HETERODOX PRODUCTION AND COST THEORY OF THE BUSINESS ENTERPRISE

By

Professor Frederic S. Lee

2009 Conference of the Association for Heterodox Economics

9-12 July 2009

Professor Frederic S. Lee
Department of Economics
University of Missouri-Kansas City
5100 Rockhill Road
Kansas City, Missouri 64110
USA
E-mail: leefs@umkc.edu

HETERODOX PRODUCTION AND COST THEORY OF THE BUSINESS ENTERPRISE

Heterodox economists long complained about having no systematic alternative to neoclassical production and cost theory. This paper deals with this complaint. That is, it presents a theory of production and costs of the business enterprise that is a complete alternative to the neoclassical theory of production and costs of the firm. The business enterprise produces an array of outputs, that is, goods and services or product lines. A product line may consist of a single main product with numerous derivative but secondary and/or by-products; or it may consist of a conceptually distinct product that is a differentiated array of products. In either case, the structure of a single product in a product line is hard to isolate because fixed investment goods and labor power skills may be used to produce more than one product; and the costing of the product is difficult because of the problem of allocating various common shop costs. To overcome this, the product line is defined in terms of its core or main product—that is, a product line consists of a single homogeneous product. As a going enterprise, when producing any product line, it engages in sequential acts of production through historical time and as a result incurs sequential costs of production also through time. These acts of production and the costs incurred in producing a product line are determined by the underlying relatively enduring structures of production and costs. The structure of production consists of plant segments-plant, shop technique of production, and the enterprise technique of production; and correspondingly, the cost structure of the product line consist of direct costs, shop expenses and enterprise expenses (jointly called non-direct costs). The basic analysis of the structure of production and costs is a two dimensional comparative analysis in which production and costs are examined relative to different flow rates of output (or degrees of capacity utilization). Hence it concentrates on the "virtual" movement of inputs and costs and the flow rate of output. From this it is possible to delineate a time-oriented structure of production and costs with regard to sequential and different variations in the flow rate of output. Thus the starting point are the structures of production and costs as they relate to plant segments, plants, and direct costs; followed by delineating the structures of production and costs regarding the shop and enterprise techniques of production and shop and enterprise costs, and the structure of production and average total costs for the product line; and concluding by utilizing the structures of production and costs to analyze sequential acts of production and costs through time.

Accounting Rules

The business enterprise adopts and develops cost and financial or, more generally, managerial accounting practices that are necessary for it to be a going concern. So long as the enterprise remains a going concern, its accounting practices remain relatively enduring, although changing in minor ways in light of changes in technology, inputs used in production, and the information needs of management. If an enterprise is not a going concern, it is a terminal venture in that it has a specific starting and ending date. Consequently, accounting for expenditures as deductions or one-time expenses against revenue, revenue, and business income is straightforward. Moreover, the question of the value of the fixed investment goods and depreciation never arises. That is, the fixed investment goods are valued at the beginning of the venture and then revalued at the terminal date. Their initial value is their historical costs while their liquidation value at the terminal date is added to the profit account for distribution (Litherland 1951). An enterprise as a strictly terminal venture is largely incommensurate with a going concern economy; rather it is compatible with an exchange economy where repeatable and ongoing economic activities and provisioning *process* are absent. As for the going concern, the accounting practices must ensure an accurate delineation of costs which must be recovered if the enterprise is to be a going concern. More specifically, because a going enterprise engages in continuous sequential acts of production, its income or profits has to be calculated periodically, which is denoted as the *accounting period* and is generally taken to be a calendar year, and in a manner that permits distributing part of it as dividends without impairing the enterprise's productive capabilities. This means it is necessary to treat inputs (which are producible and reproducible) that contribute to the production of the output as reoccurring costs as oppose to one-time expenses against total revenue to arrive at profits.[1] In this manner, the expenses of resources, goods, services, labor power skills, depreciation of fixed investment goods used directly and/or indirectly in production are costs that are recouped so that the enterprise can repeat production. This implies that the fixed investment goods are not viewed as commodities to be sold on the market for revenue purposes; rather as a going concern, enterprises view them as essential non-commodities for maintaining the going plant whose historical value is considered a recoverable cost to be changed against revenue before determining business income.

The accounting practices essential to a going concern deal with (1) the tracing of the direct and overhead material, services, resources, and labor skills inputs relevant to the production of a unit of output, (2) the categorization of costs into direct (variable) and overhead (fixed) costs, (3) the determination of the cost of producing a unit of output, (4) depreciation, and (5) the determination of profits associated with a particular product and the business income for the enterprise as a whole. Evidence from archives of business enterprises show that prior to 1700 merchants utilized

[1] That is, costs are defined in terms of the going enterprise, so that what constitutes costs are reoccurring expenses derived from the use of reproducible intermediate inputs, labor power skills, and fixed investment goods. Such costs are objective and irreducible to a homogeneous unit such as labor or subjective disutility. Moreover, non-produced items that are not utilized on a reoccurring basis are not costs but expenses that are charged against revenue. Therefore, scarce factor inputs are not costs in the context of the going enterprise, which means that the category of costs of the going enterprises is conceptually distinct from the category of costs in neoclassical theory in that the

accounting systems to keep records of purchases and sales; and after that, industrial enterprises drew on these systems to keep records of purchases and sales, and to document the internal movement of inputs in the production process. In particular, sophisticated cost accounting systems for tracking direct inputs and direct costs in the production of a specific good have been in use since the 1700s. At almost the same time, enterprises developed accounting procedures that differentiated between direct and overhead inputs and costs, began identifying and measuring/quantifying them, and devised procedures to allocate the overhead costs among the various goods produced.[2] Thus by 1900 comprehensive accounting systems of various degrees of sophistication were in general use and remain so to the present day. With developed cost accounting systems in hand, enterprises are able to engage in costing of a good, that is, to arrive at its unit (or average) direct or direct plus overhead cost.[3] Costing systems utilized historical-estimated, or standard costs and employed various methods (based on, for example, output, direct costs, direct labor costs, labor hours, or machine hours) for the allocation of overheads.[4] However, changes in technology, the production of new goods and

---

former is not based on relative scarcity. [Kurz and Salvadori, 2005]

[2] The term 'direct' input/cost refers to inputs and costs that are directly associated with the production of a good; while 'overhead' input/cost refers to an input/cost that is not directly associated with the production of a good. Direct and overhead are not the same as variable and fixed; and accountants and business enterprises generally did not use those concepts until the latter part of the twentieth century, when the accounting profession began acquiring the concept from economics. While not identical, they are in practice pretty much the same and for theoretical purposes with regard to pricing and the determination of profits, the differences are not important. Thus for this book, direct and overhead will be used.

[3] The costing of unit direct costs (or what is called 'direct' or 'marginal' costing is done only under special circumstances, if the accounting procedures employed do not permit more detailed costing to take place, or if management is not interested in a better understanding of its costs. But in general, enterprises undertake total (absorption) costing which includes both direct and overhead costs.

[4] There are three types of costing procedures, historical-estimated and standard costing. In the former, costs are determined by methods that range from a perfunctory guess to a very careful computation based upon past experience; in either case, past costs are used as the basis to determine the costs of a good that will be produced in the future. In the latter, costs are determined in advance of production by a process of scientific fact-finding that utilizes both past experience and controlled experiments. However, in spite of the differences, both estimated and standard costing arrive at the

services, the need for new and better product line cost information, and competitive pressures have pushed enterprises to alter their cost accounting and costing systems although not significantly, but their function of collecting cost information and use for estimating product line costs has remained unchanged—as long as enterprises remain going concerns, costing accounting and costing systems will remain relatively stable and hence relatively enduring structures .[5] [Chatfield 1973; Garner 1954; Jones 1985; Boyns and Edwards 1995; Boyns, Edwards, and Nikitin 1997; Fleischman and Parker 1997; Lee 1998: Appendix A; Al-Omiri and Drury, 2007]

Business enterprises have always made financial decisions, such setting prices, whether to produce a good or close down a product line, or undertake an investment project; and tying costing systems to the financial decisions (which occurred as early as 1700) helped immensely in making the decisions. This long historical emergence was, in part, due to an interlinked problem qua controversy grounded in the nature of a going concern. In particular, profits are defined as the difference between revenue and costs for a particular period of time, such as the accounting period, but whether that definition is consistent with the nature of the going concern depends on how expenditures on fixed investment goods are accounted for. From 1700 into the early 1900s, expenditures on fixed investment goods paid for and expensed out of revenues or profits and not included as a cost component, that is depreciation, of a product. Being treated as a current expense and hence not added to the capital account, the capitalized value of the enterprise did not change. More significantly, it also meant that the enterprise's cost structure did not include all the costs to be a going concern—that is, it did not include the cost of the fixed investment goods needed for

---

costs of producing a good that will be used in setting the price in the same way. Hence in this chapter and the next reference will only be made to costing.

