modelling - Results

1.4.1 Model 1A Identical Waged Income, Stochastic on Consumption

- Earnings – uniform distribution
- all agents have identical productive ability
- consumption stochastic from normal distribution
- consumption constant and identical over long run
- **All agents absolutely identical**
- perfect fit to GLV for wealth (as expected)





- also gives power tail:

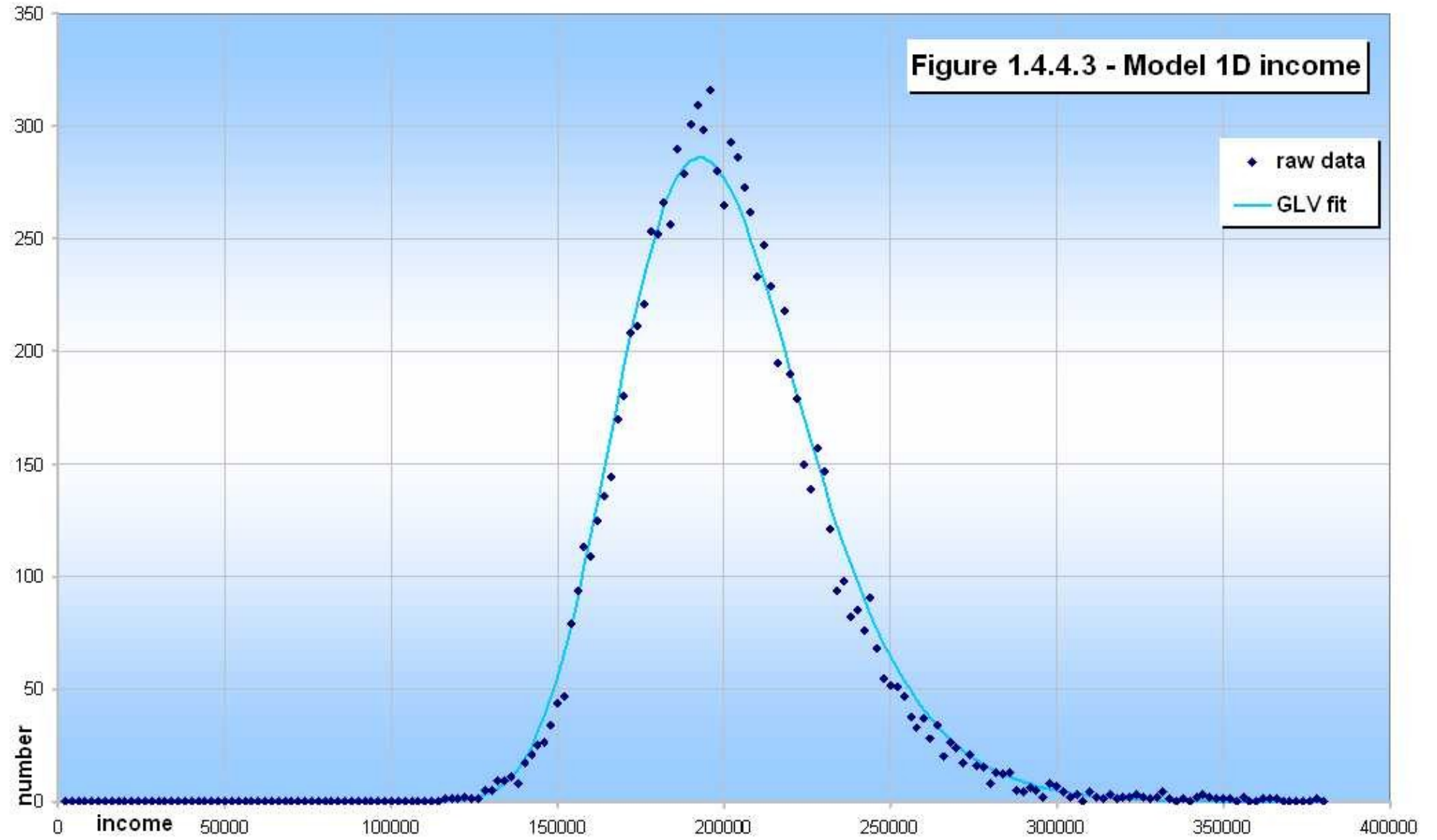
- **highly unequal wealth distribution produced from identical agents**
- Wealth distribution is a simple result of statistical mechanics; of entropy.
- The fundamental driver forming this distribution of wealth is not related to ability or utility in any way whatsoever.
- Distribution on returns instead of consumption produces similar results
- Income not analysed as agents move up and down in the distribution very rapidly

1.4.2 Model 1B Distribution on Waged Income, Identical Consumption, Non-stochastic

- Earnings – normal distribution at start of run (not stochastic)
- Consumption rate – uniform distribution
- dull model – output distribution is identical to input distribution
- **distribution of consumption / savings rates is key to wealth condensation effects**

1.4.4 Model 1D Distribution on Consumption and Waged Income, Non-stochastic

- Earnings – normal distribution at start of run (not stochastic)
- Consumption rate – normal distribution at start of run (not stochastic)
- Produces GLV for wealth distribution
- Produces apparent GLV distribution for income



- actually a combination of two underlying distributions:
 - GLV distribution of income from wealth – which is proportional to wealth (via r)
 - and normal distribution of earnings income – defined in model
- result looks like GLV

1.5 Wealth & Income Modelling - Discussion

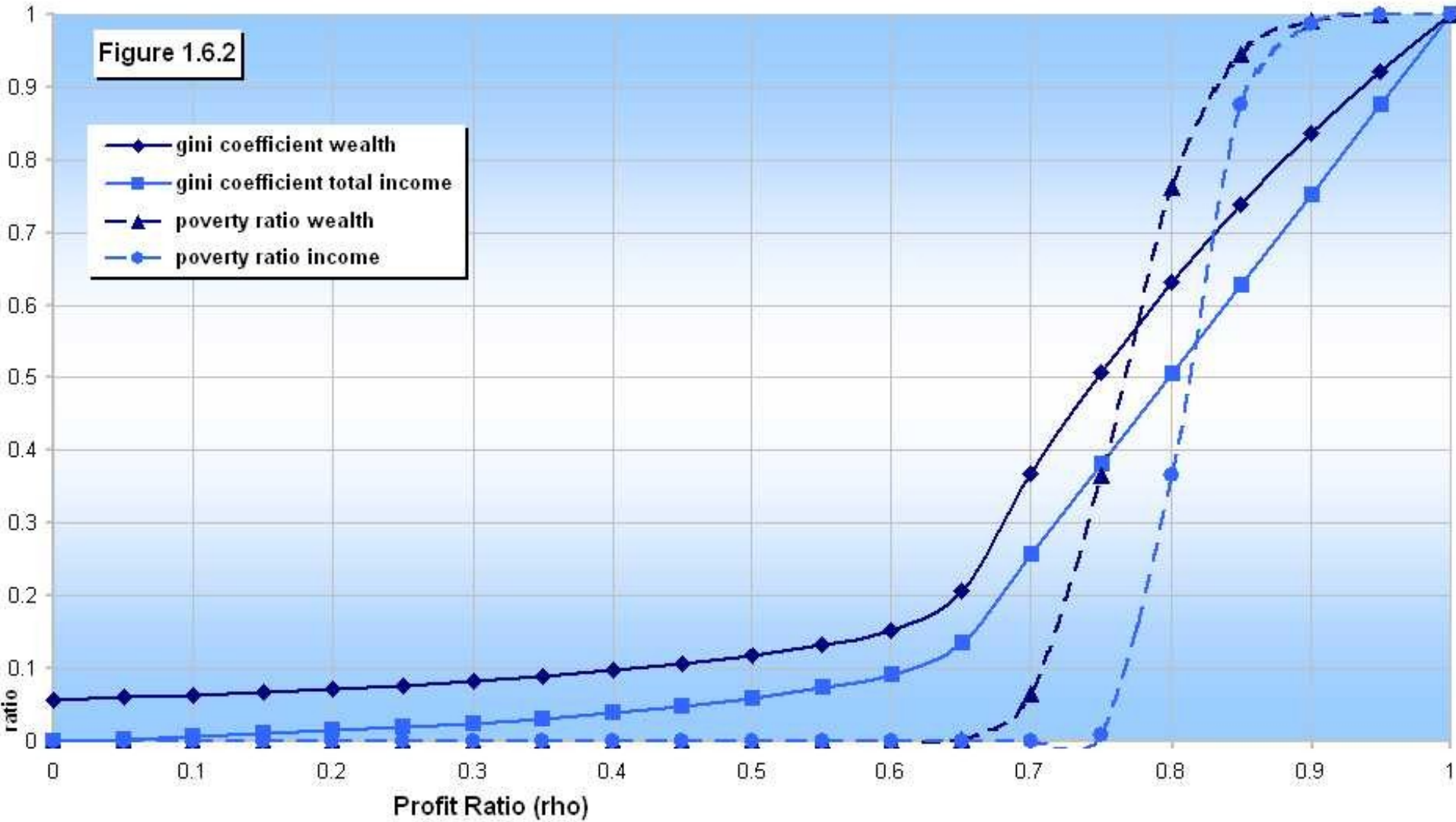
- Output distributions for wealth and income are much more unequal than input earnings / consumption distributions.
- Wealth condensation model – caused by statistical mechanics
- System involving capital: changes normal distributions into power tail distributions
- natural split between wealth owning class and working/middle class dependent on earnings

- rather than 'predator-prey' model better to think as grazing model – sheep graze grass, humans 'graze' wool from sheep.
- ownership of capital allows 'grazing'
- Rupert Murdoch grazes on many people due to ownership of many newspapers
- Apex grazer is Bill Gates, can graze on Murdoch as Murdoch companies use Windows software
- The more capital you have got, the more grazing you get to do.

