UTILITY COMPARISON AND HIERARCHICAL NEEDS: BRINGING MASLOW INTO THE WORLD OF ECONOMICS

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A MAJOR SHORTCOMMING in Neoclassical theory is its inability to describe how an economy's performance is influenced by the distribution of goods and services among its members. We know that countries with high Gini Ratios such as Guatemala and Honduras tend to less developed, while the Western democracies have Gini ratios that are much lower[1]. It has been noted in many times in the development literature that when resources such as land are concentrated in the hands of a small elite they are used less efficiently[2].

A source of this shortcoming is Neoclassical Theory's inability to explain how an agent's choices are conditional on his social and economic circumstances. Neoclassical theory regards a consumer's choices as the product of his tastes and preferences. Such are regarded unique to each individual and not varying predictably from person to person. The data shows however that consumer choice does tend to vary in a predictable manner with income. Engle's law has been empirically verified many times over the past two centuries[3]. Consumer expenditure data analyzed by Richard Stone more than 50 years ago has shown consumers to satisfy their want for basic goods such as food, before spending significantly on goods satisfying less urgent needs[4].

Arguing that the law of diminishing marginal utility applied to wealth in general, late nineteenth century Marshallians advocated progressive taxes as that they provide greater social welfare than taxation at a fixed rate[5]. Lionel Robbins, in his celebrated essay of 1932 argued that such reasoning was predicated on ones ability to make interpersonal comparisons of emotional satisfactions. Since such cannot be measured they cannot be compared, hence such reasoning is not scientifically valid[6].

The tasks of this paper are three fold and will be accomplished in three sections. In the first section the problem of interpersonal comparison will be examined. Here, the technical problems

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[1] See data in World Development Indicators (1998)
[2] Todaro and Smith (2003) pp. 430-31
[3] Nicholson (1995) p.139
[4] See Stone (1954)
[5] Hands (2001) p.36
[6] Robbins (1952) pp.139-40

associated with interpersonal comparison of preferences will be discussed. Next we will take up, the methodological problem of assuming preference to be the source of consumer choice.

In the second section, the Aristotelian concept of Use Value will be re articulated as an alternative to preference orderings and utility. While use value, along with what shall be called *Marginal Value* will be functionally similar to utility and marginal utility, they will be immune to the problems examined in the first section of this paper. It will be argued that, at least conceptually, it will be possible to aggregate the use and marginal value functions of society's members to determine the exchange behavior of a typical individual. Furthermore, the aggregation process should average out any arbitrary characteristics of choice that do not depend on the individual's external circumstances.

The third section of this paper will be an illustrative application of the methods in Section 2. In this section example use and marginal value functions for the typical consumer will be constructed based on Maslow's theory of hierarchical needs rather than empirical data. It will be shown that the consumer behavior predicted by the example functions will be consistent with Engle's law and Richard Stone's results.

1) THE PROBLEM OF INTERPERSONAL COMPARISON IN THE NEO-CLASSICAL FRAMEWORK

The difficulty with making interpersonal comparisons begins of course with measurement. More importantly though, such difficulty follows from ambiguity as to what such a comparison consists of. The literature often refers to interpersonal comparison of *utility* which is the cardinal forerunner of preference. Utility however has come to mean radically different things to different people. When the concept was first proposed by Jeremy Bentham in 1789, it was meant as a measure of an individual's *wellbeing,* useful to a third party observer interested in crafting social institutions. In Bentham's view, the effectiveness or "goodness" of a law is to be judged on its ability to provide the greatest utility to the greatest number of people. Bentham had no interest in creating a theory of human motivation, his interest was in jurisprudence[7]. In *Chapter V* of his *Introduction to the Principles of Morals and Legislation*, Bentham, describes utility as consisting of "interesting perceptions"[8] of various pleasures, and the absence of their opposing pains. We can group Bentham's pleasures into two categories; *Material Satisfactions*, those most directly associated with consumption of goods, and *Psychic Satisfactions*, which include things such as pleasures of good memories, and pleasant expectations of the future[9]. By classifying utility as a perception, Bentham has essentially equated wellbeing with *satisfaction*, an equation that is subject to challenge as shall be discussed shortly.

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[7] Warnock (2003) p.4
[8] Bentham (2003) p. 44
[9] Bentham (2003) pp.44-47

Utility took on a completely different meaning in the hands of William Stanley Jevons nearly a century later. Jevons conceived of utility as a physiological response to stimulus, which diminished as the stimulus continued. There is no evidence that Jevons ever studied Bentham, though he would have been familiar with utilitarianism, as would have any intellectual of his time.[10] Jevons credits his inspiration to the work in experimental psychology of Gustav Fectner, some ten years earlier. In the tradition, which follows Jevons, utility has been taken to be the motivation behind consumer choice.

At present when one speaks of utility, he could mean either wellbeing or satisfaction of a desire. If he means *wellbeing*, he could mean the *sensation* of wellbeing, or *actual wellbeing* as determined by an objective standard that takes into account things such as health, nutrition, and education. If by utility one means *satisfaction*, he could mean satisfaction as would obtain from winning a debate, or from consuming goods and services.

In many ways these concepts of utility overlap. On can be easily tempted to address all of these in a common utility maximization framework. As will be argued shortly, it is generally a mistake to do so, a mistake that contributes to the impossibility results that confound the Social Choice literature.

It is the literature of Social Choice in which the technical problems of interpersonal comparison has been addressed. In this case, utility has come to mean satisfaction of one's *political will*, and may or may not pertain to one's consumption. Generally, though not always, such satisfaction is equated with wellbeing. We will consider the social choice literature first before discussing the relationships between utility, wellbeing, and consumer choice.

1.1) Interpersonal Comparison of Preference and Social Choice

The social choice problem can be stated generally as follows: Given a community of N individuals, with M alternatives between which they may choose. Each individual has for himself rank-ordered the alternatives according to his own preference. The social choice problem then is to define the means of aggregating the individual's preferences into a single ordering that represents the entire group. Conceivably, the ordering of a single member (an autocrat) could be chosen for the whole society, though few would be persuaded of the ethical merit of such a process. An ordering could also be imposed exogenously as might be the case in a conservative theocracy. More ethically justifiable by far would be processes that consider the desires of all society's members, either equally or in some weighted fashion.[11] The process of aggregation is called a *Social Welfare Functional* (SWFL)

To the class of more common SWFLs, belong those of the utilitarian type. These are intuitively similar to what Bentham proposed. Given any pair of alternatives x and y we may say that x is

[10] Black (1972)
[11] For an overview of this literature see d'Aspremont and Gevers (2002) pp.465-76

socially preferred to y if and only if on average, the utility of all of societies members would be greater if x were chosen than if y were chosen. Symbolically, this can be written as[12]:

$$x \succ y \quad \Leftrightarrow \quad \sum_{i=1}^{N} \frac{1}{N} u_i(x) > \sum_{i=1}^{N} \frac{1}{N} u_i(y) \qquad (1.1\text{-}1)$$

The function $u_i(x)$ represents the utility experienced by the i[th] member of society, should alternative x be chosen.