[5] In recent decades various studies have noted the relative stability in accounting practices used by enterprises. They show that enterprises slowly make marginal changes while retaining basic practices, even when faced with a changing environment. [Emore and Ness 1991; Bright et. al 1992;

ongoing and future production. So when the fixed investment goods wore out or became technologically obsolete and thus needed to be replaced, a 'cost-recovery' fund for their replacement purchase did not exist.

Enterprises dealt with the problem through adopting replacement accounting in which replacement (which could include repairs) investment was charged directly against revenues before profits were determined; having repairs to the fixed investment goods (which is a form of investment) charged directly against revenues before profits were determined; or establishing a depreciation fund of money based on assigned depreciation rates (based on reducing balance, straight line, or some other basis) to different categories of fixed investment goods based on their historical costs, which involved a charge against revenue before profits were determined or directly against profits.[6] However, the demand by shareholders of the enterprise for immediate dividends (which is their monetary access to the social provisioning process) irrespective of the negative impact on the going concern capabilities of the enterprise to provide an ongoing stream of dividends and hence an ongoing access to the provisioning process resulted in a change in the way expenditures on fixed investment goods were dealt with.[7] Instead of being expenses charged against revenue, they initially are expenditures out of profits that become a cost of production.[8] To include depreciation as a cost of production, it is first necessary to value the fixed investment goods, which is generally done at

---

Granlund, 2001]

[6] Allocations to the depreciation fund often varied directly with profitable years (Stone 1973-74; Edwards 1980).

[7] There was another controversy which involved whether 'interest' on the paid in 'capitalized value' of the enterprise was a cost or not. In some partnerships, interest charges were included as costs in order "to ensure that individuals were properly remunerated for differential capital contributions rather than to produce a more accurate costing of business operations' (Edwards 1989: 312; also see Stone 1973-74; Hudson 1977). While this case seems to be the basis of mainstream arguments that includes normal profits as costs, generally interest charges are not considered costs.

[8] This means that fixed investment goods are not seen as commodities to be sold to raise revenue, but as a cost of production to be recovered.

historical cost (that is in terms of money). Then a method of depreciation, such as straight-line or accelerated, is deployed to determine the amount of depreciation to be allowed as a cost of production. Once depreciation is a cost of production, the accounting working rules of the enterprise ensure that, in principle, all inputs are traceable, all costs identified and allocated, and the determination of business income or profits can be done without affecting the going plant of the enterprise.[9] [Edwards 1980, 1986, 1989; Boyns and Edwards 1997; Napier 1990; Tyson 1992; Fleischman and Parker 1997; Wale 1990]

Structure of Production and Costs

The business enterprise produces an array of outputs, that is, goods and services or product lines. As a going enterprise, when producing any product line, it engages in sequential acts of production through historical time and as a result incurs sequential costs of production also through time. These acts of production and the costs incurred in producing a product line are determined by the underlying relatively enduring structures of production and costs. The structure of production consists of plant segments-plant, shop technique of production, and the enterprise technique of production; and correspondingly, the cost structure of the product line consist of direct costs, shop expenses and enterprise expenses (jointly called overhead costs). The basic analysis of the structure of production and costs is a two dimensional comparative analysis in which production and costs are examined relative to different flow rates of output (or degrees of capacity utilization). Hence it concentrates on the "virtual" movement of inputs and costs and the flow rate of output. From this it is possible to delineate a time-oriented structure of production and costs with regard to sequential

[9] The old method of expensing the purchases of fixed investment goods meant that the capitalized value of the enterprise did not alter. Consequently, the concept of the rate of profit under this system had no precise meaning, making it useless as a theoretical concept. Although the introduction of depreciation partially redresses this issue, the use of historical cost makes the rate of profit a backward looking concept, hence ill-suited as a theoretical tool. [DEVELOP THIS]

and different variations in the flow rate of output. Thus the starting point are the structures of production and costs as they relate to plant segments, plants, and direct costs; followed by delineating the structures of production and costs regarding the shop and enterprise techniques of production and overhead costs, and the structures of production and costs for the product line; and concluding by utilizing the structures of production and costs to analyze sequential acts of production and costs through time.

Technology, Plants, and Direct Costs

The basic aggregate unit of production is an establishment which houses or encompasses the activities immediately involved in the production of the product line—it is denoted as the *plant*. Given the plant, production can be further delineated in that more than one plant may be used to produce the product line and/or that each plant may consist of a number of plant segments, each also capable of producing the product line. Whether the plant is an emergent technological establishment, divided into separate plant segments, or a hybrid of the two depends on the technology constituting the plant.[10] Although the production of a product line may consist of many processes and stages, the enterprise's cost accounting procedures are capable of tracking each direct intermediate and labor power input and their amount used in production. Consequently, the array of direct inputs used in production of a unit of output constitutes and represents the technology used in production.

Plant Segment, Plant, and the Structure of Production

For the segmented plant (SP), the primary unit of production is the *plant segment* (PS) which is defined as the technical specifications of direct intermediate inputs of resources, goods, and services and labor power skills needed to produce a given amount of output, *g*, of a product line in a specific period of time. This usage of direct inputs, is, in turn, uniquely determined by the specifications of the plant segment and underlying fixed investment goods (**K** = $k_1, \ldots, k_k$), and the social/labor conditions surrounding production. Moreover, the fixed investment goods used in production of *g* is uniquely related to it in that it is specifically tailored to produce *g* per period of calendar time. The period of time used in the specification of the PS is called the *production period* and it denotes the amount of calendar time needed to produce *g*, starting with the first input and ending with the output. Therefore, given the fixed investment goods and their operating specifications, the unit of output, and the production period, the plant segment is delineated as follows:

(4.1) Plant segment (PS): $g \leftarrow \sum_{\substack{i=1 \\ v=1}}^{h,z} a_i \times l_v \; : \; \mathbf{K} = k_1, \ldots, k_k.$

where a_i is a direct intermediate input technical coefficient and is the amount of the i-th input needed

to produce g amount of output;

l_v is a direct labor power input technical coefficient and is the amount of the v-th labor power input needed to produce g amount of output; and

g is the flow rate (or amount of) of output per production period.

Since each PS is a recipe of rigidly fixed ingredients for producing a single batch or amount of output—that is each technical coefficient is fixed, it is impossible for any one PS to produce more than *g* per production period. Consequently, to increase flow rate of output of a product line at a point in time, the enterprise must bring on line additional plant segments complete with their specific complement of fixed investment goods, implying that the plant consists of more than a single plant

---

[10] For evidence of the three types of plants, see Lee (1986).

segment to produce the product. This characterization of production and of the flow rate of output means that the PS is not particular to any production period, but exists for all production periods, thus making it a component of the structure of production; and that the PS is unaffected by the passage of time or by repeated usage through time even though it must exist in time. As a result, this relatively enduring structural property permits the PS to be used over and over again under the guise of sequential production. In this manner, the fixed technical coefficients are flow coefficients and *g* is a flow of output denominated in terms of a single production period.[11]

Consider the case for the segmented plant when the plant segments of a plant are not identical, meaning that each PS consists of different amounts of the same inputs or of different inputs.[12] If m plant segments are being used, where $1 \leq m \leq$ maximum number of plant segments in the plant, then we have

(4.2) Segmented plant: $$q_m = \sum_{j=1}^{m} g_j \leftarrow \sum_{j=1}^{m} \sum_{\substack{i=1 \\ v=1}}^{h,z} a_{ij} \text{ x } l_{vj} \ : \mathbf{K}_{sp} = k_{1m}, \ldots, k_{km}.$$

where q_m is the plant's aggregate flow rate of output for *m* plant segments;

k_{km} is the quantity of the k-th fixed investment good associated with the m plant segments that constitute the segmented plant; and

$\mathbf{K}_{sp}$ is the vector of fixed investment goods associated with the segmented plant.