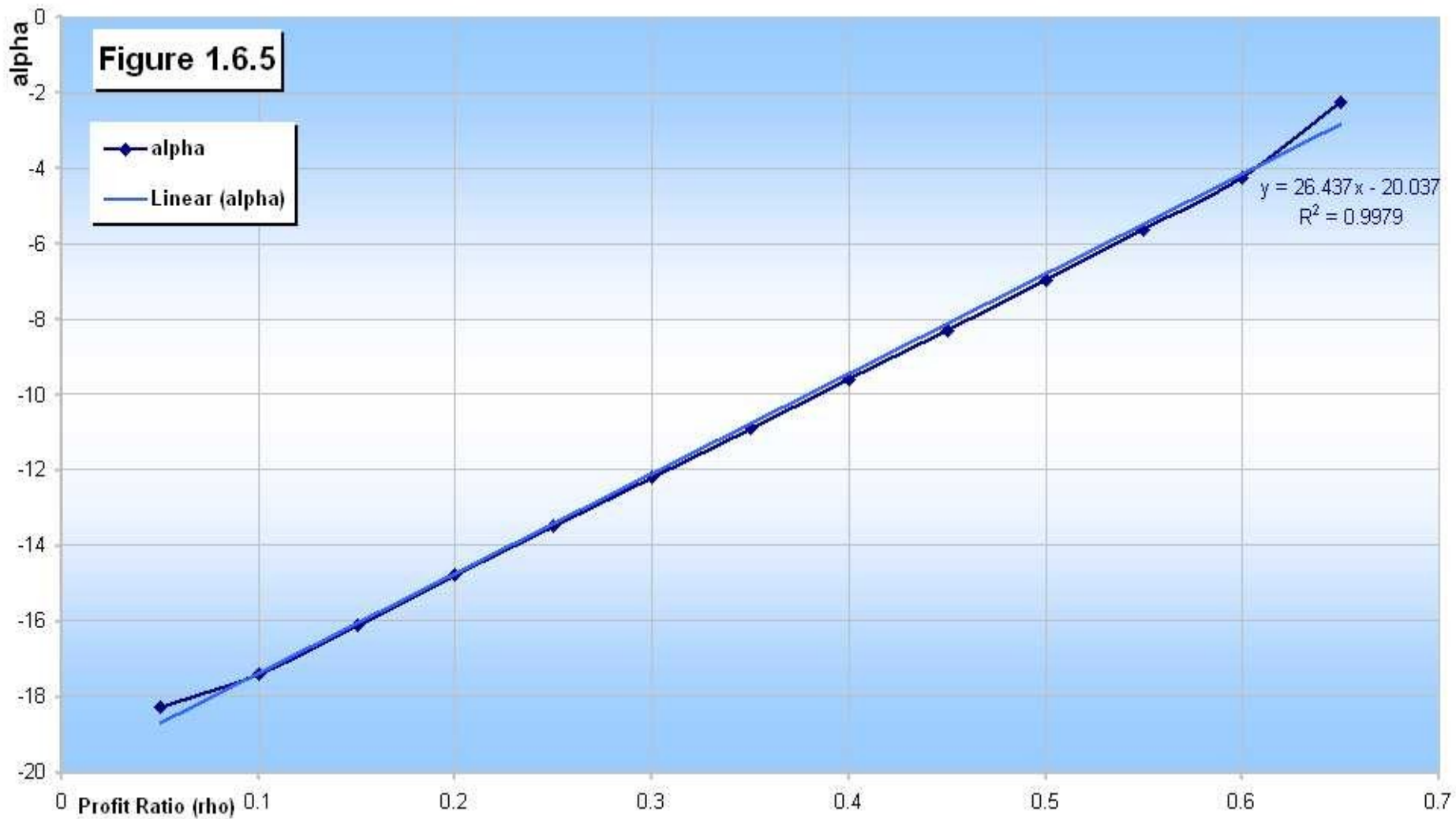
Don't need any of the following:

- Different initial endowments
- Savings rates that change with wealth
- Different earning potentials
- Economic growth
- Expectations (rational or otherwise)
- Behaviouralism
- Marginality
- Utility functions
- Production functions

1.6 Enter Sir Bowley - Labour and Capital



- when $\rho = 0$, $\beta = 1$, Gini index is zero
- when $\rho = 1$, $\beta = 0$, all earnings are returned as capital
 - the individual with the highest saving propensity, becomes the owner of all the wealth
 - Gini index goes to 1.



- power tail exponent for wealth varies linearly with profit ratio / Bowley ratio

- following formulae extracted empirically from data
 - (not proved analytically):

$$\alpha = \frac{1.36(1 - \rho)}{v^{1.15}} \quad (1.6d)$$

$$\alpha = \frac{1.36\beta}{v^{1.15}} \quad (1.6e)$$

v is the variance of the normal distribution of consumption rates

- increase in profit ratio / decrease in Bowley ratio has two effects on income distribution
 - simple change in income shares – bad
 - change slope of power tail – very, very, bad indeed
- Second effect much more important than first

- Power law appears to be result of two growth rates
cf. [Newman 2005], which gives a general formula for α as:

$$\alpha = 1 - a/b \quad (1.6f)$$

- In wealth model:

$$\text{Profit ratio} \quad \rho = \frac{\text{direct returns to capital}}{\text{total income from capital}}$$

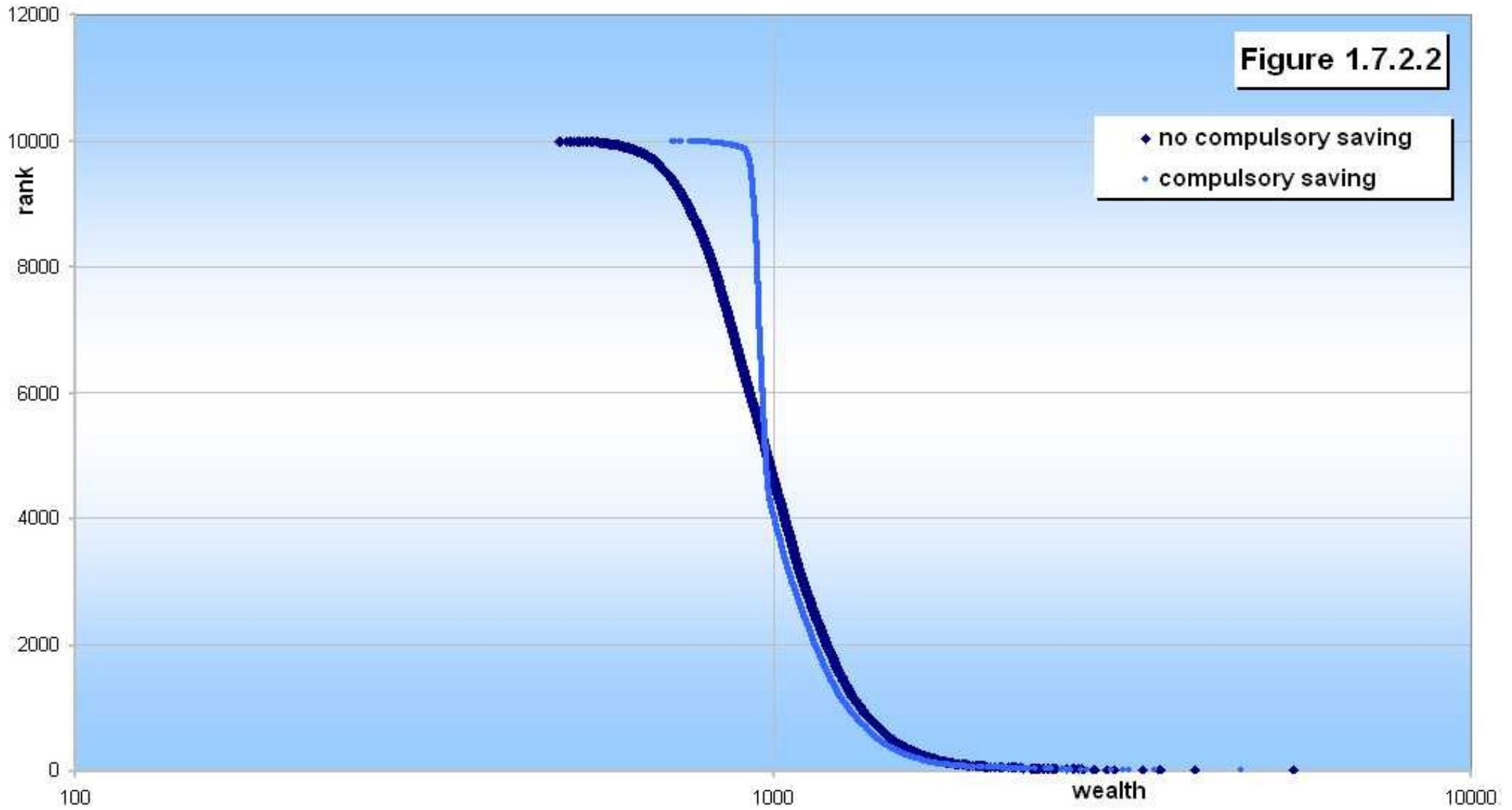
$$\text{Profit ratio} \quad \rho = \frac{r}{\Gamma} \quad (1.3w)$$

ρ is the growth rate that capitalists get on capital, divided by the growth rate that everybody (capitalists and workers) gets on capital. (Consumption balances growths.)

1.7 Modifying Wealth and Income Distributions

1.7.2 Compulsory Saving

- If any agent's current wealth was less than 90% of the average wealth, that agent was obliged to decrease their consumption rate by 20 percent.
- Moving ownership of a portion of the means of production into the hands of the poor



- Poverty largely eliminated
 - still have power tail for the most talented
 - rich are not taxed
 - poor are compelled to save.
-
- In practise use system like Chilean / Singapore / Australian compulsory pensions
 - but for all ages
 - give extra assistance for low earners

2. Companies Models

- Allow W to differ from K
- Share prices can be different to company fundamental values
- shareholders are myopic – shares valued on previous dividends
 - as financial pricing:

$$\text{Present Value} = \frac{\text{Dividend}_1}{r}$$

r is the relevant market interest/profit rate; Dividend_1 is the latest dividend payment, and capital growth is ignored

[Brealey et al 2008, chapter 5].