Equation 1.1-1 only has meaning if utility can be identified and quantified. The use or ordinal preferences begs this question. Rather than say, x gives Frank more pleasure than y, we simply say Frank *prefers* x to y, without quantifying the strength of preference. For the sake of Equation 1.1-1, we can represent preferences numerically with *any* function $u_i(.)$ so long as $u_i(x) > u_i(y)$ whenever the i[th] individual prefers x to y.

For pair-wise comparisons equation 1.1-1 works fine. We can say that society prefers x to y if more of its members prefer x to y than vice versa. If more than two alternatives are being considered however, this process breaks down according to Condorcet's *Paradox of Voting*. The paradox is illustrated as follows: Consider three individuals; Fred, Mary, and George, who are asked to rank, according to their preference, three bundles of goods; a, b, and c. Fred prefers a to b and b to c, Mary prefers b to c to a, and George prefers c to a to b. These orderings are summarized below:

$$\begin{aligned} &Fred: && a \succ b \succ c \\ &Mary: && b \succ c \succ a \\ &George: && c \succ a \succ b \end{aligned}$$

The three are asked to vote their preference among each pair of alternatives. By a two-thirds majority, Fred and George prefer a to b. By the same majority Fred and Mary select b over c. Finally, Mary and George select c over a. We find that the social "ordering" (shown below) is inconsistent. It is unable to select any alternative as "best" or "worst".

$$Society: \quad a \succ b, \quad b \succ c, \quad c \succ a$$

This paradox is what has ultimately led to Kenneth Arrow's famous impossibility theorem. This problem cannot be circumvented unless we consider the relative intensities of the individual's preferences.

We know from experience that people in fact do consider the intensity of preference as the following example shows: Consider three individuals who are deciding a time to meet for lunch.

[12] There are numerous variants to the utilitarian SWFL given here. For a full discussion see See d'Aspremont and Gevers (2002) p.470

Two would prefer to meet at 12:30 as opposed to 11:30, but their degree of preference is slight. The third individual has an important business engagement that does not end until noon, and strongly prefers not to cut it short. One would expect the first two individuals to defer to the third and agree to meet at 12:30.

Agents' intensities of preference have been measured experimentally and several means of "cardinalizing" agent's preference have been devised.[13] Such methods however do not produce scales of measurement that are consistent from one agent to another. A method of cardinalization, which Sen discusses in *Collective Choice and Social Welfare* [14] involves scaling the agents' preferences so that his most preferred bundle is assigned a utility level of "1" while his least preferred bundle is assigned a utility of "0". Intermediate bundles are assigned utility levels in between based on empirical testing. Such utility values cannot be compared between agents. Generally, agents will not have the same best and worst alternatives, nor can it be assumed that the intensity of preference for the best over the worst alternative is the same between agents. Additionally, should a new "best" or "worst" alternative appear for any agent, the intensity values for all other alternatives would change. In this case adding the agents' intensities of preference will not produce a consistent or meaningful result. Sen provides proof, similar to that of Arrow, that cardinally measured preferences cannot be aggregated if the preferences are not measured on comparable scales.[15]

With no reliable scale for quantifying preference, neoclassical theory leaves us with no means of interpersonally comparing utility. This is as much the case in questions of consumer choice as in social choice. The problems of aggregation are the same. The unfortunate news is that this leaves us with no quantifiable measure of social wellbeing. The ordinality of preferences leaves us with Pareto Efficiency as the only standard.

1.2) Utility, Choice, and Wellbeing

In the neoclassical literature, whether it be that of social choice or consumer choice, the choices one makes, and the wellbeing one enjoys both ensue from his utility. *Choice* is the result of solving the utility maximization problem, while *wellbeing* results from attainment of what has been chosen. Problems of logic result whenever one uses a non-observable quantity such as utility, to explain an observed result such as ones choices. While it may be quite reasonable to assume that some consequent B (i.e. the choice one makes between commodities) is *usually* the result of some unobserved antecedent A (i.e. the maximization of pleasure), it is quite a stretch to presume that A *always* causes B, an even greater stretch to presume that *only* A causes B.

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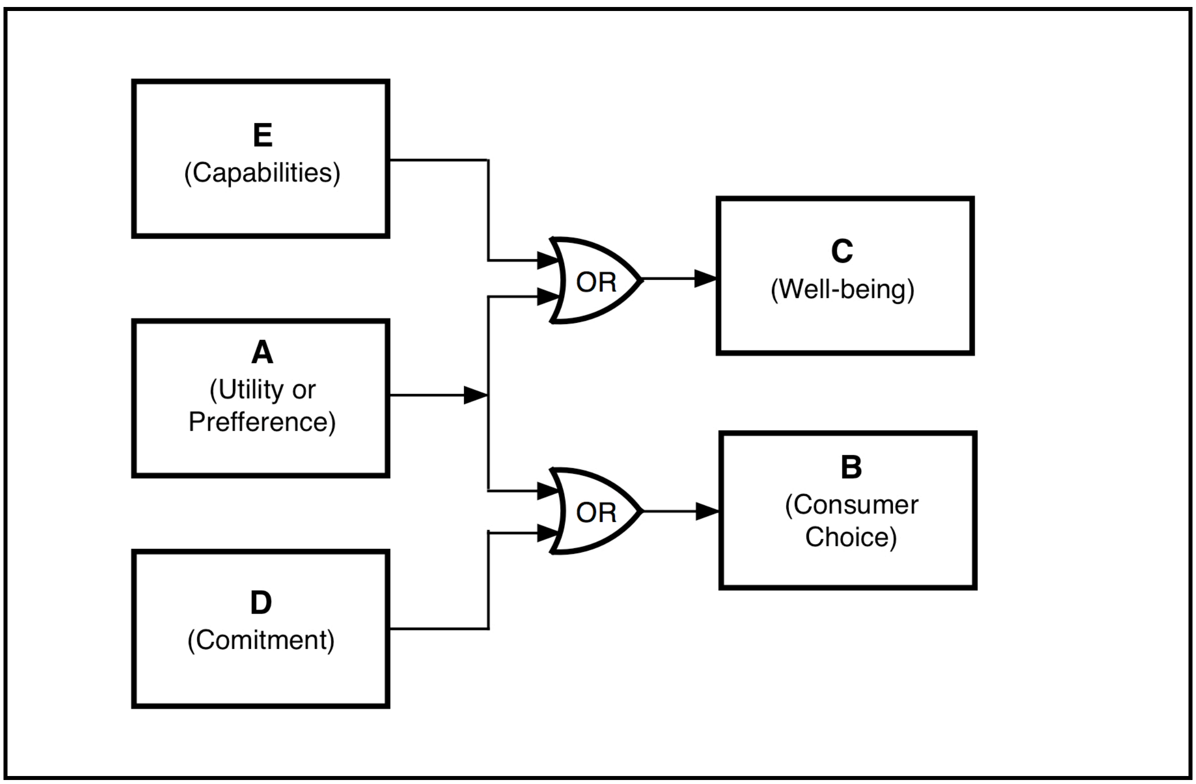
[13] Sen (1970) pp. 98-103
[14] Sen (1970)
[15] Sen (1970) pp. 128-30