---

[11]It also can be noted that the characterization of the PS sweeps away the property of single (or multiple) input-output variation—that is the marginal products for intermediate and labor power inputs do not exist. Since an increased flow rate of output requires additional plant segments complete with their specific complement of **K** fixed investment goods, it is impossible to argue that an increase in the flow rate of output can occur by simply increasing one, some, or all the direct intermediate and labor inputs. Consequently, not only are marginal products, the law of variable proportions, and 'convexity' inapplicable to this analysis, but the traditional distinction between fixed and variable inputs is also undermine.

[12]It is possible that technically different plant segments can produce different flow rates of output, but this will be ignored.

From equation (4.2), the average amount of direct intermediate and labor power inputs used to produce a unit of output at a given flow rate of output is derived by dividing by q_m:

(4.3) average plant segment (APS): $$1 = \frac{q_m}{q_m} \leftarrow \sum_{\substack{i=1\\v=1}}^{h,z} a^*_i \times l^*_v \; : K_{mu}$$

where $a^*_i = \sum_{j=1}^{m} a_{ij}/q_m$ is the i-th average plant intermediate production coefficient at q_m;

$l^*_v = \sum_{j=1}^{m} l_{vj}/q_m$ is the v-th average plant labor power production coefficient at q_m; and

$K_{mu} = q_m/q_{max}$ represents the degree of capacity utilization of the plant where q_{max} is the plant's maximum flow rate of output when all plant segments at utilized.

The average plant segment and its production coefficients (which are input-output ratios) represent the plant's structure of production at different flow rates of output or capacity utilization. If the plant segments are different, then production coefficients will vary, as will the APS, as capacity utilization increases. However, if the plant segments of the plant are all identical, the outcome of an increase in the flow rate of output or K_{mu} is

(4.4) $$q_m = \sum_{j=1} g_j \leftarrow \sum_{j=1}^{m} \sum_{\substack{i=1\\v=1}}^{m\;\;h,z} a_{ij} \times l_{vj} \; : \mathbf{K}_{sp}$$

or $$q_m \leftarrow q_m \sum_{\substack{i=1\\v=1}}^{h,z} (a_i \times l_v) = \sum_{\substack{i=1\\v=1}}^{h,z} (a_i q_m \times l_v q_m) \; : \mathbf{K}_{sp}.$$

since $q_m = mg = m$. From equation (4.3) and (4.4), the average plant segment of the segmented plant is:

h,z h,z

(4.5) APS: $1 \leftarrow \frac{q_m}{q_m} \sum_{\substack{i=1 \\ v=1}} (a_i \times l_v) = \sum_{\substack{i=1 \\ v=1}} (a^*_i \times l^*_v) : K_{mu}$

since $a_i q_m/q_m = a^*_i = a_i$ and

$l_v q_m/\underline{q_m} = l^*_v = l_v$.

So, when plant segments are identical, the intermediate and labor power production coefficients do not vary with the flow rate of output, thus making them equal to their respective technical coefficients of the individual plant segments. Consequently the plant's structure of production, as represented by the APS, does not vary with capacity utilization.

The technological emergent plant is equivalent to a single plant segment plant; hence it can be delineated as follows:

(4.1.1) Emergent plant : $q \leftarrow \sum_{\substack{i=1 \\ v=1}}^{h,z} a_i \times l_v : \mathbf{K}_{ep} = k_1,\ldots, k_k$.

where a_i is a direct intermediate input technical coefficient and is the amount of the i-th input needed

to produce q amount of output;

l_v is a direct labor power input technical coefficient and is the amount of the v-th labor power

input needed to produce q amount of output;

q is the plant's flow rate (or amount of) of output per production period.

The emergent plant is either on-line or not, that is it is either operating at a full capacity or not operating at all. Finally, although the hybrid plant takes many technological forms, it can be represented as amounts of labor power skills that are given for all degrees of capacity utilization, with the intermediate inputs rigidly fixed per unit of output, and as an array of fixed investment goods that can operate at varying degrees of capacity utilization:

(4.6) Hybrid plant : $q \leftarrow \sum_{\substack{i=1 \\ v=1}}^{h,z} a_i q \text{ x } l_v$: $\mathbf{K}_{hp} = k_1, \ldots, k_k; K_{mu}$.

(4.7) Average hybrid plant (AHP): $1 \leftarrow \frac{q}{q} \leftarrow \sum_{\substack{i=1 \\ v=1}}^{h,z} (a^*_i \text{ x } l^*_v)$: K_{mu}

where $a^*_i = a_i q/q$; and

$l^*_v = l_v/q$.

So, when K_{mu} increases to the technically specified full capacity utilization of all fixed investment goods, the intermediate production coefficient remains constant while the labor power production coefficient declines, which means the hybrid plant' structure of production also varies.

To summarize, the basic aggregate unit of production is the plant. Whether it is a segmented, emergent, or hybrid plant, production is a recipe of fixed ingredients that results in fixed technical coefficients. Hence, the intermediate and labor power inputs are not individually productive; instead to be productive all inputs must be used together along with the associated fixed investment goods. When the capacity utilization of the plant increases, the resulting production coefficients may increase, decrease, or remain constant, even though the underlying technical coefficients are fixed; and the changes are a result of the technology embodied in the plant, not the outcome of some law of production. So how a plant's structure of production, as represented by APS and AHP, varies with changes in K_{mu} can only be determined by empirical investigations, not by assumption. [Lee, 1986; Dean, 1976; Eichner, 1976]

Plant Segment, Plant, and the Structure of Average Direct Costs

With the introduction of intermediate input prices and wage rates, the plant segment becomes the direct costs of the product line produced by the plant

(4.8) Plant segment direct cost of production (PSDCP): $\sum_{\substack{i=1 \\ v=1}}^{h,z} a_i p_i + l_v w_v$

where p_i is the price of the i-th direct intermediate input; and

w_v is the wage rate of the v-th direct labor power input.

From this and drawing on (4.2) and (4.3), we have

(4.9) Segmented plant average direct costs (SPADC): $\sum_{\substack{i=1 \\ v=1}}^{h,z} a^*_i p_i + l^*_v w_v$

where $a^*_i p_i = \sum_{j=1}^{m} a_{ij} p_i / q_m$ is the plant average direct intermediate costs of the i-th input (PADMC);

$l^*_v w_v = \sum_{j=1}^{m} l_{vj} w_v / q_m$ is the plant average direct labor power costs of the v-th input (PADLC).

If the plant segments differ and assuming that the lowest PSDCP is used first, then SPADC will vary as K_{mu} varies since the production coefficients (a^*_i, l^*_v) vary and will increase as K_{mu} increases. In contrast if all plant segments are identical, then SPADC will not vary as K_{mu} increases since each production coefficient (a^*_i, l^*_v) will not vary. Thus the plant's structure of costs, represented by SPADC, PADMC, and PADLC, will vary and increase or not as K_{mu} varies, depending on its underlying structure of production. In the case of the emergent plant, its direct costs of production are equal to its plant average direct costs:

(4.10) EmPDCP = EmPADC: $\sum_{\substack{i=1 \\ v=1}}^{h,z} a_i p_i + l_v w_v$

Finally, for the hybrid plant, its direct costs of production and plant average direct costs are

h,z

$$(4.11)\quad \text{HPDCP: } \sum_{\substack{i=1\\v=1}} a_i q p_i \text{ x } l_v w_v \ : \ K_{mu}.$$

$$(4.12)\quad \text{HPADC: } \sum_{\substack{i=1\\v=1}}^{h,z} (a^*_i p_i \text{ x } l^*_v w_v) \ : \ K_{mu}$$

where $a^*_i p_i = a_i q p_i/q$ is the plant average direct intermediate costs of the ith input; and

$l^*_v w_v = l_v w_v/q$ is the plant average direct labor power costs of the vth input.