Other assumptions:

- managers of companies act to preserve the stability of dividend payouts
- managers act to preserve the capital of their companies
- liquidity, and so company size and book to market values are assumed to be irrelevant
- liquidity is constant throughout the modelling process
- risk is identical, and zero, for all companies in the model.
- Given above assumptions of zero risk and high liquidity; following Fama & French [Fama & French 1992], this leaves short term returns as the only factor that investors use to value companies.
- Note, are now looking at $K(j)$ and $w(j)$ (note, not $w(i)$)

Figure 2.2.1

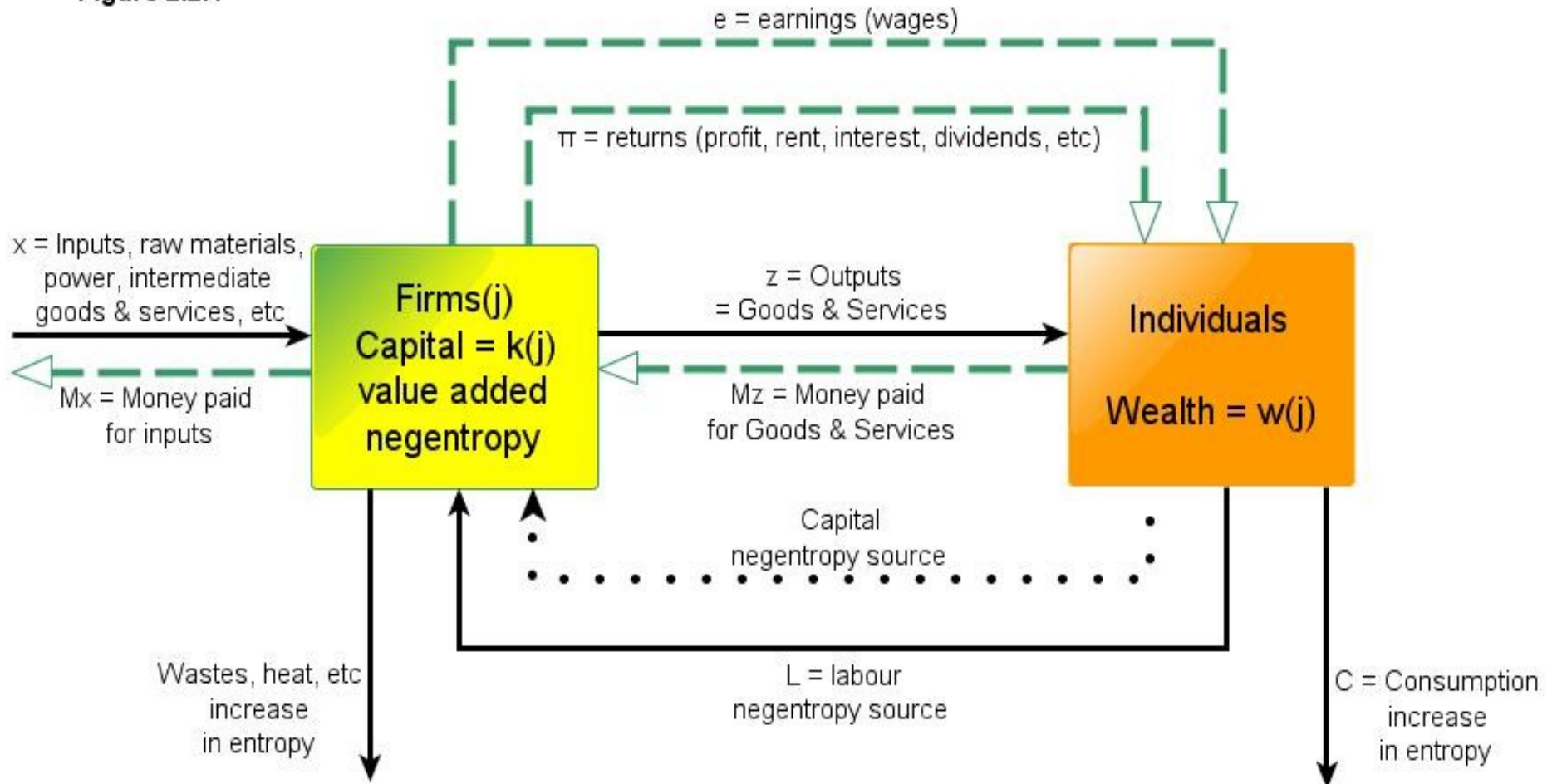
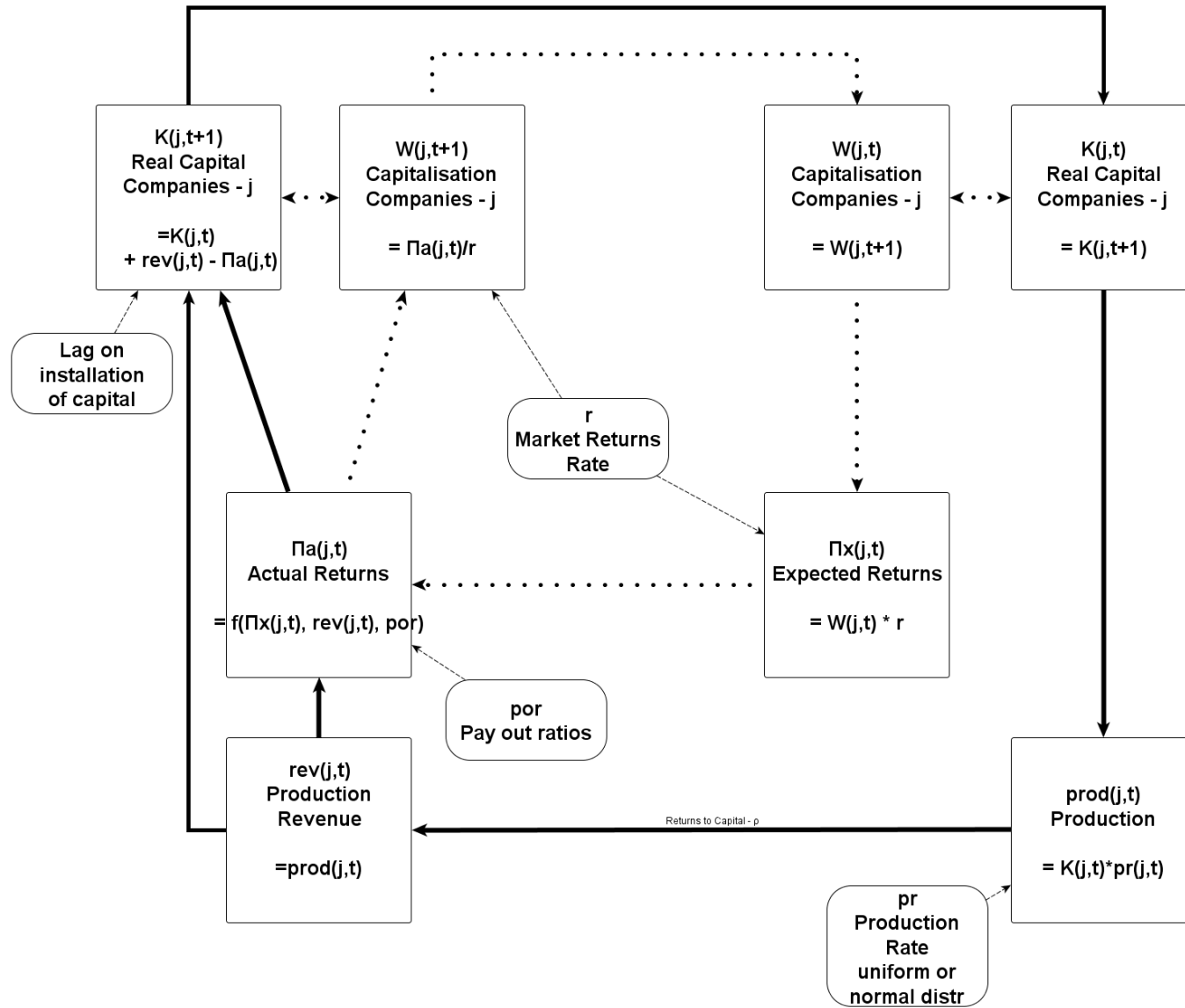


Figure 2.2.2
Companies Model
Iteration Diagram



Two cycles for capital:

- 'Sraffian', 'real' capital cycle – black heavy arrows
 - production of commodities by means of commodities
 - 'real' goods with intrinsic value
- 'Minskian', 'financial' capital cycle – in dotted arrows
 - valuation by revenue stream
- key box is 'Actual Returns' πx
 - function of 'real' production, and
 - function of 'financial' expected returns

- So:
 - $W(j,t)$ is a function of $K(j,t)$, and
 - $K(j,t)$ is a function of $W(j,t)$
- Gives a (General) Lotka-Volterra system with two different types of stock
 - Real capital K
 - Capitalisation W

- Labour and earnings ignored
- Production rate; pr - defined distribution – uniform or normal
 - Leontief coefficient
- market expected returns on capital; r – constant
- 'Capital hoarding' via 'payout ratios' – actual returns reduced to keep capital in company

Formula for iterations:

$$K_{j,t+1} = K_{j,t} + K_{j,t} \text{prodrate}_{j,t} - f([W_{j,t}r], [K_{j,t}], \text{por})$$

$$W_{j,t+1} = \frac{1}{r} f([W_{j,t}r], [K_{j,t}], \text{por})$$

Stock-flow diagrams (after Godley)