As is shown in Figure 1.2-1, Erroneous results can occur when one presumes A produces some other condition C (such as wellbeing) in addition to B. Observance of B in this case would *necessarily* imply the presence of C, since both follow from A. If there were another condition D (which we have so far ignored) which gives rise to B but not C, any conclusion we might draw regarding C based on an observance of B would be incorrect, though valid given our assumptions. In practice, should B ever be observed in the absence of C, we *should* be led to conclude that the "theory" shown in Figure 1.2-1 is incorrect. It is easy however to "strategically immunize"[16] such a theory with a claim that observances of C are inherently unreliable.

Sen among others has called into question the connection between the pleasure or the perception of wellbeing and its actual achievement. According to Sen: "Well being is ultimately a matter of valuation, and while happiness and fulfillment of desire may well be valuable for a person's well being, they cannot - on their own or even together – adequately reflect the value of well being."[17]



<u>Fig 1.2-1)</u> Antecedent A produces consequents B and C, while antecedent D produces B only. Observation of B will cause one to erroneously to conclude that C must also be present if one assumes that D does not exist.

[16] For a discussion of immunizing stratagems, see Blaug (1992) pp. 17-21
[17] Sen (1987) p.46. See also Sen (1980 and 1985)

In particular, Sen points to what Bentham called the "pleasure of relief" and the "pleasure of malevolence" as examples of a lacking correspondence between perceived and actual wellbeing. In the case of relief, a battered housewife may experience great joy from a small mercy far out of proportion to the suffering that is her daily lot[18]. Bentham's "pleasure of malevolence", which is what one might enjoy while witnessing a public execution, can be seen as a special case of the pleasure one might derive from being "nosey". In *Collective Choice and Social Welfare.* Sen asks if the welfare of an agent A is really enhanced if his neighbor B sleeps on his side as opposed to his back, should A have such preference over B's behavior[19]. Extension of this argument to the enjoyment one might obtain from discrimination against minorities is quite apparent.

On the converse, it is also quite apparent that what produces actual wellbeing may not always produce a sense of pleasure. This is particularly true when the healthful benefits of some good are not understood by its consumer. A farmer in a developing country may well reject western medical treatment when he fails to understand its benefits. Indulgence in unhealthy pleasures, particularly when it comes to diet, is something all of us have experienced. Sen argues that wellbeing proceeds from achievement of *capabilities* such as health, education, and longevity that do not necessarily stimulate one's pleasure center[20]. Figure 1.2-1 allows for the possibility that wellbeing can result from either perceived wellbeing or the attainment of capabilities.

Few would argue that that pleasure *frequently* motivates a consumer's choice. To see if utility is the *only* motivation, we need to compare what Bentham calls the "pleasure of benevolence", and what Sen calls *commitment* If one could make the argument that people behave ethically because it gives them pleasure to do so (or pain in the form of guilt if they do not), then one might be able to assert that utility is the sole motivator. Sen does not accept this. He leaves open the possibility for agents to make choices based on ethical commitment, which he defined as involving "a counter-preferential choice, one where the alternative chosen provides less utility to the chooser"[21]. Ian Little recognized this in his critique of Arrow's seminal work in social choice[22] when he observed that one may prefer a distribution is that is more to his benefit, yet oppose policies leading to that distribution if he finds them unjust.[23]

Commitment needs to be distinguished from benevolence or *sympathy*, in that the wellbeing of a sympathetic person is directly impacted by the wellbeing of another. For example "If knowledge of the torture of others makes you sick, it is a case of sympathy; if it does not make you feel personally worse off, but you think it is wrong and you are ready to do something to stop it, it is a case of commitment."[24] If we allow for the possibility of committed behavior, we can no longer consider utility to be the sole determinant of an agent's choices. Figure 1.2-1 shows utility as contributing to both choice and wellbeing, but uniquely determining neither.

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[18] Sen (1987) p. 45
[19] Sen (1970) p. 79
[20] SEN (1999)
[21] Sen (1977) p.328
[22] Arrow (1951)
[23] Little (1952)
[24] Sen (1977) p.326

2) THE USE VALUE APPROACH

At this point, it appears that two methodological steps need to be taken. First, questions of consumer choice, and the value derived from it, need to be considered separately from the other questions of social choice. Second, we need to frame the discussion in terms of value *itself* without attempting to attribute too a cause such as utility.

2.1) The Concepts of Use value and Marginal Value

The concepts of value to be developed in this section are very old. Rather than creating something entirely new, what is done here is to reflect on, distil, and re articulate ideas that have been with us in some cases throughout Western History.

The notion that the value a consumer places on a good depends subjectively on the need it fills, goes back to at least to the ancient Greeks. The term *Oeconomicus,* from which our discipline gets its name, comes from two Greek words *oιxoζ* (house) and *vo'μoζ* (rule). *Oeconomicus,* which is usually translated "household management", refers to the art of arraigning the family's material goods for the purpose of providing the good life. According to Aristotle: "Property is part of the household, and the art of acquiring property is a part of managing the household; for no man can live well, or indeed live at all, unless he is provided with necessaries."[25] It is clear that such "property" is sought because it is immediately useful, and that the required quantities are limited.[26] In *Rhetoric*, in which Aristotle addresses the art or reasoning in general, he mentions qualities such as durability, security, and capacity to serve men in all seasons as sources of use value. In 1905, Oskar Krauss[27] suggested that Aristotle had anticipated the concept of marginal utility. For this he relies on Aristotle's *Topics*, 118: "A thing is more desirable if, when added to a lesser good, makes the whole a greater good. Likewise also, you should judge by means of subtraction: for the thing upon whose subtraction the remainder is [made] a lesser good may be taken to be a greater good."