So as K_{mu} increases PADMC is constant since the intermediate production coefficient is constant, while PADLC declines as K_{mu} increases because the labor power production coefficient declines; thus as K_{mu} increases up to full capacity utilization, PADC declines because of its underlying structure of production. To summarize, the plant's structure of average direct costs, as represented by SPADC, HPADC, PADMC, and PADLC, can vary in any direction with changes in K_{mu} depending on its underlying structure of production. [Lee, 1986]

Multi-Plant Production and Enterprise Average Direct Costs of Production

Business enterprises may employ up to *p* plants to produce a product line. Thus the number of plants actually used in production depends on the total flow rate of output as well as the flow rate of output of each plant. Consequently the shape of the enterprise's average direct costs curve depends on which plants are being utilized and the degree of utilization of each plant. Focusing on the k-th plant and assuming full capacity utilization, we have

$$(4.13.1)\qquad \text{Segmented plant:}\qquad q_{maxk} \leftarrow \sum_{\substack{i=1\\v=1}}^{h,z} a_{imk} \text{ x } l_{vmk} \ : \ \mathbf{K}_{spk}; K_{mumax}$$

$$(4.13.2)\qquad \text{Emergent plant: } q_{maxk} = q \leftarrow \sum_{i=1}^{h,z} a_{imk} \text{ x } l_{vmk} \ : \ \mathbf{K}_{epk}; K_{mumax}$$

$v=1$

$$(4.13.3)\qquad \text{Hybrid plant}: \; q_{maxk} \leftarrow \sum_{\substack{i=1\\v=1}}^{h,z} a_{imk} \text{ x } l_{vmk} = \sum_{\substack{i=1\\v=1}}^{h,z} a_i q_{maxk} \text{ x } l_{vmk} \;:\; \mathbf{K}_{hpk};\, K_{mumax}$$

where q_{maxk} represents the maximum flow rate of output of the k-th plant;

a_{imk} is the amount of the i-th intermediate input needed to produce the maximum flow rate of output of the k-th plant;

l_{vmk} is the amount of the v-th labor power input needed to produce the maximum flow rate of output of the k-th plant; and

K_{mumax} is full capacity utilization of the k-th plant.

Thus the enterprise's average direct inputs structure of production (EADSP) for the product line is

$$(4.14)\quad \text{EADSP}: [1/q_e] \sum_{\substack{i=1\\v=1\\k=1}}^{h,z,p} a_{imk} \text{ x } l_{vmk} = \sum_{\substack{i=1\\v=1}}^{h,v} a^*_{imk} \text{ x } l^*_{vmk} \;:\; \mathbf{K}_d = \sum_{k=1}^{p} \mathbf{K}_{spk}, \mathbf{Ke}_{pk}, \mathbf{K}_{hpk};\; K_{mue}$$

where $q_e = \sum_{k=1}^{p} q_{maxk}$ is the enterprise's flow rate of output for k plants;

$a^*_{imk} = a_{imk}/q_e$ is the i-th intermediate input production coefficient at q_e;

$l^*_{vmk} = l_{vmk}/q_e$ is the v-th labor power input production coefficient at q_e;

$\mathbf{K}_d$ is the array of fixed investment goods across all plants that are 'directly' used in the production of the product line; and

$K_{mue} = q_e/q_{emax}$ is the degree of capacity utilization of the product line where q_{emax} is the enterprise's maximum flow rate of output when all plants are used.

If the plants are identical, then the production coefficients (a^*_{imk}, l^*_{vmk}) are constant as K_{mue}

increases; but if the plants are not identical then the production coefficients vary as K_{mue} increases.

Adding intermediate input prices and wage rates to EADSP results in the enterprise average direct costs of production for the product line:

$$(4.15)\quad EADC = \sum_{\substack{i=1\\v=1}}^{h,z} a^{*}_{imk}p_i \text{ x } l^{*}_{vmk}w_v \;:\; \mathbf{K}_d; K_{mue}$$

where $a^{*}_{imk}p_i$ is the enterprise average direct intermediate costs of the i-th input (EADMC); and

$l^{*}_{vmk}w_v$ is the enterprise average direct labor power costs of the v-th input (EADLC).

As noted above, if the plants are identical, then the production coefficients are constant as K_{mue} increases, resulting in constant EADC, EADMC, and EADLC.[13] However, if the plants are not identical, then they will change as K_{mue} changes. That is, if technical and organization knowledge has changed over time, then each plant may be different in terms of intermediate and labor power inputs used and the flow rate of output. Consequently, it is not possible to determine the order in which the various plants are used to produce the output without first comparing their average direct costs (Gold, 1981). Assuming that the business enterprise tries to produce any flow rate of output as cheaply as possible, it will use plants with lower PADC at full capacity utilization first and plants with higher PADC later:

$$(4.16)\quad PADC_1 \leq ... \leq PADC_k \leq \ldots \leq PADC_p$$

where $PADC_k$ is the plant average direct costs of the k-th plant at full capacity utilization; and

$PADC_p$ is the highest cost and last plant used by the business enterprise.

Consequently, as capacity utilization increases and more plants are brought on line, EADC will increase due to the use of more costly plants:

---

13 This assumes that input prices do not change with changes in the usage of the inputs. But if p_i is based on the quantities of the input used in that for large amounts of a_{imk}, p_i is reduced, then EADC

(4.17) if $PADC_k < PADC_{k+1}$ then $PADC_{k+1} > EADC_k$ and $EADC_{k+1} - EADC_k > 0$

where $PADC_{k+1}$ is plant average 'incremental' costs.

If variations of K_{mue} takes place within the k-th segmented plant, the production coefficients (a^*_{imk}, l^*_{vmk}) will increase even though the plant segments are all identical so that the plant coefficients (a^*_i, l^*_v) are constant as plant capacity utilization increases, which implies that enterprise average direct costs will increase even though average direct costs within the plant are constant. That is, assume that the least costly plant segments are used first and the most costly later,

(4.16.1) $$PSADC_1 \leq \ldots \leq PSADC_m \leq \ldots \leq PSADC_{max}$$

where $PSADC_m$ is the average direct costs of the m-th plant segment;

$$PSADC_m: \sum_{\substack{i=1 \\ v=1}}^{h,z} a^*_{im}p_i + l^*_{vm}w_v;$$

$a^*_{im}\, p_i = a_{im}p_i/g_m$ is the plant segment average direct intermediate costs of the i-th input;

$l^*_{vm}w_v = l_{vm}w_v/g_m$ is the plant segment average direct labor power costs of the v-th input; and

g_m is the flow rate of output of the m-th plant segment.