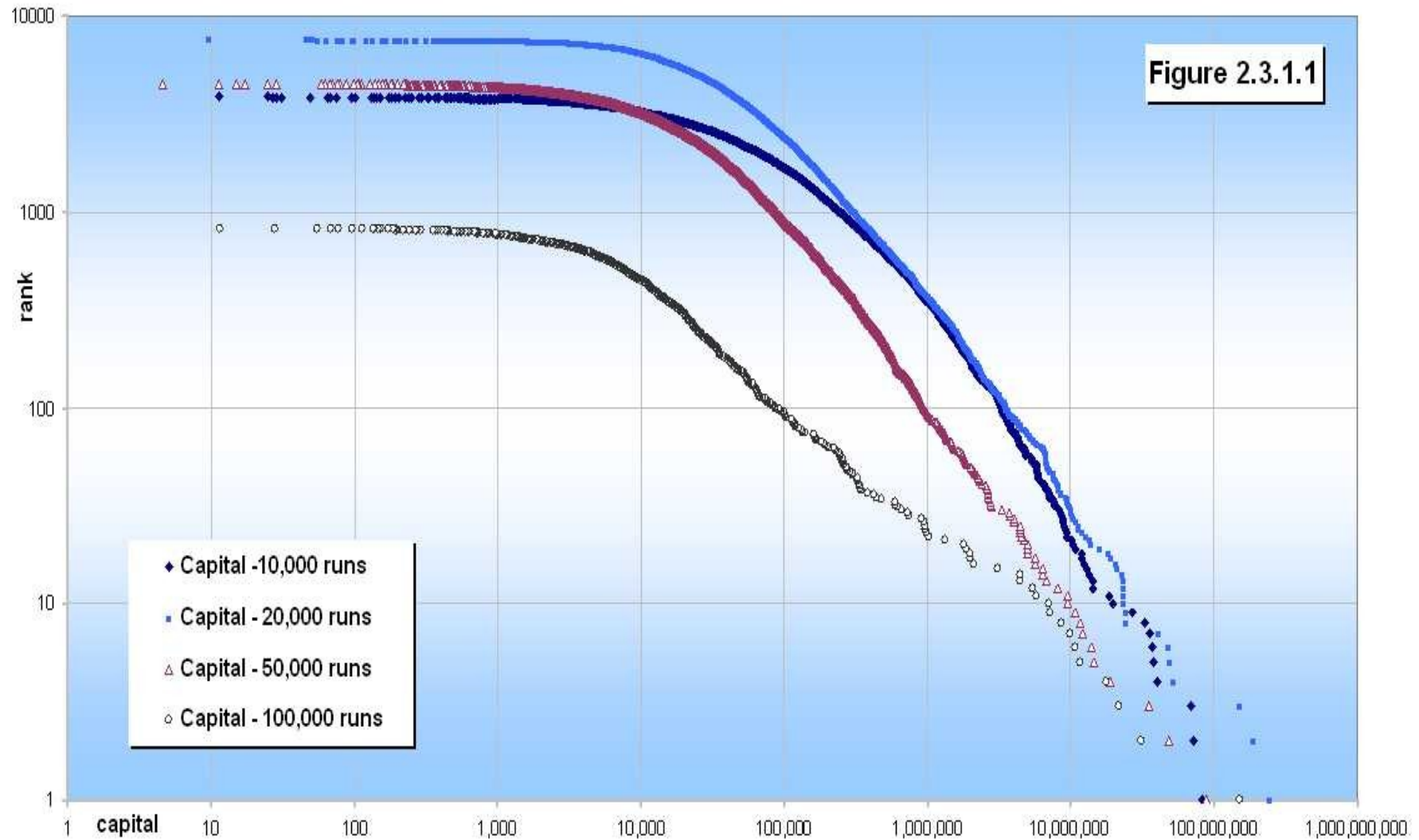
	Households			Firm - A				Firms - others			
	Real Capital	Current Account	Shares	Current Account	Inventories	Real Capital	Capitalisation	Current Account	Inventories	Real Capital	Capitalisation
Net Worth time=t	0	0	+Q _t	0	0	K _{A,t}	-Q _t	0	0	K _{oth,t}	na
Labour* earnings	-NEP _e = work	+e ^{expected}		-e ^e	+K _A .pr _e						
Capital Profits	-NEP _π	+π ^{expected} =Q _t r		-π ^e	+K _A .pr _π						
Consumption* (from earnings)											
Consumption (from profits)											
B2B											
Change in real Capital											
Net Worth time=t+1	0	0	+Q _{t+1}	0	0	K _{A,t+1}	-Q _{t+1}	0	0	K _{oth,t+1}	na

* line cancels to zero, NEP – negentropy, pr_π – is stochastic

* line cancels to zero, NEP – negentropy, S – sales, pr_n – is stochastic

	Households			Firm - A				Firms - others			
	Real Capital	Current Account	Shares	Current Account	Inventories	Real Capital	Capitalisation	Current Account	Inventories	Real Capital	Capitalisation
Net Worth time=t	0	0	$+Q_t$	0	0	$K_{A,t}$	$-Q_t$	0	0	$K_{oth,t}$	na
Labour* earnings	$-NEP_e$ = work	$+e$		$-K_A \cdot pr_e$	$+K_A \cdot pr_e$						
Capital Profits	$-NEP_n$	$+n^{actual}$ =f(Q,K,pr_n,por)		$-n^{actual}$ =f(Q,K,pr_n,por)	$+K_A \cdot pr_n$						
Consumption* (from earnings)	$+S_e$	$-C_e$		$+C_e$	$-S_e$						
Consumption (from profits)	$+S_n$	$-C_n$		$+C_n$	$-S_n$						
B2B				$+C_f$	$-S_f$			$-C_f$	$+S_f$		
Change in real Capital					$+\Delta IN$	$-\Delta K$			$-\Delta IN$	$+\Delta K$	
Net Worth time=t+1	0#	0	$+Q_{t+1}$ $+(n^a/r)$	0	0	$K_{A,t+1}$ $=K_{A,t} - \Delta K$	$-Q_{t+1}$ $-(n^a/r)$	0	0	$K_{oth,t+1}$ $=K_{oth,t} + \Delta K$	na

2.3.1 Model 2A Fully Stochastic on Production, No Capital Hoarding



- Production rate stochastic, same for each company over long run
- No capital hoarding
- **Companies are identical**
- **produces power tail distribution from identical companies**
- power tail correct value; approx = 1
- fails over long term, as companies removed from low end of distribution

- Capital hoarding produces stable models, but wrong power exponent
- Ian Wright models produce better outputs – from similar assumptions – uses company removal and formation

- Markets inefficient

- poorly performing companies downgraded until returns equals capitalisation – not eliminated

- value investors can spot these companies - Graham/Buffet strategy explained

Don't need:

- Economic growth
- Population changes
- Technology changes
- Different initial endowments (of capital)
- Shocks (exogenous or endogenous)
- Marginality
- Utility functions
- Production functions

3. Commodity models

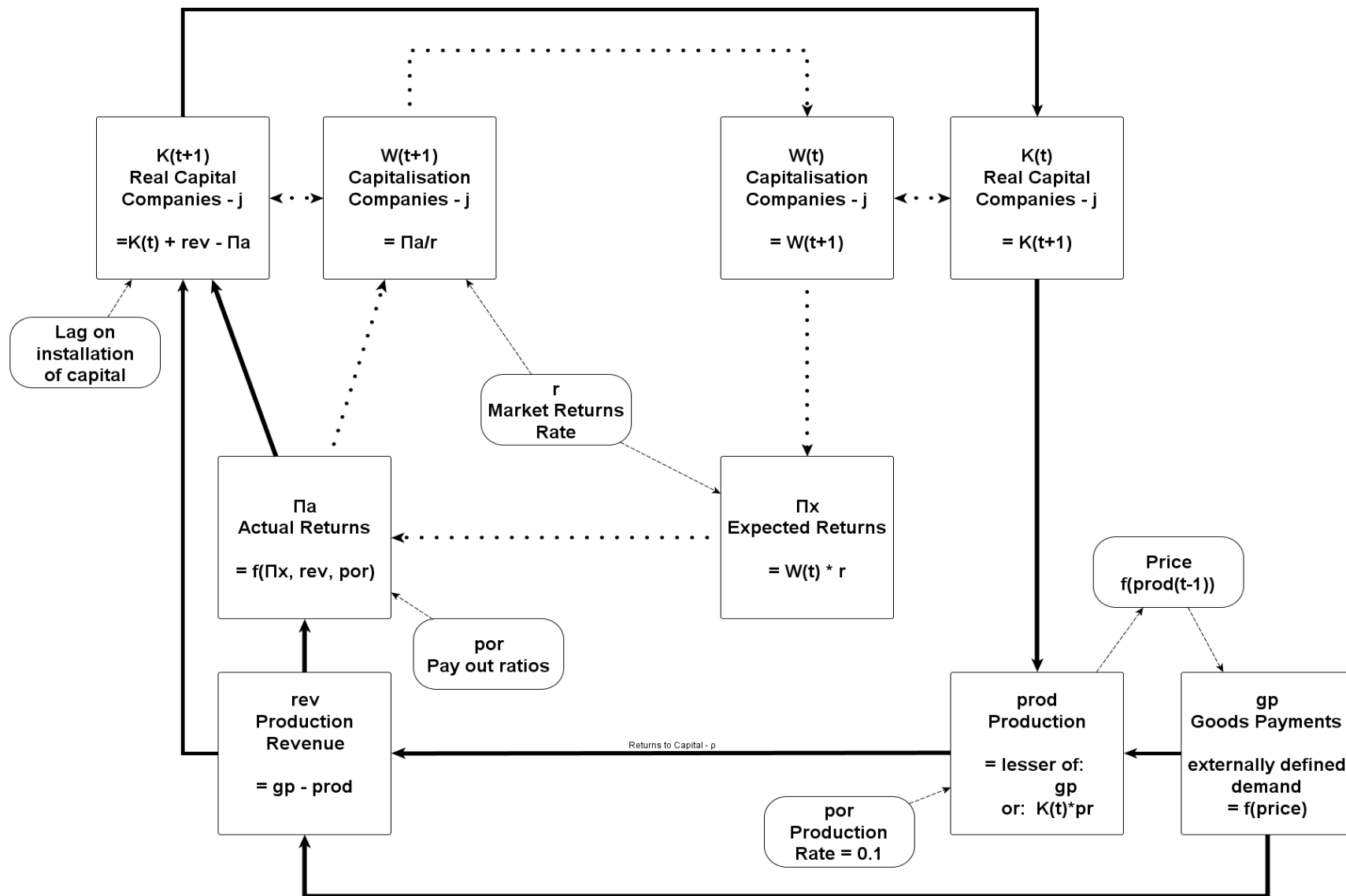
Commodities – oil, copper, coffee, etc

- characterised by long term low prices with occasional large spikes
- demand stable and price insensitive
- non-substitutable
- long delays to installing new capital