The Greeks also recognized that the value one places on a good is in part due to its desirability. They would have roundly rejected the notion that value or wellbeing were determined by satisfaction. According to Democritus: "If only a few goods are desired, these will seem to be many, because a restrained demand makes poverty equivalent to wealth."[28]
Epicurus, a successor of Aristotle and an anticipator of Bentham's calculus of pleasure and pain, taught his followers to discipline their minds so as to temper their desires with judgment. "If you wish to make a person wealthy, do not give him more money, but diminish his desire."[29] The

[25] Aristotle (2003a) p.7
[26] Aristotle (2003a) p.7
[27] See Footnote #2 in Spengler (1955) p.371 for the citing of the German language text of Krauss' work.
[28] Gordon (1975) p.15
[29] Spiegel (1991) p.38

Greeks were very aware that the pursuit of desire was dangerous if not restrained by wisdom. Solon cautioned his citizens against the destructive power of greed: "But men of the city themselves, hearkening to the call of wealth, are minded by their folly to destroy a mighty city…For they no not how to check their greed or to order the good-cheer that they have, in the quiet enjoyment of the feast."[30]

The Greek concept of value, which we shall call *Use Value* is quite apt, and can be stated in summary as; *the value of an object as subjectively determined by user, according to his economic circumstances at the time.* Such value however is a deliberate assignment, not the result of some internal force to which the consumer is compelled to respond.

This concept of value is implicit in European (continental) economic thought throughout the middle ages and into the renaissance[31]. By the late eighteenth century, French economic thinkers had begun using the term "utility", though it's meaning was somewhat different from that of Bentham. According to J. B. Say "Price is the measure of utility that a thing has in men's judgment, i.e. the satisfaction men receive from consuming it."[32] Members of the French Engineering School of the early nineteenth century elaborated further; that the value one placed on a good was measurable in terms of the price he was willing to pay to acquire it[33]. This means of determining value is not adequate by itself. The value one would place on a good for purchase will naturally depend on the quantity of it the purchaser already has. Additionally, we know that the quantity one will buy varies inversely with the price. To resolve this, we introduce the notion of *Marginal Value,* which corresponds to marginal utility or what Jevons called the *degree of utility*.

Jevons' great achievement was in determining the relationship between total utility and the degree of utility, and in finding the relationship between the degree of utility and the price the consumer would be willing to pay. We are accustomed to thinking of marginal utility as derived from total utility. From Jevons' discussion it is apparent that it would have been more natural to do it the other way around. Jevons' argument runs as follows:

> "Let us imagine the whole quantity of food which a person consumes on average during a twenty–four hour period to be divided into ten equal parts. If the food be reduced by the last part, he will suffer but little; if a second part be deficient, he will feel the want distinctly; the subtraction of the third tenth part will be decidedly injurious; with every subsequent subtraction of a tenth part his sufferings will be more and more serious until he will at length be on the verge of starvation. Now if we call each of the tenth parts an increment, the meaning of these facts is, that each increment of food is less necessary, or possesses les utility than the previous one."[34]

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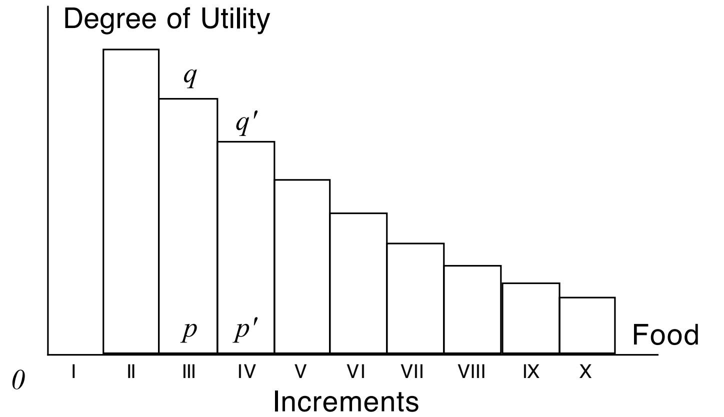
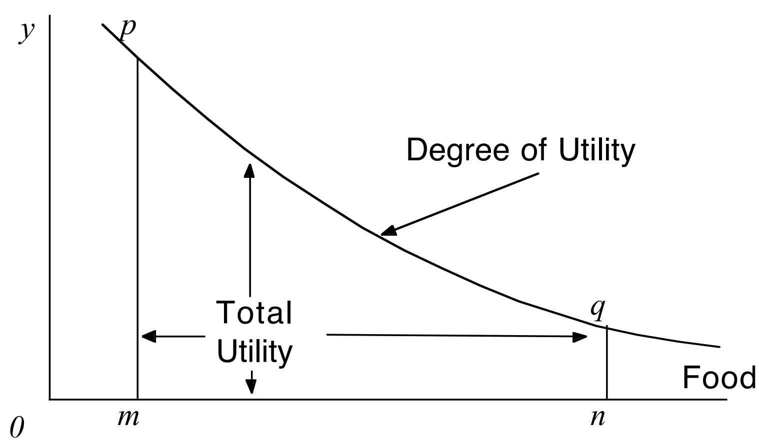
[30] Gordon (1975) p.8
[31] For a discussion of this see De Roover (1958) and Kauder (1953)
[32] Ingrao and Israel (1990) p.74
[33] Ingrao and Israel (1990) p.72-76
[34] Jevons (2003) p.420



<u>Figure 2.1-1</u> Jevons' illustration of the varying degrees of utility experienced from consumption of equal increments of food[35].

In Figure 2.1-1, reproduced from *Theory*, Jevons represents the utility obtained from each increment by the area of its corresponding rectangle. The *total* utility experienced is the sum of the areas of the rectangles. In Figure 2.1-2 he argues that if the number of increments becomes arbitrarily large and their sizes arbitrarily small, the *total utility* becomes the area under the curve corresponding to the *degree of utility*, between the first and the last increments of food consumed.



<u>Figure 2.1-2</u> As the increments of food consumed become arbitrarily many and arbitrarily small, the total utility experienced becomes the integral of the degree of utility, from the first increment to the last increment of food consumed[36].

[35] Adapted from Jevons (2003) p.417
[36] Adapted from Jevons (2003) p.418

From Jevons' discussion of exchange, it is apparent that the quantity that corresponds to observed behavior is the degree of utility, not total utility.

> "Imagine that there is one trading body possessing only corn, and another possessing only beef. … Suppose for a moment, that the ratio of exchange is approximately that of ten pounds of corn for one pound of beef: then if, to the trading body that possesses corn, ten pounds of corn is less useful than one pound of beef, that body will desire to carry the exchange further. Should the other body possessing beef find one pound less useful than ten pounds of corn, this body will also be desirous to continue the exchange. Exchange will go on until each body has obtained all the benefit that is possible, and loss of utility would result if more were exchanged. Both parties, then, rest in satisfaction and equilibrium, and the degrees of utility have come to their level, as it were." [37]

Jevons presumption that utility derived from a physiological source is certainly not necessary to his argument. His notion of utility is essentially the same as that of the French, and nearly the same as the Greek concept of Use Value. Using Jevons' framework, we can define *Marginal Value* or *Marginal Price* as the *price one would be willing to pay for* *<u>one more increment</u>* *of the good*. We have no reason to determine *why* he would pay that price; we simply observe what he is willing to do. The use value the consumer obtains from a finite quantity of the good is simply the mathematical integral of the marginal value from the first to the last increments purchased.