Therefore if $PSADC_m \leq PSADC_{m+1}$ and $PADC_{k-1} < PADC_k$, then $PSADC_{m+1} > EADC_{km}$ and $EADC_{km} < EADC_{km+1}$. So if segmented plants have different costs irrespective of whether the plant segments within a plant have the same costs, enterprise average direct costs will increase as K_{mue} increases. On the other hand, if the EADC is based on hybrid plants, then it will exhibit spiked costs even if overall costs are constant (Blinder, et. al, 1998, p. 103). That is, if $HPADC_k = HPADC_{k+1}$ at full capacity utilization so that $EADC_k = EADC_{k+1}$ for k = 1…p, then if $HPADC_{k+1}$ is partially utilized ($q < q_{maxk+1}$), $EADC_{k+1q} > EADC_{k+1}$ but tends to equality as q approaches q_{max}, implying declining plant average incremental costs. Spiked costs can also occur if the EADC is increasing

---

will decline as K_{mue} increases.

and K_{mue} increases, but they will not be as pronounced.[14]

The outcomes of the above analysis of enterprise average direct costs are that (1) under single plant production both EADC and its incremental costs can be constant, increase, or decrease as K_{mue} increases; (2) under multi-plant production EADC can be constant or increase as K_{mue} increases while its incremental costs can be increasing, decreasing, or constant; and (3) average direct intermediate and labor costs can increase, decrease, or remain constant as K_{mue} increases. These varied outcomes are due to the possibility that plants (and plant segments) can have the same or different technology that generates a structure of production whose production coefficients can vary or remain constant as K_{mue} varies. In particular, over time technical and organizational innovations occur that become embedded in the technological makeup of a plant and produces a lower PADC. The lower costs may arise, for example, from large-scale production through the use of specialized equipment, better organization of production flows, and use of different kinds of skilled or unskilled labor power. But the point is that technical and organizational knowledge continually changes and supersedes the existing knowledge. Hence the difference between the technological makeup of plants is not just time, but a wholly new unforeseen body of technical and organizational knowledge that makes for greater cost reductions per unit of output; thus it is possible to view a plant as a particular time-specific embodiment or 'vintage' of technical and organizational knowledge.[15] Since the older vintage plants have higher PADC, an increasing EADC is a result of technological

---

[14] It is possible that the segmented and hybrid plants have different relative costs at different degrees of capacity utilization. Thus it is possible that all plants are in use but at different K_{mu} for a given K_{mue}. While it complicates the theory of costs of the business enterprise, it does not fundamentally alter it. [Westfield, 1955]

[15] This differentiation between plants is not compatible with the neoclassical economies of scale which is based on proportional increases in the inputs and the absence of technological change and new knowledge. [Gold, 1981]

progress;[16] and in contrast, if technological progress is absent, then EADC is constant so that vintage plants are the same as new plants. There, it is the existence of technological progress which creates vintage plants that makes the EADC increase as K_{mue} increases, not the existence of inefficient technology; and an assumption of constant EADC is an assumption of technological stagnation or at least the absence of technological progress. [Lee, 1986; Eichner, 1976; Gold, 1981; Salter, 1966]

Shop Technique of Production and Shop Expenses

As noted above, the costs a business enterprise incurs in the production of a product line are divided into direct costs and overhead costs. The former is specified in terms of a production period while the latter is specified in terms of an accounting period which consists of a number of production periods. Overhead costs, in turn, are divided into two categories, *shop expenses* and *enterprise expenses* (which will be dealt with below). Shop and enterprise expenses can be further divided into indirect costs and depreciation. *Indirect costs* consist of the labor power and intermediate input expenses required to supervise and manage the production of a product line; hence they must be able to accommodate many different flow rates of output in a single production period and a succession of flow rates of output over a number of production periods. That is, for a business enterprise to engage in sequential acts of production over time as well as to be able to vary how much it produces in any production period, it must continually incur labor power and intermediate input expenses which permit this. Shop expenses are those non-direct costs associated with the production of a particular product line in a plant and across plants and generally include the

[16]This statement may have exceptions if changes in wage rates, profit mark ups, technology, and social conditions of work generate an array of input prices and wage rates that results in $PADC_{k+1} < PADC_k$. This reordering of vintage plants is comparable to the reswitching of techniques of production in the capital controversies.

salaries of foremen, support staff and supervisors; the intermediate inputs needed to maintain the support staff and the technical efficiency of the plant(s) used directly in production; and the depreciation allowance associated with the plant(s).

Shop Technique of Production

Each plant involved in the production of a product line utilizes an array of labor power and intermediate inputs in conjunction with an array of fixed investment goods ($\mathbf{K}_{se}$) to oversee production for the accounting period which can be thought of as the plant's managerial technique of production (PMTP). Although the technical coefficients that make up the PMTP are not rigid, they are specified at the same time the technology of plant is determined. Assuming the number of production periods in the accounting period to be *f*, the PMTP for the k-th plant is the following:

(4.17) Plant managerial technique of production (PMTP): $\mathrm{PMTP}_k = \sum_{\substack{r=1 \\ s=1}}^{b,c} a_{rk} \times l_{sk} \;:\; \mathbf{K}_{sek} = k_{k1}, \ldots, k_{kk}$

where a_{rk} is the r-th plant managerial intermediate input technical coefficient in absolute amount for the accounting period;

l_{sk} is the s-th plant managerial labor power input technical coefficient in absolute amount for the accounting period; and

$\mathbf{K}_{sek}$ is the array of fixed investment goods associated with PMTP.

The technical coefficients are made up of flows of inputs over successive production periods that constitute the accounting period and their amount for any f-th production period is given and sufficient to manage any degree of capacity utilization of the plant—which implies that incremental variations in the amount of any coefficients has no impact on the degree of capacity utilization. While the flow of the managerial inputs need not be absolutely uniform over the production periods, their variations cannot be too great and in the end they have to add up to the absolute amounts needed for the accounting period. To simplify the analysis, it is assumed that the managerial inputs are uniformly distributed over the *f* production periods; therefore the $PMTP_k$ for the f-th production period can be represented as

$$(4.18) \quad PMTP_{kf} = 1/f \sum_{\substack{r=1\\s=1}}^{b,c} a_{rk} \times l_{sk} = \sum_{\substack{r=1\\s=1}}^{b,c} a_{rkf} \times l_{skf}$$

Since $PMTP_{kf}$ can accommodate any variation in its flow rate of output, the average $PMTP_{kf}$ is

$$(4.19) \quad APMTP_{kf} = (1/q)(1/f)\sum_{\substack{r=1\\s=1}}^{b,c} a_{rk} \times l_{sk} = \sum_{\substack{r=1\\s=1}}^{b,c} a^*_{rkf} \times l^*_{skf}$$

where $a^*_{rkf} = a_{rkf}/q$ is the r-th average plant managerial intermediate input production coefficient for the f-th production and q flow rate of output; and

$l^*_{skf} = l_{sk}/fq$ is the s-th average plant managerial labor power input production coefficient for the f-th production period and q flow rate of output.

Thus, as K_{mu} increases, $APMTP_{kf}$ varies and the average plant managerial production coefficients for the f-th production period decline, reaching their lowest value when the plant is at full capacity utilization.

If the enterprise uses more than one plant in the production of a product line, it has more than a single PMTP. As a group they are the shop technique of production (STP) and represent the enterprise's 'technical organization' of its managerial supervision of the production of the product line:

$$(4.20) \quad STP = PMTP_1 + \ldots + PMTP_p = \sum_{k=1}^{p} PMTP_k = \sum_{k=1}^{p} \sum_{r=1}^{b,c} a_{rk} \times l_{sk} \;:\; \mathbf{K}_{se} = \mathbf{K}_{se1} + \ldots + \mathbf{K}_{sep}$$

s=1

(4.20.1) $$STP = \sum_{\substack{r=1 \\ s=1}}^{b,c} a_{rp} \text{ x } l_{sp} \ : \mathbf{K}_{se}$$

where $a_{rp} = \sum_{k=1}^{p} a_r$ is the r-th shop intermediate input technical coefficient for the accounting period;

$l_{sp} = \sum_{k=1}^{p} l_s$ is the s-th shop labor power technical coefficient for the accounting period.

Because STP is based on PMTP, its technical coefficients are made up of flows of inputs over successive production periods that constitute the accounting period and their amount for any f-th production period is given and sufficient to manage any degree of capacity utilization for the product line, K_{mue}. Since managerial inputs are assumed to be evenly distribute over the production periods that constitute the accounting period, the shop technique of production for the f-th production period is

(4.21) $$STP_f = (1/f)STP = \sum_{\substack{r=1 \\ s=1}}^{b,c} a_{rpf} \text{ x } l_{spf}.$$

Finally, for any production period, the STP can accommodate variations in the flow rate of output in terms of bringing a plant (or plant segment) on line or closing a plant (or plant segment) down. Therefore the average shop technique of production (ASTP) for the f-th production period is

(4.22) $$ASTP_f = (1/q_e)STP = \sum_{\substack{r=1 \\ s=1}}^{b,c} a^*_{rpf} \text{ x } l^*_{spf}$$

where $a^*_{rpf} = a_{rpf}/q_e$ is the r-th average shop intermediate input production coefficient for the f-th production period when the enterprise's flow rate of output is q_e; and

$l^*_{spf} = l_{spf}/q_e$ is the s-th average shop labor power input production coefficient for the f-th production period when the flow rate of output is q_e.