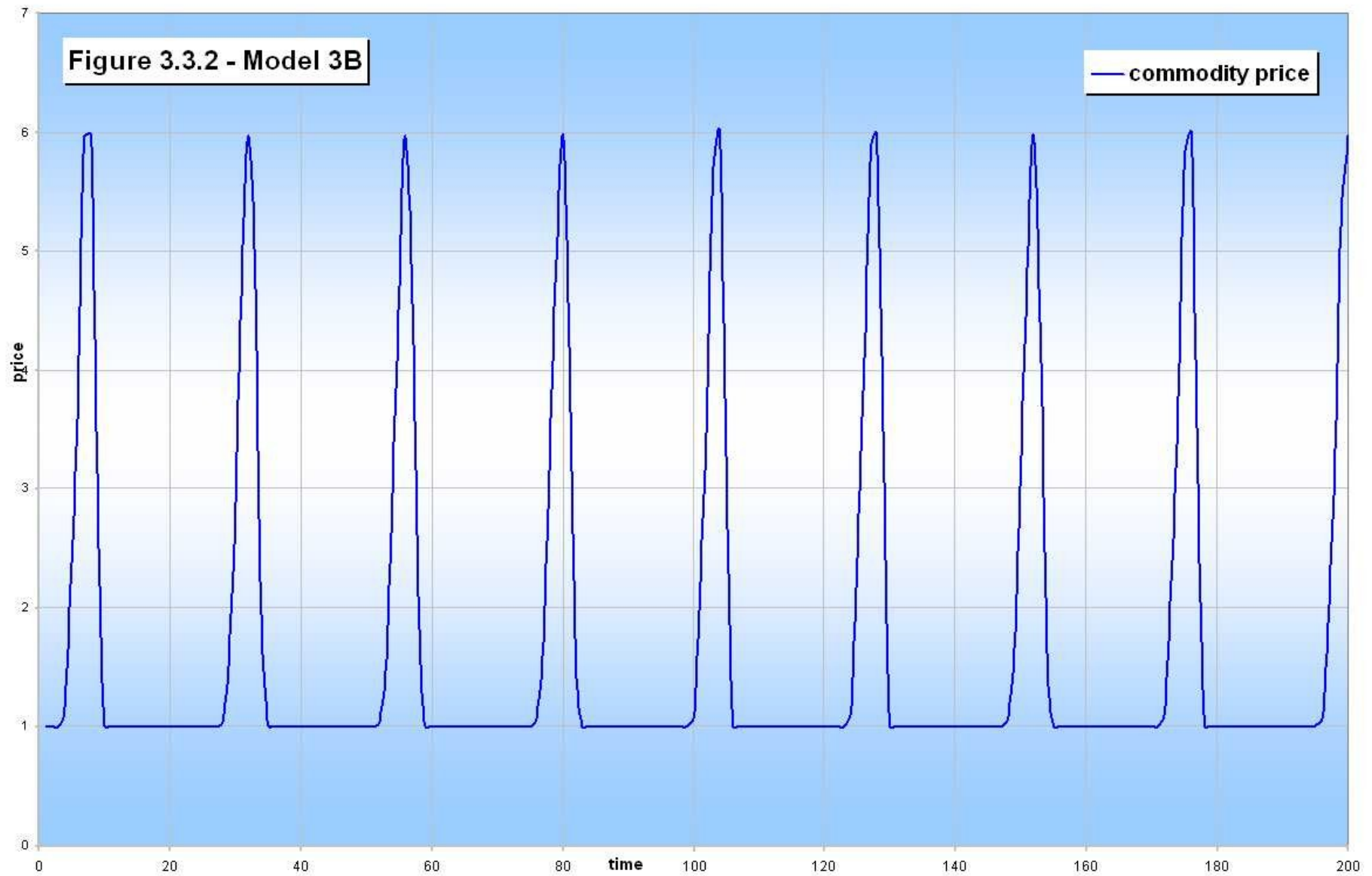
- need dynamics not comparative statics
- demand fixed constant

- use copper (not oil), so costs of commodity don't effect economy as a whole
- then input costs are independent of commodity price
- input costs, labour and machines, are linear function of production
- price is highly dependent on supply
 - if supply more than demand; price is input cost
 - if supply less than demand; price rises rapidly

Figure 3.2.2
Commodities Model
Iteration Diagram



Results – lag on installation of capital, no capital hoarding:



- unstable system
- wide cyclical variations in prices
- real price of copper, based on inputs, should be 1 unit
 - only at bottom of cycle.
- behaviour is chaotic, not stochastic. Random changes are generated endogenously. No stochastic generator in this model
- Lotka-Volterra model, not a General Lotka-Volterra model
- build up of capital is too much for the economy to support
- build up of capital in the commodity sector is inherently unstable

- problems are deep in the maths of the system
- blaming investors or speculators for misjudging their investments is as sensible as blaming foxes for procreating when there are a lot of rabbits

- Diminishing returns and marginality are not necessary.
- model does not average to the correct input prices even over the long term
- correct input prices are instead associated with the bottoms of the cycles.
- market is inefficient. Average prices are substantially higher than they would be if they had the opportunity to settle to long-term static 'cost-plus' prices.

4. Minsky goes Austrian à la Goodwin – Macroeconomic Models

Assume:

- produced goods have real intrinsic values
- market prices can vary from these values
- Consumption is a fixed proportion of consumers' paper wealth, as income models
- Companies have real capital which can produce a fixed proportion of output, and needs a proportional supply of labour, as all models above.

- price of paper wealth assets is defined by the preceding revenue stream; as in the myopic companies model above.
- management in companies can be capital preserving, as companies model above.
- can be delays in installing capital as commodities model above.
- population is constant
- technology productivity is constant. Production rate is fixed; production is proportional to real capital installed.

- Labour required is proportional to real capital installed
 - Leontief coefficient of production
- The price of labour is non-linear according to supply. That is real wage rates go up when there is a shortage of labour, and go down when there is a surplus of labour. Labour is a genuinely scarce resource.
 - Labour behaves like a 'commodity'

- If demand is less than amount of goods that can be provided (with current installed capital) then price of goods is input costs
- If demand is greater than amount of goods that can be provided, then cash paid is shared amongst goods available. So have temporary goods price inflation, and super-profits for companies – used to install more capital

- Consumers can receive more income than they spend in consumption, alternatively can spend less.
 - So have a cash-balance H for excess income
 - assumed held in non invested cash account
 - non realistic
 - if H +ve, consumers have spare cash
 - if H -ve, consumers have debts
 - H is Godleian buffer / float variable
 - allows imbalances generated to be accommodated
- total wealth W is the sum of the capitalisation Q and cash-balance H , so:

$$C = W \Omega \quad (4.2a) \quad \text{or:}$$

$$C = (Q+H) \Omega \quad (4.2b)$$

	Households			Firms			
	Real Capital	Current Account	Shares	Current Account	Inventories	Real Capital	Capitalisation
Net Worth time=t	0	$+H_t$	$+Q_t$	0	0	K_t	$-Q_t$
Labour earnings	$-NEP_e$ = work	$+e$		$-e$ $=f(K, K^2)$	$+K.pr_e$		
Capital Profits	$-NEP_n$	$+n^a$ $=f(Q, K, K^2, pr_n, por)$		$-n^a$ $=f(Q, K, K^2, pr_n, por)$	$+K.pr_n$		
Consumption	$+S$	$-C$		$+C$ $=W.\Omega$	$-S$		
Change in real Capital					$+\Delta IN$	$-\Delta K$	
Net Worth time=t+1	0	$+H_{t+1}$ $=H_{t+1}+e+n^a-C$	$+Q_{t+1}$ $+(n^a/r)$	0	0	K_{t+1} $=K_t-\Delta K$	$-Q_{t+1}$ $-(n^a/r)$

- Model can show stable or complex behaviour:

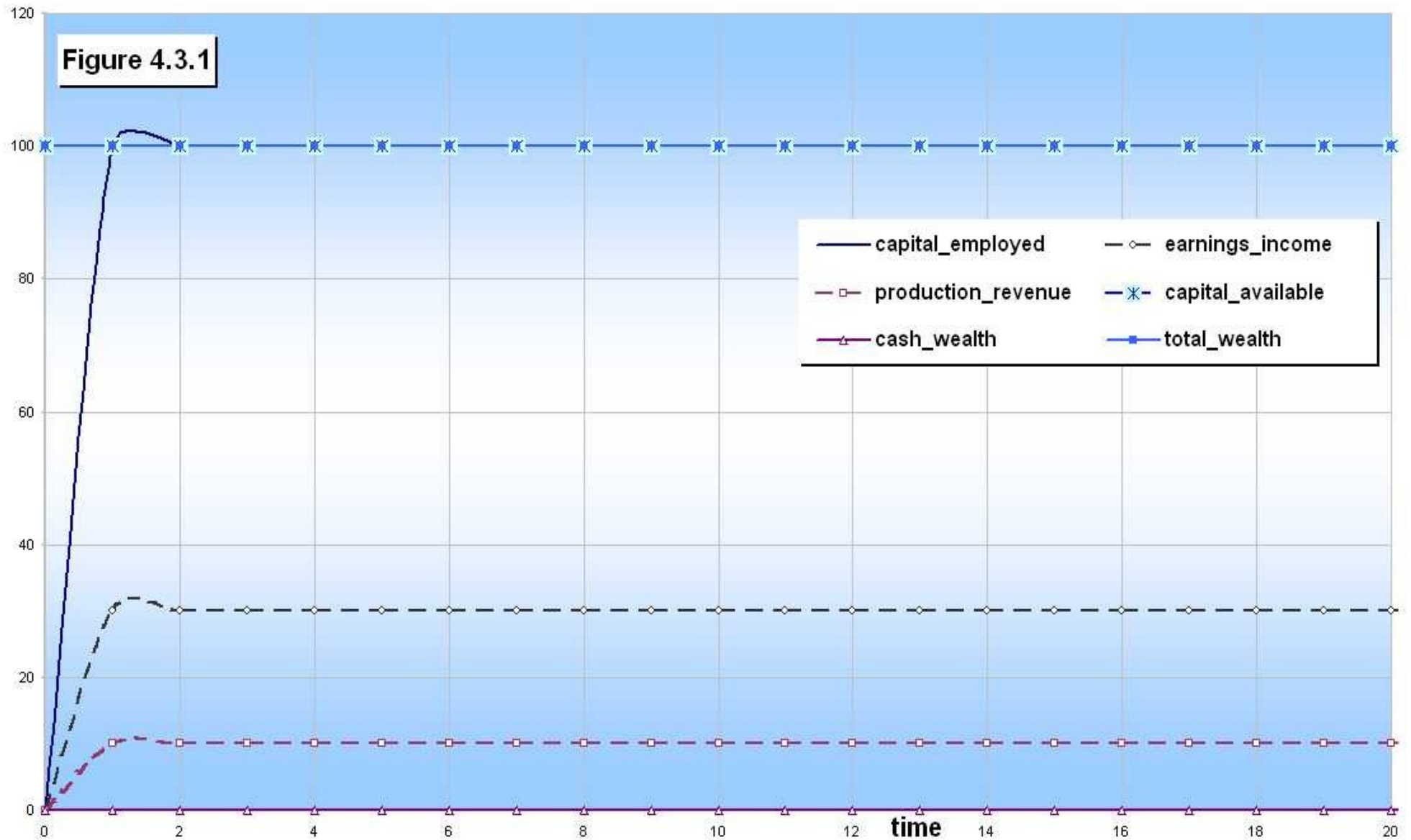
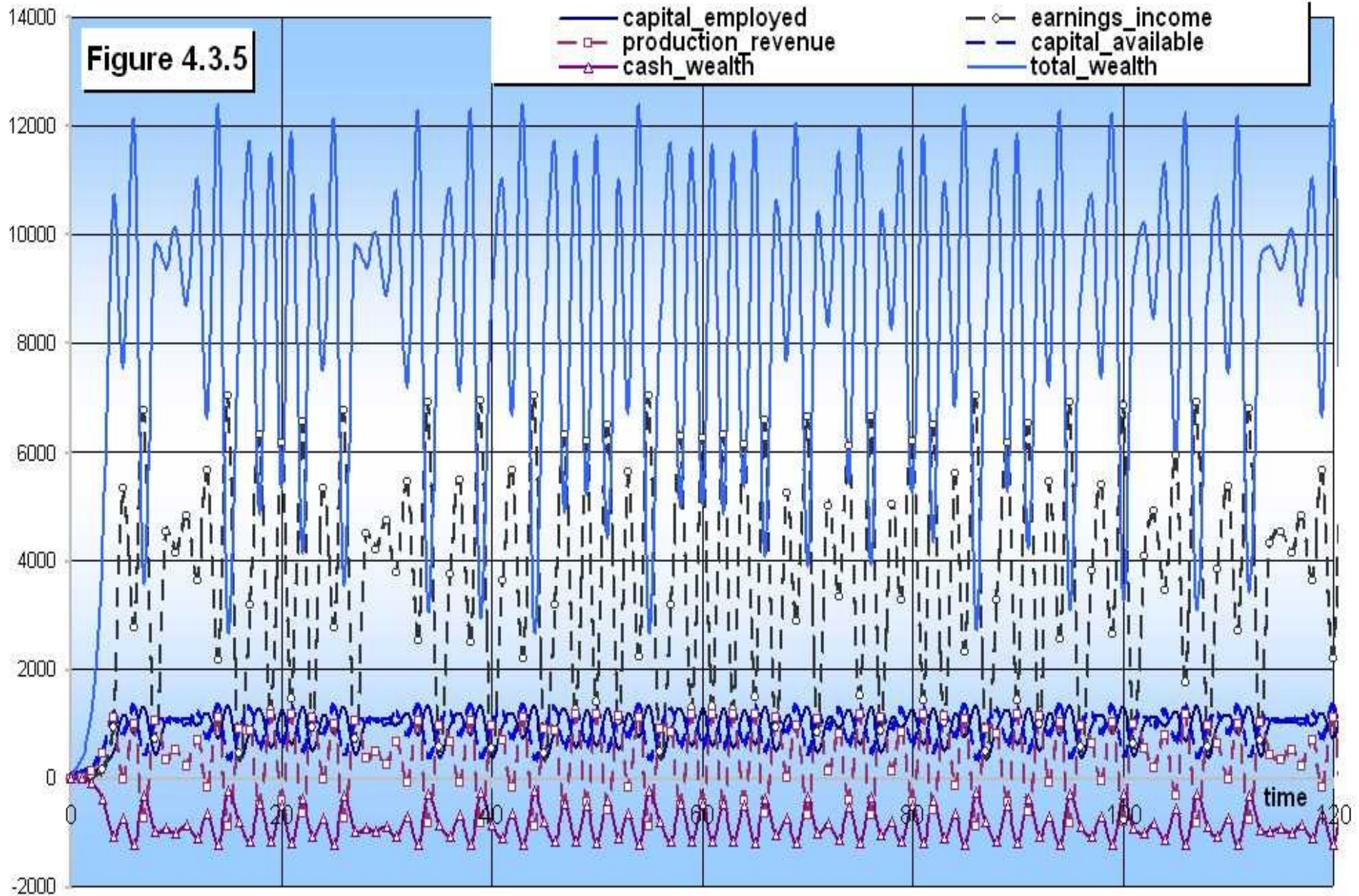


Figure 4.3.5



- even with simple behaviour long term equilibrium can be very different according to initial conditions
 - Dynamic systems can have many different equilibria
 - Keynes was right – economy does not balance automatically

Bowley ratios from the models:

Figure 4.3.7	β
Model 4A	0.75 (exactly)
Model 4B	0.92
Model 4C	0.78
Model 4D	0.85

Not far off reality

Don't need:

- Stochasticity (in any form) (model is chaotic, not stochastic)
- Economic growth
- Population changes
- growth in labour force
- Technology changes
- Productivity growth
- Investment
- Saving
- Accelerators
- Multipliers

(cf Goodwin)

Also don't need:

- inflation (long-term)
- Shocks (exogenous or endogenous)
- Different initial endowments (of capital or wealth)
- Utility functions
- Production functions
- governments
- fiat money
- fractional reserve banking
- speculators
- Ponzi finance
- debt deflation

(cf Neoclassical / RBC / Austrian / Minsky)

- **Basic instability due to pricing of paper assets on past / predicted future cash flow**
 - **So W can be different to K**
 - Minsky and the Austrians were right
 - Creation of liquidity and monetary growth are endogenous to the basic pricing mechanisms of the finance system.
 - Endogenous creation of financial wealth feeds back into the creation of more real capital
 - so creating more financial wealth.
 - So creating more real capital,.....etc
 - endogenous creation of financial wealth gives apparently secure paper assets against which debt can be secured
 - debt allows yet more capital creation.

4.5 A Present for Philip Mirowski? – A Bowley-Polonius Macroeconomic Model

"I mean the stability of the proportion of national dividend accruing to labour, irrespective apparently of the level of output as a whole and of the phase of the trade cycle. This is one of the most surprising, yet best-established, facts in the whole range of economic statistics.....Indeed... the result remains a bit of a miracle." [Keynes 1939]

"...no hypothesis as regards the forces determining distributive shares could be intellectually satisfying unless it succeeds in accounting for the relative stability of these shares in the advanced capitalist economies over the last 100 years or so, despite the phenomenal changes in the techniques of production, in the accumulation of capital relative to labour and in real income per head." [Kaldor 1956]

- The source of the value of the Bowley ratio in the model was investigated empirically
- while holding the cash balance at zero; the following formula was 'discovered' from the model:

$$\begin{aligned}\beta &= \text{Bowley ratio} \\ &= 1 - \frac{r}{\Omega} \quad (4.5k)\end{aligned}$$

- this can be derived trivially:

$$Y = e + \pi \quad \Rightarrow \quad \text{by definition, so:} \quad e = Y - \pi$$

$$\frac{e}{Y} = 1 - \frac{\pi}{Y} \quad \Rightarrow \quad \text{so:} \quad \beta = 1 - \frac{\pi}{Y} \quad \text{but:}$$

$$\text{Consumption} = \text{Income} \quad \Rightarrow \quad C = Y \quad \text{so:}$$