Consider again Figure 2.1-1. Let r_i represent the marginal value placed on (i.e. the marginal price paid for) the i[th] increment of food. The use value of consuming all 10 increments would be:

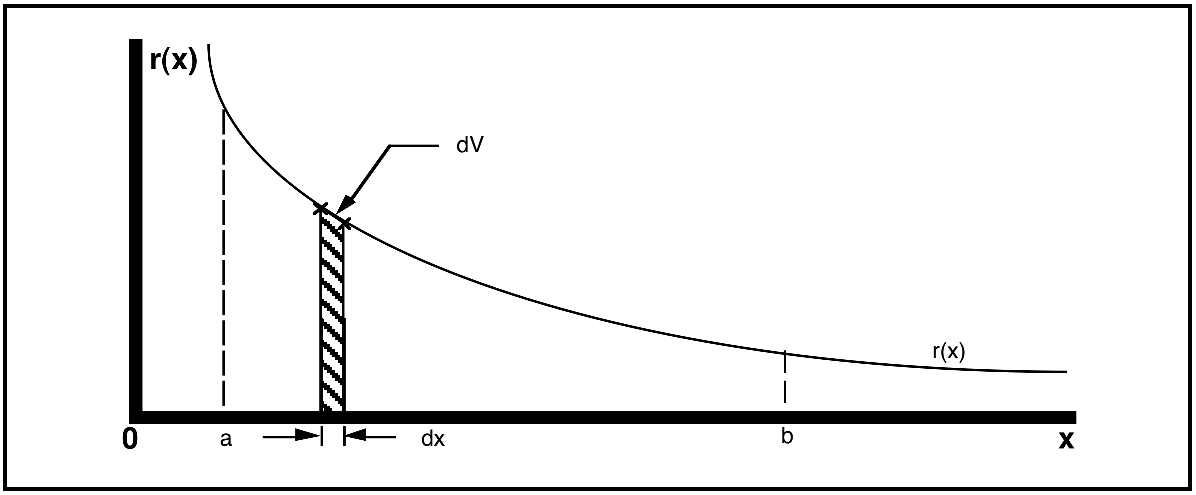
$$V = \sum_{i=1}^{10} r_i \qquad (2.1\text{-}1)$$

If we make the increments arbitrarily small, r_i becomes a *function* of x. We say that, if the consumer possesses (or has already consumed) a quantity x of the good, the price he would be willing to pay for an additional increment would be $r(x)$. Assume now that the consumer starts with some quantity a of the good. Consider also that he purchases a finite quantity $b-a$ in many tiny increments from a perfectly discriminating monopolist (In this case, he pays his marginal price for each increment.) The use value the consumer places on the quantity $b-a$ is:

$$V(b-a) = \int_a^b r(x)dx \qquad (2.1\text{-}2)$$

This is as illustrated in Figure 2.1-3

[37] Jevons (2003) p.430



<u>Figure 2.1-3 Marginal Price (Marginal Value) and Use Value</u>. The use value placed on the quantity of commodity *b-a* is the integral of the reservation price function from *a* to b

The quantity a that the consumer had in his possession initially influences $V(b-a)$ as is accounted for in the lower limit of integration. If the consumer were to acquire an additional quantity $c-b$ the value $V(c-b)$ would differ from $V(b-a)$ even if $c-b=b-a$. This is of course due to the diminished marginal value resulting from the prior purchase.

Since use value and marginal value are functionally similar to utility and marginal utility, the results of neoclassical theory as pertains to exchange and equilibrium are preserved. What has changed is that both marginal value and use value are *measurable in terms of the medium used for exchange*. Since the scale can be made common to all consumers. Interpersonal comparison is possible. Since use value and marginal value exist as a matter of definition, their connection to the consumer's well being is not automatic. Additionally, the results of interpersonal comparison must be interpreted with some care.

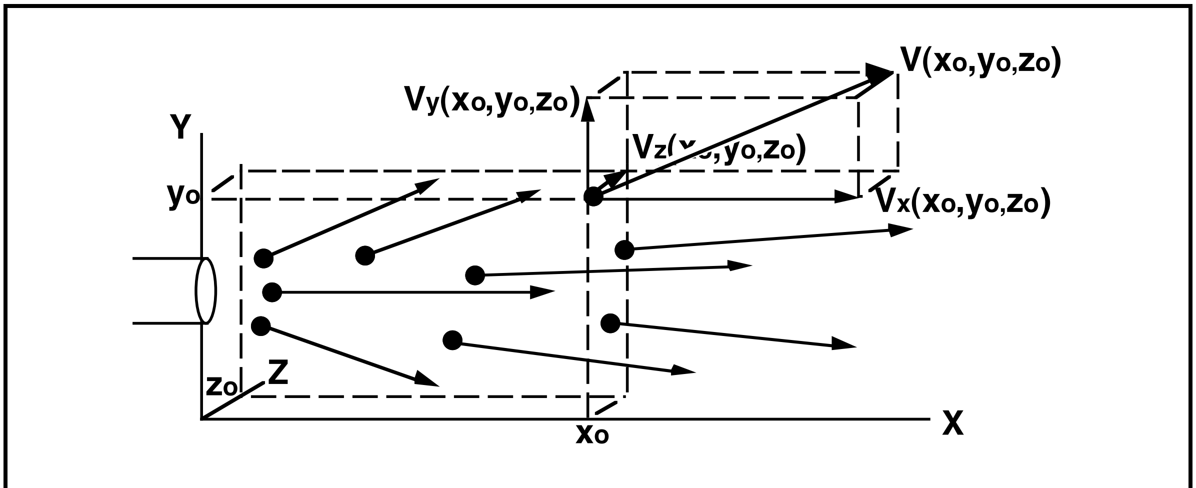
2.2) Vector Calculus and the Problem of Integrability

Matters become considerably more difficult when multiple goods are involved. We will need to employ the methods of vector calculus as it is used in basic physics[38]. Neoclassical economists are accustomed to regarding vectors as linear arrays of objects that are not necessarily related. In elementary Physics, vectors are regarded as *single numbers* that may be expressed with multiple components. Vectors can be used to express quantities such as velocity, which have both *magnitude* and *direction*. The velocity of a vehicle is not merely "60 mph." it is "60 mph. *in a northeasterly direction*". The force of impact when two automobiles collide is determined by both the speed and direction of travel of the vehicles involved. A vector describing motion (or a

[38] For a thorough discussion of vector calculus see Davis and Snider (1975) and Lovelock and Rund (1989)

position) in three dimensions will typically require three components to express, one for each coordinate axis. Vectors can be represented graphically with arrows having length corresponding to their magnitude, and pointing in the appropriate direction.