Thus, as K_{mue} increases, $ASTP_f$ varies and the average shop production coefficients decline, reaching their lowest value when K_{mue} reaches full capacity utilization.

Indirect Costs: Cost of the Shop Technique of Production

With the introduction of intermediate input prices and salaries, the STP_f becomes indirect costs or the cost of the shop technique of production (CSTP):

$$(4.23)\quad CSTP_f = \sum_{\substack{r=1\\s=1}}^{b,c} a_{rpf}p_r + l_{spf}w_s \;:\; \mathbf{K}_{se}$$

where $CSTP_f$ is the cost of shop technique of production for the f-th production period;

p_r is the price of the r-th intermediate input;

w_s is the salary of the s-th labor power input;

$a_{rpf}p_r$ is the shop intermediate costs of the r-th input; and

$l_{spf}w_s$ is the shop labor power costs of the s-th input.

The $CSTP_f$ shows that indirect costs are cost flows over the production periods that constitute the accounting period; but they are also invariant with respect to different flow rates of output in the f-th production period. Therefore, the average $CSTP_f$ and the average intermediate and labor input costs will vary inversely with the flow rate of output or degree of capacity utilization:

$$(4.24)\quad ACSTP_f = CSTP_f/q_e = \sum_{\substack{r=1\\s=1}}^{b,c} a^*_{rpf}p_r + l^*_{spf}w_s$$

$$(4.24.1)\qquad \Delta ACSTP_f/\Delta q_e < 0$$

(4.24.2) $\Delta SAMC_f/\Delta q_e < 0$

(4.24.3) $\Delta SALC_f/\Delta q_e < 0$

where $a^*_{rpf}p_r = a_{rpf}p_r/q_e$ is the shop average intermediate costs of the r-th input ($SAMC_f$); and

$l^*_{spf}w_s = l_{spf}w_s/q_e$ is the shop average labor power costs of the s-th input ($SALC_f$).

Since costs of the shop technique of production are contractual expenditures that cannot be deferred (like depreciation for instance), they have to be paid on a regular basis. Thus, although fixed with regard to variations in the flow rate of output within a production period, they are not deferrable over production periods; rather they have to be paid-out on a regular, sequential basis.

Depreciation

As noted above, depreciation of fixed investment goods is a cost that is incurred in the production of a product line. To determine it, the fixed investment goods involved in its production have to be identified. From equations 4.1, 4.2, 4.1.1, 4.6, 4.14, 4.17, and 4.20, the array of fixed investment goods associated with the production of the product line is

4.25 $\mathbf{K}_{dse} = \mathbf{K}_d + \mathbf{K}_{se}$.

With the fixed investment goods associated with the production of the product line identified, their individual values are determined based on their historical costs. Then using straight-line or declining charges methods, the depreciation allowance of each fixed investment good for the accounting period is determined from whence they are aggregated into a single value amount for the accounting period, D_{dse}. Distributing D_{dse} equally across all production periods, depreciation allowance for the f-th production period is $D_{dsef} = D_{dse}/f$. Since D_{dsef} is invariant with respect to variations in the flow rate of output, average depreciation costs and hence the shop depreciation production coefficient varies inversely with as the flow rate of output or degree of capacity utilization:

(4.26) $d^*_{dsef} = D_{dsef}/q_e$

(4.26.1) $\Delta d^*_{dsef}/\Delta q_e < 0$

where d^*_{dsef} is the shop depreciation production coefficient for the f-th production period when the flow rate of output is q_e.

Shop Expenses

Shop expenses (SE) for the f-th production period is obtained by adding together D_{dsef} and $CSTP_f$:

(4.27) $SE_f = \sum_{\substack{r=1 \\ s=1}}^{b,c} a_{rpf}p_r + l_{spf}w_s + D_{dsef}.$

Since $CSTP_f$ and D_{dsef} are cost flows, SE_f is also a cost flow. Thus it cannot be seen as "fixed" even though it is invariant with respect to different flow rates of output. Average shop expenses (ASE) for the f-th production period is

(4.28) $ASE_f = SE_f/q_e = \sum_{\substack{r=1 \\ s=1}}^{b,c} (a^*_{rpf}p_r + l^*_{spf}w_s) + d^*_{dsef} \; : \mathbf{K}_{se}; K_{mue}$

and as the degree of capacity utilization increases, ASE_f declines (equations 4.24.1 and 4.26.1)

(4.28.1) $\Delta ASE_f/\Delta q_e < 0.$

Enterprise Technique of Production and Enterprise Expenses

Because an enterprise is generally a multi-product producer and a going concern, it incurs expenses that are common to all of its product lines and are necessary if it is to stay in existence as a going concern and hence are identified as enterprise expenses. In general, these costs are associated with those activities which the enterprise must engage in order to co-ordinate the production flows of its various product lines, to sell its various product lines, and to develop and implement enterprisewide investment and diversification plans and which include the salaries of management, stationary, selling and other office expenses, and the depreciation of the central office fixed investment goods. This array of labor power and intermediate inputs in conjunction with an array of fixed investment goods ($\mathbf{K}_{ee} = k_{ee1},\ldots, k_{eek}$) are used to manage the enterprise as a whole for the accounting period which includes the various degrees of capacity utilization for any one product line and all product lines; and it can be thought of as the *enterprise technique of production* (ETP):

$$(4.29) \quad ETP = \sum_{\substack{u=1 \\ e=1}}^{o,y} a_u \times l_e \ : \mathbf{K}_{ee}$$

where a_u is the u-th enterprise intermediate input technical coefficient for the accounting period; and

l_e is the e-th enterprise labor power technical coefficient for the accounting period.

The technical coefficients are made up of flows of inputs over the accounting period that are not synchronized with the production periods of the various production lines, which would not be possible in any case since they are not necessarily the same. Therefore, it is not possible, as with the STP, to allocate the flow of the inputs to any and all product lines; rather the allocation is done in terms of money.

With the introduction of intermediate input prices and yearly salaries, the ETP becomes indirect costs or the cost of the enterprise technique of production (CETP):

$$(4.30) \quad CETP = \sum_{\substack{u=1 \\ e=1}}^{o,y} a_u p_u + l_e s_e$$

where p_u is the input price of the u-th enterprise intermediate input;

$a_u p_u$ is the enterprise intermediate costs for the u-th input for the accounting period;

s_e is the yearly salary of the e-th labor power input; and

$l_e s_e$ is the enterprise labor power costs for the e-th input for the accounting period.

Given the CETP for the accounting period, it is allocated to each of the enterprise's z product lines. Once a given percentage of CETP, αCETP, is allocated to the z-th product line for the accounting period, it is then allocated equally over all the production periods. Therefore, the CETP for the enterprise's z-th product line and the f-th production period is

$$(4.31)\quad CETP_{zf} = (\alpha_z)(1/f)(\sum_{\substack{u=1\\e=1}}^{o,y} a_u p_u + l_e s_e) = \sum_{\substack{u=1\\e=1}}^{o,y} a_{uzf} p_u + l_{ezf} s_e$$

where $a_{uzf}p_u$ is the enterprise intermediate costs for the u-th input for the z-th product line and f-th production period;

$l_{ezf}s_e$ is the enterprise labor power costs for the e-th input for the z-th product line and f-th production period; and

α_z is the percentage of CETP allocated to the z-th product line.