$$\beta = 1 - \frac{\pi}{C} \quad \text{or:}$$

$$\beta = 1 - \frac{\pi/W}{C/W} \quad \text{so:}$$

$$\beta = 1 - \frac{r}{\Omega}$$

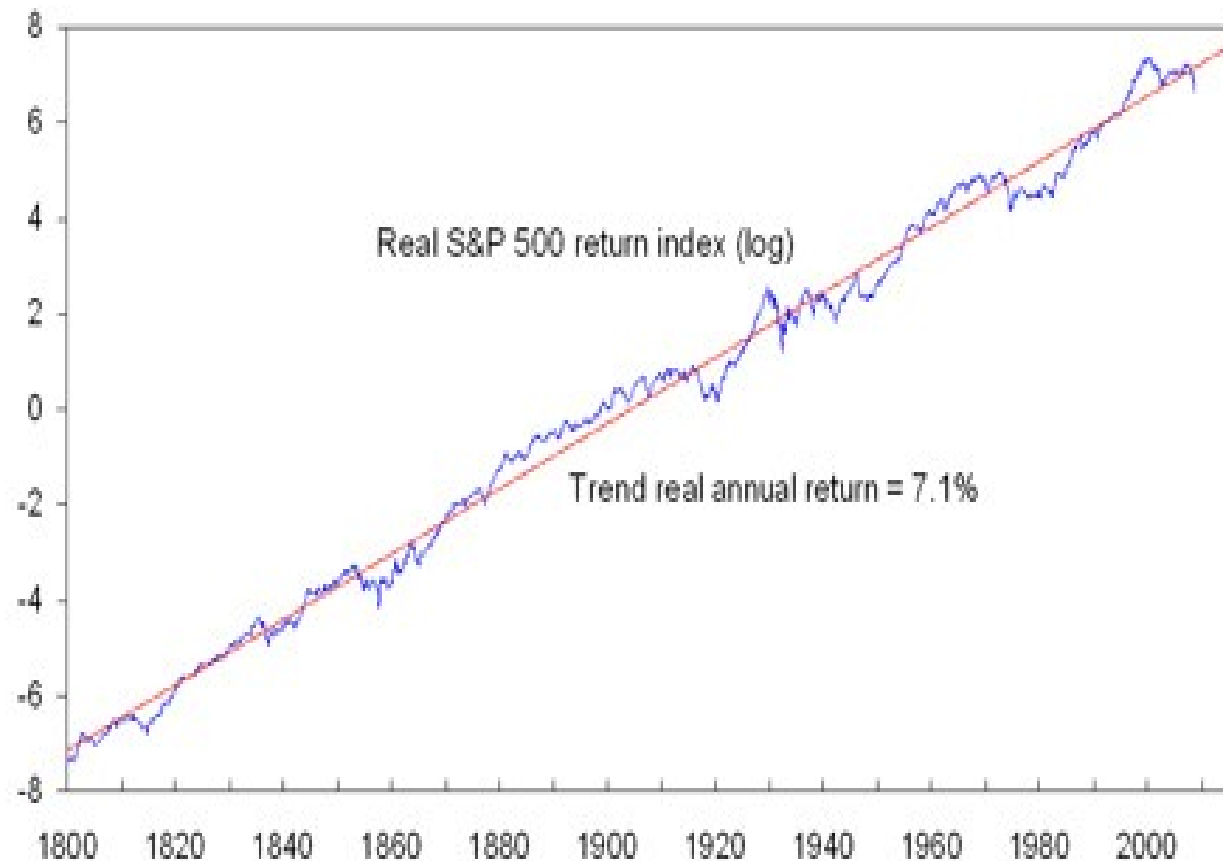
- The proportion of returns to labour is determined macroeconomically by consumption / savings rates.
- Not determined by production functions

- the consumption rate Ω defines Γ ; the ratio of total income to capital.
- r is smaller than Ω - gives Bowley ratio between 0.5 and 1.0 – matches real values

- Consumption rates known exogenous variables
 - life cycle theory – save for pensions

- Real returns known to be very stable over long term
 - though reasons not clear

LONG-TERM STOCK MARKET REAL RETURN



Source: Global Financial Data & New Star estimates

- If r and ω fixed (and also zero debt) then Bowley fixed by eqn:

$$\beta = 1 - \frac{r}{\Omega}$$

challenge to economists – is the maths flawed:

$$Y = e + \pi \quad \Rightarrow \quad \text{by definition, so:} \quad e = Y - \pi$$

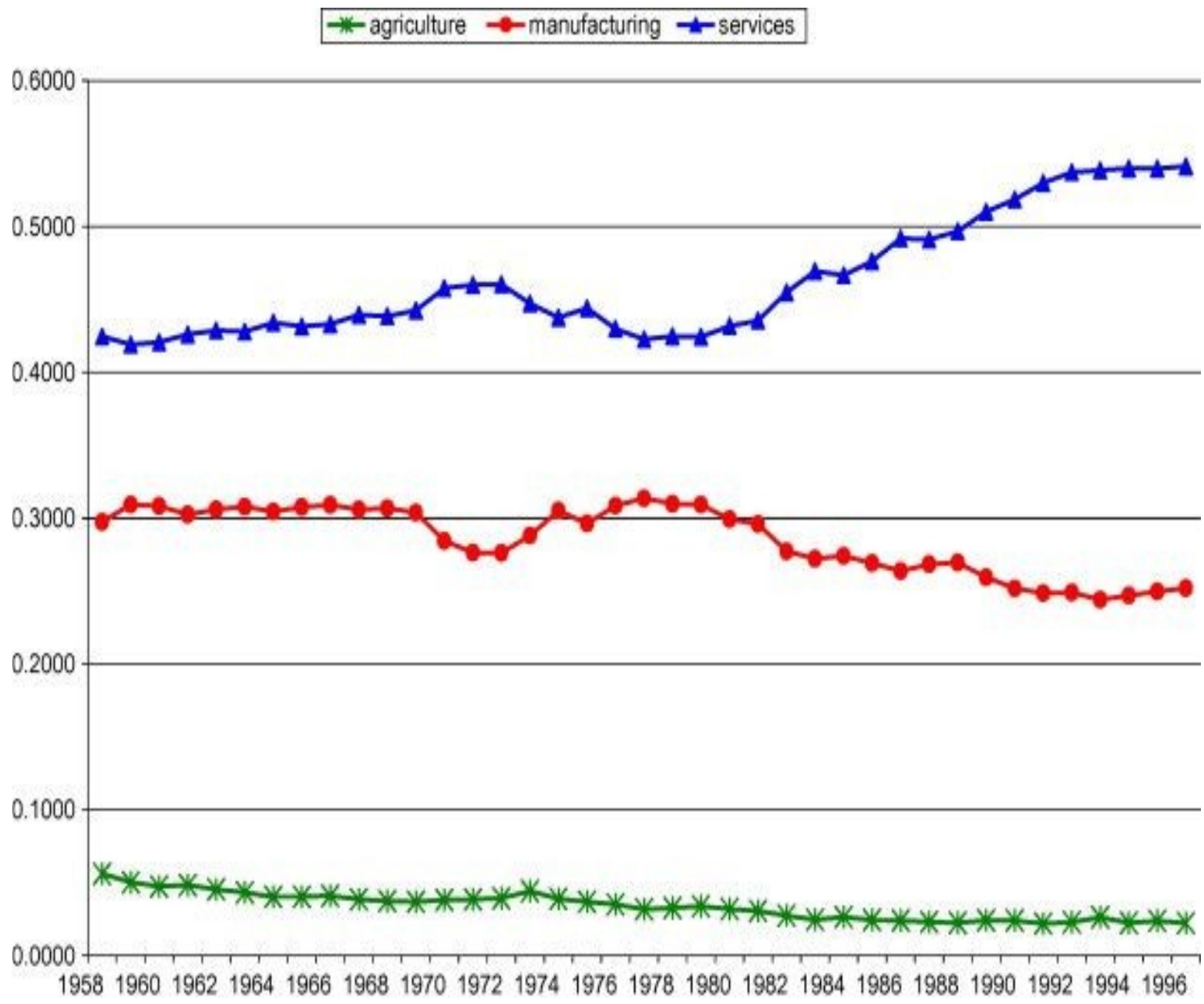
$$\frac{e}{Y} = 1 - \frac{\pi}{Y} \quad \Rightarrow \quad \text{so:} \quad \beta = 1 - \frac{\pi}{Y} \quad \text{but:}$$

$$\text{Consumption} = \text{Income} \quad \Rightarrow \quad C = Y \quad \text{so:}$$

$$\beta = 1 - \frac{\pi}{C} \quad \text{or:}$$

$$\beta = 1 - \frac{\pi/W}{C/W} \quad \text{so:}$$

$$\beta = 1 - \frac{r}{\Omega}$$



- data from Young [Young 2010] shows
 - the relative shares accruing to labour and capital can change quite significantly within individual sectors
 - shares to labour have decreased significantly in manufacturing and agriculture
 - meanwhile shares to labour have increased significantly in services
 - overall shares to labour fairly constant
 - all the cappucino bars and hairdressers have been created to keep the shares to labour in balance with consumption rates

The Bowley Ratio

Some consequence of this derivation

- At the level of the whole economy
 - labour and capital are NOT interchangeable factors
 - for a fixed amount of capital, at a fixed level of productive technology
 - There is a fixed amount of labour required
 - labour above this requirement will NOT be employed
 - no matter how far they drop their wage requirements

- Good consequences
 - share to labour 2/3rds
 - prevents all income accruing to capitalists
 - most goes to labour
 - prevented Marx's predictions coming true
- Bad consequences
 - share to labour 2/3rds
 - human labour required to generate wealth
 - keeps capital installation low
 - inefficient – better to use machines to produce goods & services
 - means we are all too busy – prevents leisure time
 - nb. Marx 2/3rds right about labour as source of value
 - remains so to this day

4.6 Unconstrained Bowley Macroeconomic Models

- If the cash-balance balance is allowed to change from zero, then Bowley ratio given by:

$$\begin{aligned}\rho &= \frac{rQ}{\Omega(Q + H)} \\ \beta &= \frac{\Omega + \Omega(H/Q) - r}{\Omega + \Omega(H/Q)} \\ &= \frac{1 + (H/Q) - (r/\Omega)}{1 + (H/Q)}\end{aligned}\quad (4.6a)$$

- also trivial to derive from basic algebra

- If the cash balance is positive and increasing; Bowley ratio heads closer to unity, good for workers, bad for capitalists.
- if H is negative (a debt) and the size of the debt is increased, then the size of both the numerator and denominator reduce, however the value of the numerator reduces more rapidly than the size of the denominator, and the Bowley ratio slowly decreases.
- (At least at first.)

- If debt is allowed to continue increasing, then the Bowley ratio drops rapidly to zero, and then shortly afterwards heads off to negative infinity.
- In the model it isn't possible to reach these points; as the Bowley ratio heads to zero the model becomes unstable, and explosive
 - the economy blows up in an bubble of excess real capital and even more excess debt.
 - *This may sound familiar.*

Note also:

- value of H has a direct effect on Bowley ratio β in eq 4.6(a)

$$\begin{aligned}\beta &= \frac{\Omega + \Omega(H/Q) - r}{\Omega + \Omega(H/Q)} \\ &= \frac{1 + (H/Q) - (r/\Omega)}{1 + (H/Q)}\end{aligned}\quad (4.6a)$$

- β has a direct effect on alpha, the exponent of the power tail in the wealth distribution in eq 1.6(e).