In physics, vector *functions* are used to show the motion of fluids such as gas escaping from a pipe as shown in Figure 2.2-1. The vector at each point within the fluid describes the velocity that a particle would travel if placed in the fluid at that point. The reader is likely familiar with vector diagrams illustrating the flow of ocean currents.



<u>Figure 2.2-1 Gas escaping from an exhaust pipe.</u> Vectors depict the speed and direction of travel of a particle placed at their point of origin.

To begin our discussion of use and marginal value, we start with the following assumption:

> <u>Assumption 1</u>: *For every bundle of goods the consumer might possess, he knows how much of any one good he would exchange for an increment of any other*.

We will denote the bundle he possesses with the vector $\vec{x}$. The set of exchanges he would make constitute his marginal value, which is now a *vector* $\vec{r}$, which is a function of $\vec{x}$.

Consider an economy in which there are three goods: apples a, bananas b, and pears p, which can be exchanged using a numeraire good n. Assume the trader has x_a apples, x_b bananas, and x_p pears in his possession. We can represent this bundle as a vector:

$$\vec{x} = x_a\hat{\varphi}_a + x_b\hat{\varphi}_b + x_p\hat{\varphi}_p \qquad (2.2\text{-}1)$$

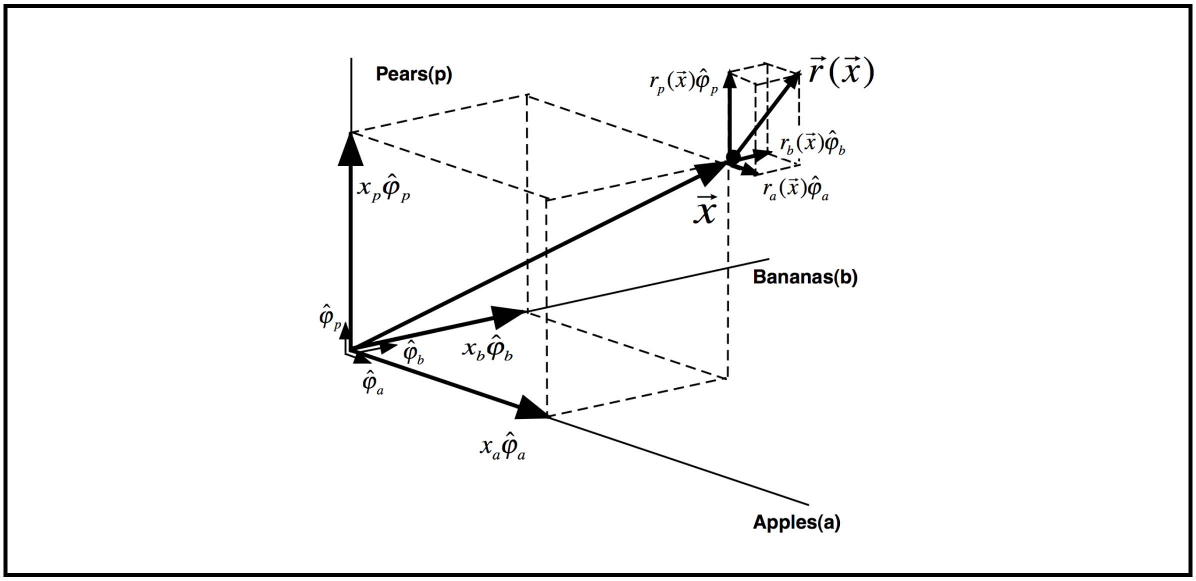


Figure 2.2-2

Each term in the sum represents the component of the vector corresponding to its respective good. Each component consists of a scalar coefficient x_i multiplied by a basis vector $\hat{\varphi}_i$ signifying the good to which it corresponds.[39]

As is shown in Figure 2.2-2, the set of all possible bundles consisting of apples, bananas, and pears can be represented by the set of points $\vec{x}$ which fills a three dimensional space. Per the assumption above, for every possible $\vec{x}$ there corresponds a vector $\vec{r}(\vec{x})$ indicating the exchanges the consumer would be willing to make if he possessed $\vec{x}$. $\vec{r}(\vec{x})$ is a vector function, sometimes called a vector *field*. A vector field can be represented graphically by joining the vector arrows associated with each point into streamlines as shown in Figure 2.2-3

The marginal value function $\vec{r}(\vec{x})$ can be written in a manner similar to the vector $\vec{x}$:

$$\begin{aligned}\vec{r}(\vec{x}) &= r_a(\vec{x})\hat{\varphi}_a + r_b(\vec{x})\hat{\varphi}_b + r_p(\vec{x})\hat{\varphi}_p \\ &= r_a(x_a,x_b,x_p)\hat{\varphi}_a + r_b(x_a,x_b,x_p)\hat{\varphi}_b + r_p(x_a,x_b,x_p)\hat{\varphi}_p\end{aligned} \qquad (2.2\text{-}2)$$

The coefficient $r_a(\vec{x})$ is a function representing the consumer's marginal value for apples. Note that this function depends on the quantity of *all* goods in the consumer's possession, (as is pedantically emphasized by the second line of the equation above).

[39] Technically, x is the vector sum of its components. Each component consists of a basis vector of magnitude 1 multiplied by a scalar.