Like with the $CSTP_f$, the $CETP_{zf}$ shows that indirect costs are cost flows over the production periods that constitute the accounting period; but they are also invariant with respect to different flow rates of output in the f-th production period. Therefore, the average $CETP_{zf}$ and the average intermediate and labor input costs will vary inversely with the flow rate of output or degree of capacity utilization:

$$(4.31)\quad ACETP_{zf} = CETP_{zf}/q_e = \sum_{\substack{u=1\\e=1}}^{o,y} a^*_{uzf} p_u + 1^*_{ezf} w_e$$

$$(4.31.1)\qquad \Delta ACETP_{zf}/\Delta q_e < 0$$

$$(4.31.2)\qquad \Delta EAMC_{zf}/\Delta q_e < 0$$

$$(4.31.3)\qquad \Delta EALC_{zf}/\Delta q_e < 0$$

where $a^*_{uzf}p_r = a_{uzf}p_r/q_e$ is the enterprise average intermediate costs of the u-th input of the z-th product line ($EAMC_{zf}$); and

$l^*_{ezf}w_s = l_{ezf}w_s/q_e$ is the enterprise average labor power costs of the e-th input of the z-th product line ($EALC_{zf}$).

Since costs of the enterprise technique of production are contractual expenditures that cannot be deferred (like depreciation for instance), they have to be paid on a regular basis. Thus, although fixed with regard to variations in the flow rate of output within a production period, they are not deferrable over production periods; rather they have to be paid-out on a regular, sequential basis.

Since the array of fixed investment goods ($\mathbf{K}_{ee} = k_{ee1},\ldots, k_{eek}$) associated with the ETP are known, the depreciation allowance for enterprise expenses, D_e, for the accounting period is determined in the same manner described above in reference to shop expenses. It is then allocated to the various product lines so that the enterprise depreciation allowance of the z-th product line for the accounting period is $D_{ez} = (\alpha_z)(D_e)$; and for the z-th product for the f-th production period, it is $D_{ezf} = (1/f)(\alpha_z)(D_e)$. Finally, although D_{ezf} is invariant with respect to variations in the flow rate of output, the enterprise depreciation production coefficient for the z-th product line and f-th production period varies as the flow rate of output varies:

(4.32) $d^*_{ezf} = D_{ezf}/q_e$

(4.32.1) $\Delta d^*_{ezf}/\Delta q_e < 0$

where d^*_{ezf} is the enterprise depreciation production coefficient of the z-th product for the f-th production period when the flow rate of output is q_e.

Finally, the enterprise expenses for the accounting period consist of the cost of the enterprise technique of production and depreciation; thus the enterprise expenses (EE) for the z-th product line in the f-th production period is

$$(4.33)\quad EE_{zf} = CETP_{zf} + D_{ezf} = \sum_{\substack{u=1\\e=1}}^{o,y} a_{uzf}p_u + l_{ezf}s_e + D_{ezf}.$$

Since each of its components are cost flows, the EE_{jf} is also a cost flow. Thus it cannot be seen as "fixed" even though it is invariant with respect to different flow rates of output. Average enterprise expenses for the z-th product line and f-th production period is

$$(4.34)\quad AEE_{zf} = EE_{zf}/q_e = ACETP_{zf} + d^*_{ezf} = \sum_{\substack{u=1\\e=1}}^{o,y} (a^*_{uzf}p_u + 1^*_{ezf}s_e) + d^*_{ezf} \;:\; \mathbf{K}_{ee}; K_{mue}$$

and as the degree of capacity utilization increases, AEE_{zf} declines (equations 4.31.1 and 4.32.1)

$$(4.34.1)\qquad \Delta AEE_{zf}/\Delta q_e < 0.$$

Structure of Production and Costs of a Product Line

The average structure of production for the j-th product line in terms of the f-th production period and for a flow rate of output of q_e (derived from equations 4.14, 4.22, and 4.31)

Structure of Production for a Product (SPP)

$$(4.33)\quad ASPP_{zf} = \sum_{\substack{i=1\\r=1\\u=1\\v=1\\s=1\\e=1}}^{h,b,o,z,c,y} a^*_{imk} \times a^*_{rpf} \times a^*_{uzf} \times 1^*_{vmk} \times 1^*_{spf} \times 1^*_{ezf} \;:\; \mathbf{K}_z = \mathbf{K}_d + \mathbf{K}_{se} + \mathbf{K}_{ee}; K_{mue}$$

We know the following: (1) that a^*_{rjkf}, a^*_{ujkf}, 1^*_{sjkf}, and 1^*_{ejkf} all decline as the flow rate of output (q_{ikf}) increases; and (2) a^*_{ijkf} and 1^*_{vjkf} can vary in any direction as output increases. Hence the movement of production coefficients as can be diverse. So over all, an enterprise's SPP consists for any array of material and service inputs and labor power (skills) whose production coefficients are jointly determined by technology and the flow rate of output. Thus the structure itself remains stable in face of variations of the flow rate of output. Hence the structure changes only when the underlying technology changes, resulting in changes in changing the material and labor power inputs. This occurs when new plants (or plant segments) are brought on line and as vintage plants (plant segments) are dropped as well as when managerial and enterprise techniques of production are altered.

When considering the structure of costs for a single product, we are essentially considering the enterprise's average total costs of production for the j-th product line, f-th production period, and flow rate of output of q_k:

(4.34) enterprise average total costs of production for the j-th product line ($EATC_j$)—see equations (4.14), (4.25), and (4.30):

$$EATC_{jkf} = EADC_{jkf} + ASE_{jkf} + AEE_{jkf}$$

$$= \sum_{\substack{i=1\\v=1\\r=1\\s=1\\u=1\\e=1}}^{h,z,b,c,o,y} a^*_{ijkf}p_i + a^*_{rjkf}p_r + a^*_{ujkf}p_u + l^*_{vjkf}w_v + l^*_{sjkf}w_s + l^*_{ejkf}s_e + d^*_{sjkf} + d^*_{ejkf}$$

(4.28) $ASE_f = SE_f/q_e = \sum_{\substack{r=1\\s=1}}^{b,c} (a^*_{rpf}p_r + l^*_{spf}w_s) + d^*_{dsef} \;:\; \mathbf{K}_{se}; K_{mue}$

(4.34) $AEE_{zf} = EE_{zf}/q_e = ACETP_{zf} + d^*_{ezf} = \sum_{\substack{u=1\\e=1}}^{o,y} (a^*_{uzf}p_u + l^*_{ezf}s_e) + d^*_{ezf} \;:\; \mathbf{K}_{ee}; K_{mue}$

$$(4.15)\quad EADC = \sum_{\substack{i=1\\v=1}}^{h,z} a^{*}_{imk}p_i \times l^{*}_{vmk}w_v \;:\; \mathbf{K}_d; K_{mue}$$

where $EADC_{jkf}$ is the enterprise average direct costs for the j-th product line, f-th production period when the flow rate of output isq_k;

ASE_{ikf} is the average shop expenses for the j-th product line, f-th production period when the flow rate of output is q_k; and

AEE_{jkf} is the average enterprise expenses for the j-th product line, f-th production period when the flow rate of output is q_k.

Restricting the structural analysis to a single production period, the relationship between $EATC_{jkf}$ and the flow rate of output can be shown in the following manner:

$$(4.35)\quad \frac{\Delta EATC_{jkf}}{\Delta q_{jkf}} \begin{array}{l} > 0 \text{ if } PADC_{jk+1} > EATC_{jkf} \\ = 0 \text{ if } PADC_{jk+1} = EATC_{jkf} \\ < 0 \text{ if } PADC_{jk+1} < EATC_{jkf}. \end{array}$$

Thus we find that the specific forms of the relationship depend on a tug-of-war between the rising incremental costs and the falling ASE_{jkf} and AEE_{jkf}. Since there is no necessary reason for the relative dominance of one side over the other, a positive, negative, or U-shaped $EATC_{jkf}$ are possible. The empirical evidence does suggest, however, that $EATC_{jkf}$ is declining as the flow rate of output increases. Still, it should be noted that whatever the shape of the average total cost curve is, the shape is solely due to technological and organizational change (LIKE NCE BUT W/DIFFERENT BASE—DISCUSS).