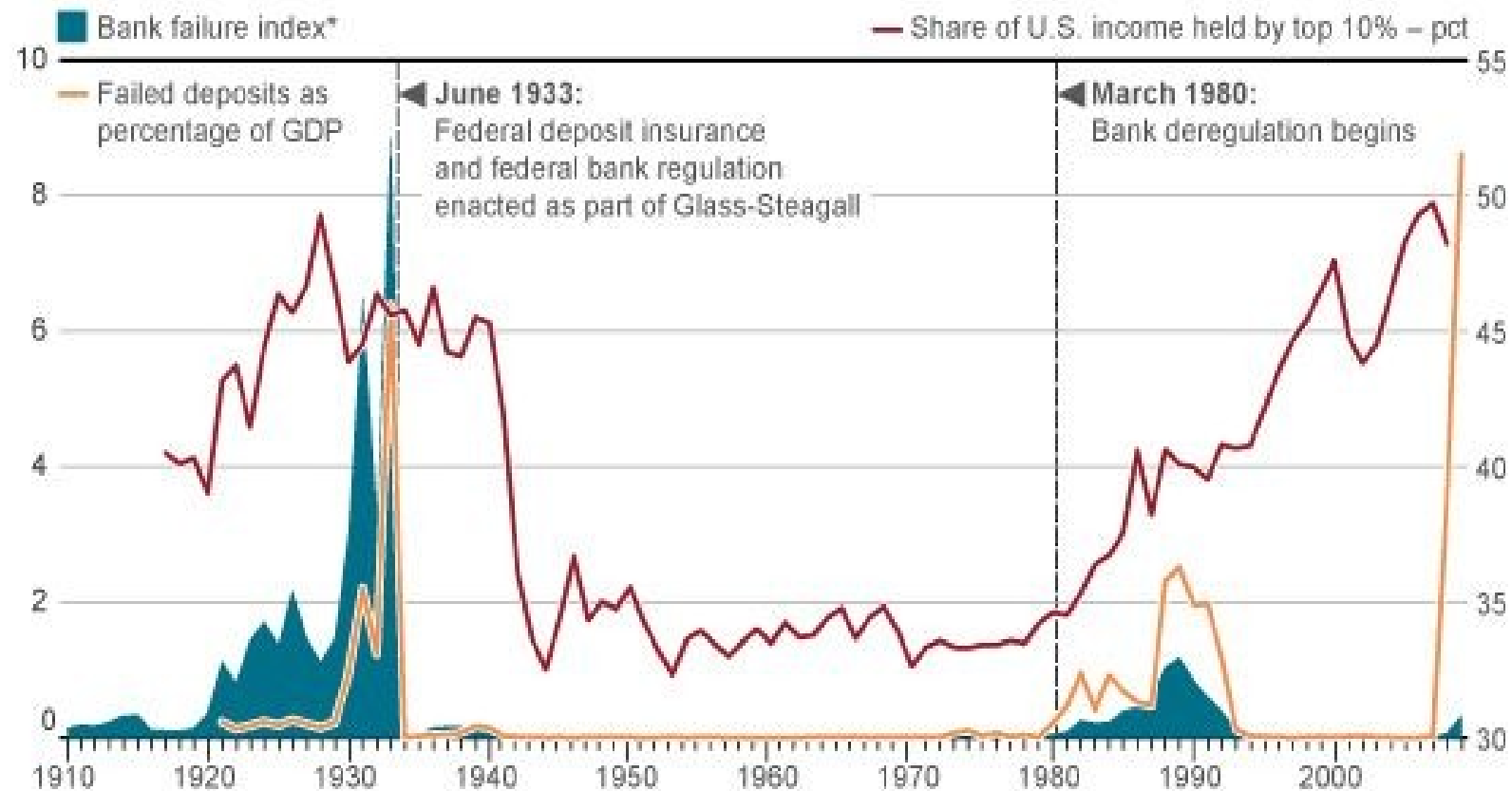
$$\alpha = \frac{1.36(1 - \rho)}{v^{1.15}} \quad (1.6d)$$

$$\alpha = \frac{1.36\beta}{v^{1.15}} \quad (1.6e)$$

- So, levels of debt have a direct effect on wealth inequality

U.S. bank failures, regulation and inequality

Research showing inequality rises during periods of minimalist bank regulation raise questions about who benefits from deregulation.



* Scales bank failures to chart size

Source: David Moss - Harvard Business School, U.S. Bureau of Economic Analysis, FDIC



- More realistically, do something with the cash H, change to savings S
- Banks take in savings S and lend to companies.
- Receive interest on loans at R
 - (equivalent rate to dividends $R = \Pi/Q$)
- Pay interest to public at r.

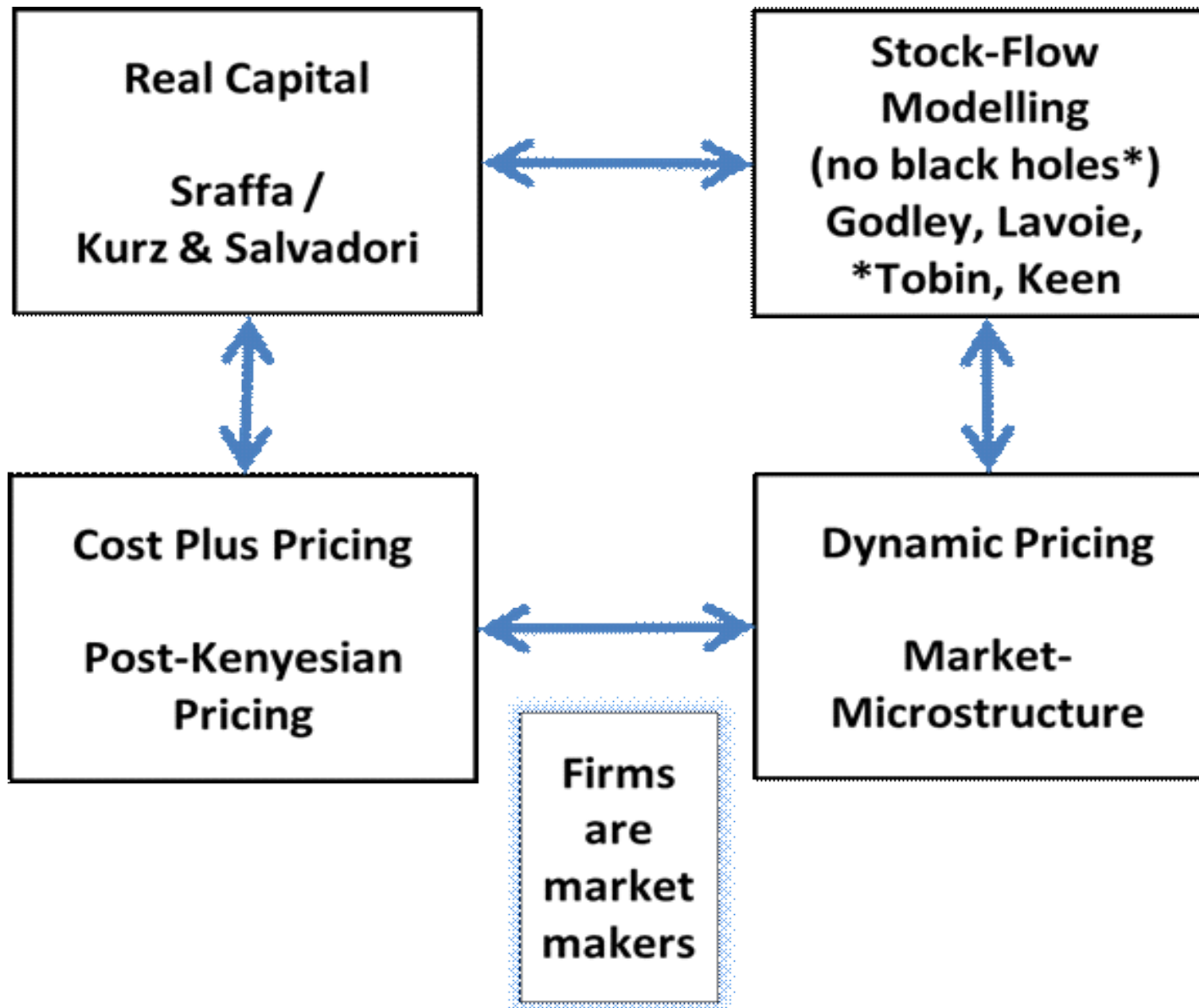
Then:

$$\rho = \frac{RQ + rS}{\Omega(Q + S)}$$

- Same conclusions arise regarding debt as above.
- Investment and Saving is a secondary loop

Future

Sraffagodlean Economics



Firms are Market-Makers

Firm - Retailer*

Post-Keynesian Pricing theory

Buys and sells goods

Has margin between buy & sell
(mark-up)

Margin large; wages, rents, etc

Uses stocks levels to guide prices

Uses flow quantities to guide prices

Market doesn't formally close

Firm tries to stabilise prices

Market-Maker

Market-Microstructure

Buy and sells stocks

Has margin between buy & sell
(margin)

Margin small, admin only

Uses stocks levels to guide prices

Uses flow quantities to guide prices

Market doesn't formally close

MM tries to stabilise prices

* manufacturer the same – only goods sold different to those bought – so margin is larger

Conclusions – Modelling

- Simple model explains
 - wealth / income distributions
 - company size distributions
 - macroeconomic cycles
 - ratio of returns to capital and labour
- Biologists have the right models

Conclusions - Policy

- The poor should be encouraged to save
- The poor should be forced to save

- The rich should be encouraged to spend*

- Income tax should depend on wealth – not income

* rich means total assets greater than £500k

- £100k for house – based on 2 sharing
- £400k gives annual income of £24k @ 6% return on investments
 - cf – 7% long-term above
- £500k in UK – don't need to work

Lord Finchley

Lord Finchley tried to mend the Electric Light
Himself. It struck him dead: And serve him right!
It is the business of the wealthy man
To give employment to the artisan.

Hilaire Belloc

Further reading:

- econodynamics.org