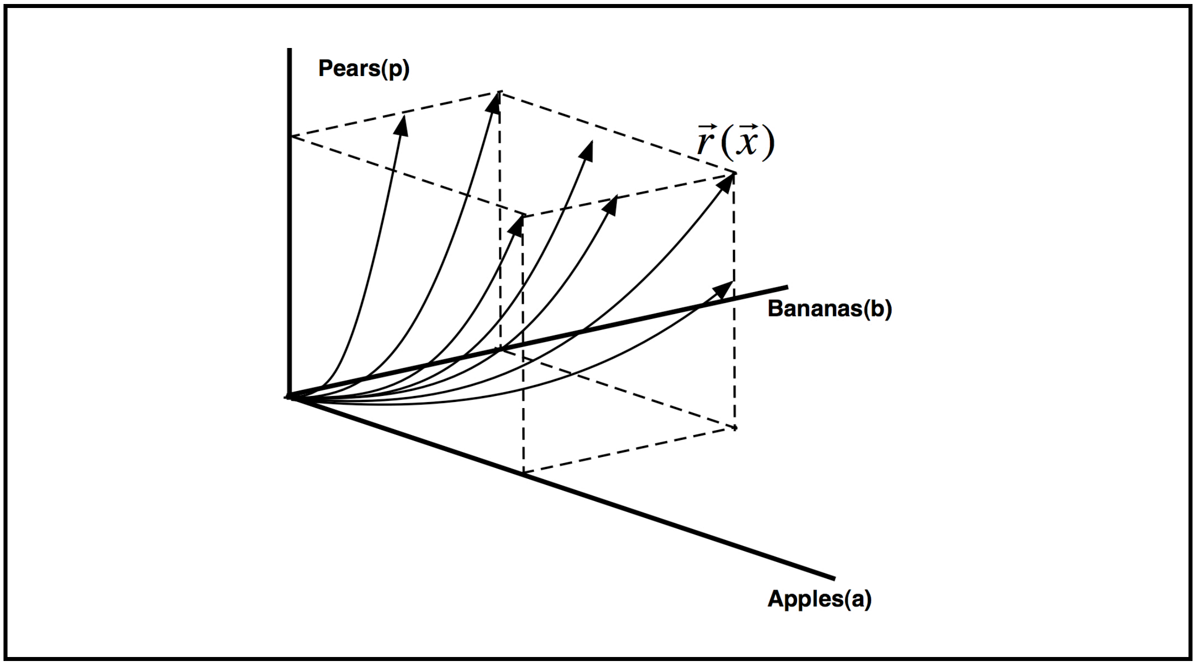


Figure 2.2-3

Finding the use value is an extension of the of the integration process done earlier. The consumer starts with some bundle $\vec{a}$ consisting of a combination of the three goods. Consider now that he purchases a new bundle $\vec{b}-\vec{a}$ in many tiny increments from the same perfectly discriminating monopolist. This time the consumer can take his increments in any one of many forms. His first increment might be all banana, his second might consist of equal parts or each good, while his third increment might consist entirely of pear. If we were to plot these increments on 2.2-3, his first increment would be a differentially short vector parallel to he banana axis, his second increment would point in a 45 degree angle from all axes, while the third parallels the pear axis. In short, the order in which the consumer incrementally acquires his goods determines the path he takes from $\vec{a}$ to $\vec{b}$ as shown in Figure 2.2-4. To find the use value of $\vec{b}-\vec{a}$ We need to perform a vector line integral along the path the consumer takes.

$$V\left(\vec{b}-\vec{a}\right)=\oint_{path}\vec{r}(\vec{x})\bullet d\vec{x} \tag{2.2-3}$$

From Figure 2.2-4 notice that every differential segment of the path $d\vec{x}$ cuts the streamlines of $\vec{r}(\vec{x})$ at a (variable) angle θ. The above integrand is a vector dot product, which is understood to mean:

$$\vec{r}(\vec{x})\bullet d\vec{x}\triangleq magnutude\left(\vec{r}(\vec{x})\right)\times magnitude\left(d\vec{x}\right)\times\cos\theta$$

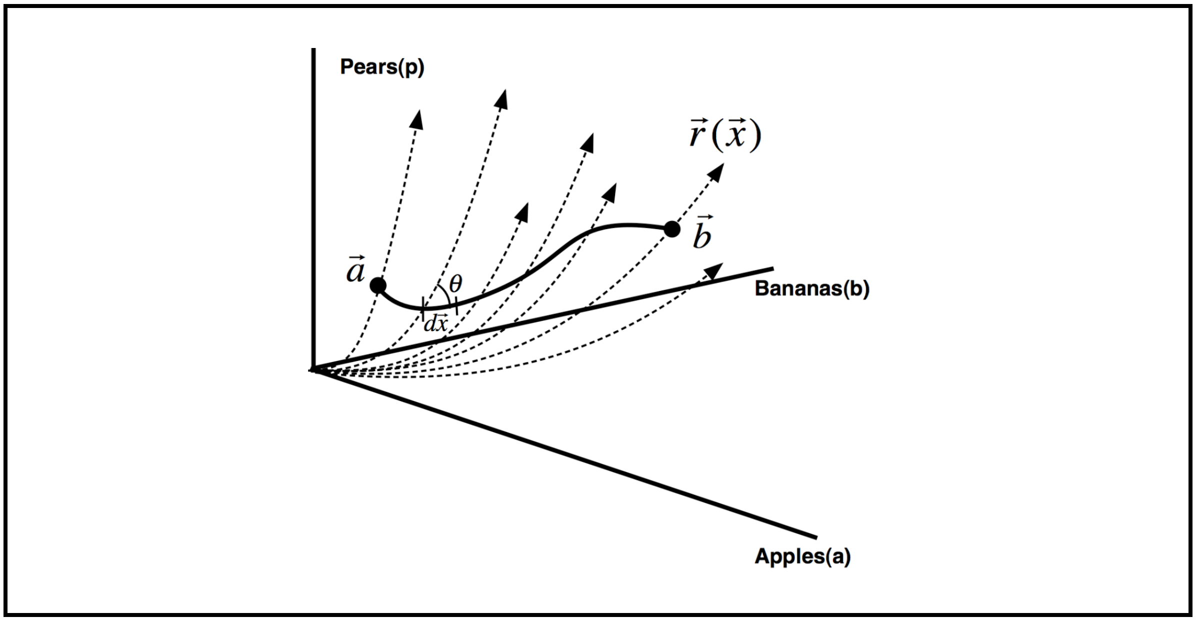


Figure 2.2-4

Unfortunately, for any general function $\vec{r}(\vec{x})$ the value of the integral will depend on the path taken. Economically, this would mean that the value a consumer places on a bundle of goods would depend on the *order in which the goods are consumed*. This is what is referred to in the literature as the *problem of integrability*. Pareto, when he first encountered the matter in 1906, argued that the order of consumption might be economically significant. Due to the complementary influences of soup and meat, the benefit gained by a man who consumes his soup before his meat might differ from the benefit gained if he were to first consume his meat[40].

Later commentators such as Edwin Wilson[41] in 1912, and sir John Hicks[42] in the 1930's regarded path dependence (i.e. non integrability) to be meaningless economically but were unable to show analytically that it must be so. Using Paul Samuelson's theory of *Revealed Preference,* Hendrik Houthakker showed in 1950 that path dependence would necessarily imply that the agents' choices were inconsistent[43]. Houthakker showed that, if the path of consumption did matter, an agent could be caused to reveal that he preferred a given bundle to itself. He ruled out such cases by imposing his *Strong Axiom of Revealed Preferences* (SARP). With an argument similar to that used by Howthakker[44] and Samuelson[45], it can be shown that, if the order of consumption matters, a consumer's use value for a given bundle can be made arbitrarily large, by repeatedly taking it away then returning it to him.