Enterprise Costs Through Time

Now we are in a position to analyze the movement of the enterprise's average direct costs over production periods, that is the movement of $EADC_{jkf}$ with respect to sequential acts of production. Since input prices, wage rates and salaries are given, variations in $EADC_{jkf}$ over sequential production occurs because of variations in the flow rate of output. To simplify the analysis, it will be assumed that all variations in the flow rate of output occur via variations in plant usage. Therefore, letting $EADC_{jkf}$ be the enterprise average direct costs for the j-th product line of the f-th production period with a flow rate of output of q_k and assuming that $q_{jk+1\ f+1} > q_{jkf}$, then

(4.38) $EADC_{jk+1f+1} > EADC_{jkf}$ since $EADC_{jkf} < PADC_{jk+1f+1}$.

That is, $EADC_j$ will increase from one production period to the next if the 'incremental costs' of producing the increased amount of output is greater than the $EADC_j$ of the preceding production period. This will be the case if the plants and plant segments employed are non-homogeneous. But if they are homogeneous for all levels of output, then the $EADC_j$ will not vary from one period to the next.[17]

Let us alter the analysis slightly in an effort to provide a clearer understanding of why $EADC_{jkf}$ varies (or does not vary) from one production period to the next. First, let us specify a particular flow rate of output that *notionally* will be the same for each production period. This flow rate of output, q_{jn}, is called a bench mark output and the $EADC_{jn}$ associated with it will be the same for each production period. Now let us impose actual movements of $EADC_{jkf}$ on $EADC_{jn}$ from which we get $EADC_{jkf}$ varying around $EADC_{jn}$ as q_{jkf} differs from q_{jn}. This is obvious due to the non-homogeneity of the plants and plant segments employed by the business enterprise. Thus, the movement of $EADC_{jkf}$ over production periods can be understood in terms of variations with respect to $EADC_{jn}$ and with respect to $EADC_{jkf}$ of the previous production period. In either case, the variations occur because of the non-homogeneity of the plant segments and plants employed which is clearly discerned in the bench-mark analysis.

Turning to enterprise average total costs and sequential production, we will find that $EATC_{jkf}$

---

[17] Thus the concept of constant $EADC_j$ in the context of sequential production cannot be sustained

is dependent on the variations in the flow rate of output, q_{ikf}. By employing the benchmark approach developed above with respect to average direct costs, we can explain the forces at work that affect the movement of $EATC_{jkf}$ over the accounting period. That is, comparing the $EATC_{jkf}$ to the benchmark $EATC_{jn}$ that is common to each production period, then the actual $EATC_{jkf}$ for any production period and its movement over the production periods can be explained. To construct a benchmark $EATC_{jn}$ that is the same in every production period, its components must be specified so as to be the same for each production period. As noted above, to obtain an $EADC_{jkf}$ that does not change for each production period, a bench-mark flow rate of output, q_{jn}, must be specified that will be common to all the production periods in the accounting period. Further, since shop and enterprise expenses are the same for each production period, the same bench-mark flow rate of output can be used to obtain bench-mark average shop and average enterprise expenses. This construction of the benchmark can be shown in the following manner:

(4.39) bench-mark enterprise average total costs:

$$EATC_{jn} = EADC_{jn} + ASE_{jn} + AEE_{jn}.$$

Since $EATC_{jn}$ remains the same for each production period, it can be used to explain, comparatively, the actual position of the $EATC_{jkf}$. That is, for the f-th production period, the difference between $EATC_{jkf}$ and $EATC_{jn}$ can be summarized as follows:

(4.40) $EATC_{jkf} – EATC_{jn} = (EADC_{jkf}-EADC_{jn}) + (ASE_{jkf}-ASE_{jn}) + (AEE_{jkf}-AEE_{jn}) >< 0.$

As noted above, $(EADC_{jkf} – EADC_{jn})$ will in general vary directly with the difference in $q_{jkf} – q_{jn}$ since the enterprise generally employs a non-homogeneous set of techniques of production. As for $(ASE_{jkf} – ASE_{jn})$ and $(AEE_{jkf} – AFE_{jn})$, non-zero results will arise when q_{jkf} differs from q_{jn}. So, in general, it is sufficient to say that the actual amount of $EATC_{jkf}$ compared to $EATC_{jn}$ and hence the

---

as a general theoretical principle.

movement of $EATC_{jkf}$ over production periods (that is over time) will depend on the technical and organizational innovations embodied in the techniques of production, that is the structure of production, and on the flow rate of output.

Conclusion

References

Addinsell, W. A. (1899) 'A New System of Cost-Taking in Trade', *The Accountant*, (May 10): 577–81.

Al-Omiri, M. and Drury, C. (2007) 'A Survey of Factors Influencing the Choice of Product Costing Systems in UK Organizations', *Management Accounting Research* 18: 399-424.

Boyns, T. and Edwards, J. R. (1995) 'Accounting Systems and Decision-Making in the Mid-Victorian Period: the case of the Consett Iron Company', *Business History*, 37(3): 28–51.

__________ (1997) 'The Construction of Cost Accounting Systems in Britain to 1900: the case of the coal, iron and steel industries', *Business History*, 39(3): 1-29.

Boyns, T., Edwards, J. R., and Nikitin, M. (1997) *The Birth of Industrial Accounting in France and Britain*, New York: Garland Publishing, Inc.

Bright. J., Davies, R. E., Downes, C. A., and Sweeting, R. C. (1992) 'The Deployment of Costing Techniques and Practices: a UK study', *Management Accounting Research*, 3(3): 201-11.

Edwards, J. R. (1980) 'British Capital Accounting Practices and Business Finance 1852-1919: an exemplification', *Accounting and Business Research*, 10: 241-58.

Edwards, J. R. (1986) 'Depreciation and Fixed Asset Valuation in British Railway Company Accounts to 1911', *Accounting and Business Research*, 16: 251-63.

Edwards, J. R. (1989) 'Industrial Cost Accounting Developments in Britain to 1830: a review article', *Accounting and Business Research*, 19(76): 305-17.

Edwards, J. R. and Baber, C. (1979) 'Dowlais Iron Company: accounting policies and procedures for profit measurement and reporting purposes', *Accounting and Business Research*, 9: 139-51.

Edwards, J. R. and Newell, E. (1991) 'The Development of Industrial Cost and Management Accounting Before 1850: a survey of the evidence', *Business History*, 33(1): 35–57.

Fleischman, R. K. and Parker, L. D. (1990) 'Managerial Accounting Early in the British Industrial Revolution: the Carron Company, a case study', *Accounting and Business Research*, 20: 211–21.

______ (1992) 'The Cost-Accounting Environment in the British Industrial Revolution Iron Industry', *Accounting, Business and Financial History*, 2(2): 141–60.

______ (1997) *What is Past is Prologue: cost accounting in the British industrial revolution, 1760–1850*, New York: Garland Publishing, Inc.

Garner, S. P. (1954) *Evolution of Cost Accounting to 1925*. University: The University of Alabama Press.

Gold, B. (1981) ‘Changing Perspectives on Size, Scale, and Returns: an interpretive survey’, *The Journal of Economic Literature* 19(1): 5-33.

Hudson, P. (1977) ‘Some Aspects of 19th Century Accounting Development in the West Riding Textile Industry’, *Accounting History*, 2: 4–22.

Jones, H. (1985) *Accounting, Costing and Cost Estimation: Welsh industry 1700-1830*. Cardiff: University of Wales Press.

Litherland, D. A. (1951) ‘Fixed Asset Replacement a Half Century Ago’, *The Accounting Review*, 26(4): 475-80.

Lukka, K. (2007) ‘Management Accounting Change and Stability: loosely coupled rules and routines in action’, *Management Accounting Research*, 18(1): 76-103.

Napier, C. J. (1990) ‘Fixed Asset Accounting in the Shipping Industry: P&O 1840-1914’, *Accounting, Business and Financial History* 1(1): 23-50.

Scapens, R. W. (1994) ‘Never Mind the Gap: towards an institutional perspective on management accounting practices’, *Management Accounting Research*, 5(3-4): 301-21.

Tyson, T. (1992) ‘The Nature and Environment of Cost Management Among Early Nineteenth Century U.S. Textile Manufacturers’, *The Accounting Historians Journal*, 19(2): 1-24.