[40] Paretto (1906)
[41] Wilson, E (1912)
[42] Hicks and Allen (1934)
[43] Houthakker, (1950)
[44] Houthakker, (1950)
[45] Samuelson (1950)

Consider a consumer who initially has some bundle x_0 , then acquires the goods needed to bring him to a new bundle x_1. The use value he derives from this process is:

$$V(\vec{x}_1 - \vec{x}_0) = \int_{\vec{x}_0}^{\vec{x}} \vec{r}(\vec{x}) \bullet d\vec{l} \qquad (2.2\text{-}4)$$

We now give the agent an additional bundle so as to bring him to $\vec{x}_2$ as shown in Figure 2.2-5. This gives him a change in use value:

$$\Delta V_{1,2} \triangleq V(\vec{x}_2 - \vec{x}_1) = \int_{\vec{x}_1}^{\vec{x}_2} \vec{r}(\vec{x}) \bullet d\vec{l} \qquad (2.2\text{-}5)$$

We repeat the process N times, bringing our agent to bundles $\vec{x}_2, \vec{x}_3, \ldots \vec{x}_n$ in sequence. From x_n , we return the agent to bundle $\vec{x}_1$. The total use value gained (or lost) in these series of exchanges is:

$$\Delta V_{1,2} + \Delta V_{2,3} + \cdots + \Delta V_{i,i+1} + \cdots + \Delta V_{N-1,N} + \Delta V_{N,1} \qquad (2.2\text{-}6)$$

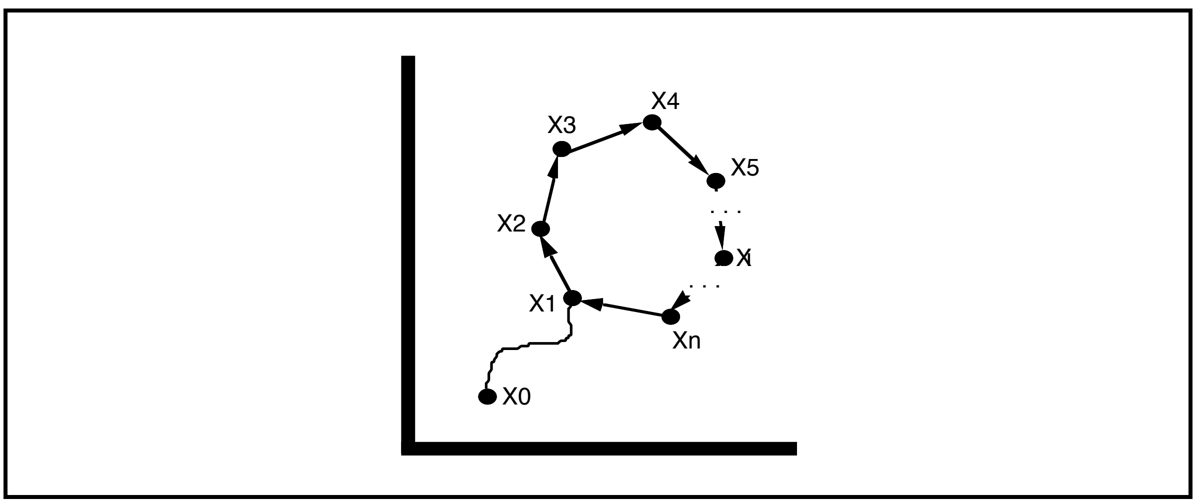


FIGURE 2.2-5

The agent now has the same bundle of goods he did before we led him around the circle of Figure 3.3-2. If the use value he derives from x_1 is any higher (lower) than $V\left(x_1 - x_0\right)$, it can be made arbitrarily high (low) by leading him around the circle the appropriate number of times. We prevent this problem with the following assumption:

> **Assumption 2:** *The Use Value obtained from any bundle of goods is a function of the consumer, the good themselves, and is independent of the sequence in which the consumer acquires them.*

If this assumption is satisfied, then the total use value gained or lost by leading the consumer around in circles must be zero. If this is true, then the use value gained by taking the consumer from x_1 to, say, x_5 as is shown in Figure 2.2-5 must be the same as if we took him counter clockwise from x_1 to x_5 by way of x_n . If Assumption 2 holds, then by *Poincare's Lemma*, a special case of *Stokes' Theorem*, the following statements must be true:

1) The marginal value function is the *gradient* of the use value function. This means that the component of the marginal value corresponding to a good is the partial derivative of the use value with respect to that good. For our three good example this can be written:

$$\vec{r}(\vec{x}) = \frac{\partial V(\vec{x})}{\partial x_a}\varphi_a + \frac{\partial V(\vec{x})}{\partial x_b}\hat{\varphi}_b + \frac{\partial V(\vec{x})}{\partial x_p}\hat{\varphi}_p \qquad \text{(2.2-7)}$$

To simplify the notation, we use the gradient operator: $\nabla \triangleq \sum_i \frac{\partial}{\partial x_i}\hat{\varphi}_i$ and write $\vec{r}(\vec{x}) \equiv \nabla V(\vec{x})$

2) The *curl* of the marginal value function is zero. This means that there can be no tendency for the streamlines of $\vec{r}(\vec{x})$ to curl, twist, or form a vortex. This condition is satisfied if, for every pair of components $r_i(\vec{x})$ and $r_k(\vec{x})$:

$$\frac{\partial r_i(\vec{x})}{\partial x_k} - \frac{\partial r_k(\vec{x})}{\partial x_i} = 0$$

Using the gradient operator this can be written simply as: $\nabla \times \vec{r}(\vec{x}) \equiv 0$

It is the zero-curl condition, sometimes called the *Antonelli* [46] condition that is important in practical work. As will be discussed in the last section of this paper, any $\vec{r}(\vec{x})$ one might wish to propose may need to be adjusted so as to satisfy the zero-curl condition.

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[46] Antonelli (1971)

2.3) Interpersonal Comparison and The Typical Agent

A convenient feature of vector fields is that they can easily be added or multiplied by a constant. Were we to have a community of N consumers, with the i[th] consumer having a marginal value function $\vec{r}^{\,i}(\vec{x})$, we could easily define an aggregate marginal value function to be the average of the individual consumer's fields, i.e.:

$$\vec{R}(\vec{x}) \triangleq \frac{1}{N}\sum_{i=1}^{N} r^i(\vec{x}) \qquad (2.3\text{-}1)$$

While this resembles the utilitarian SWFL of Equation 1.1-1 its interpretation is quite different. $\vec{R}(\vec{x})$ is the marginal value function that would characterize a "typical" member of the community. Rather than measure the community's well being, it predicts the choices the typical individual would make, given any bundle he might possess. Equation 2.3-1 is a sum of functions, not values. Like all functions, $\vec{R}$ evaluated for some bundle x_0 is the sum of the constituent functions r^i, also evaluated at x_0 as shown in Figure 2.3-1.

To put this in a social context, consider the following thought experiment that is derived from John Rawls famous "veil of ignorance" scenario. In Rawls' thought experiment, all members of the community are placed behind a "veil of ignorance" that prevents them from knowing what place they will occupy in society, their state of health, or what goods they will possess. For each possible situation they might find themselves in, they are asked to reveal the choice they would consider most just.

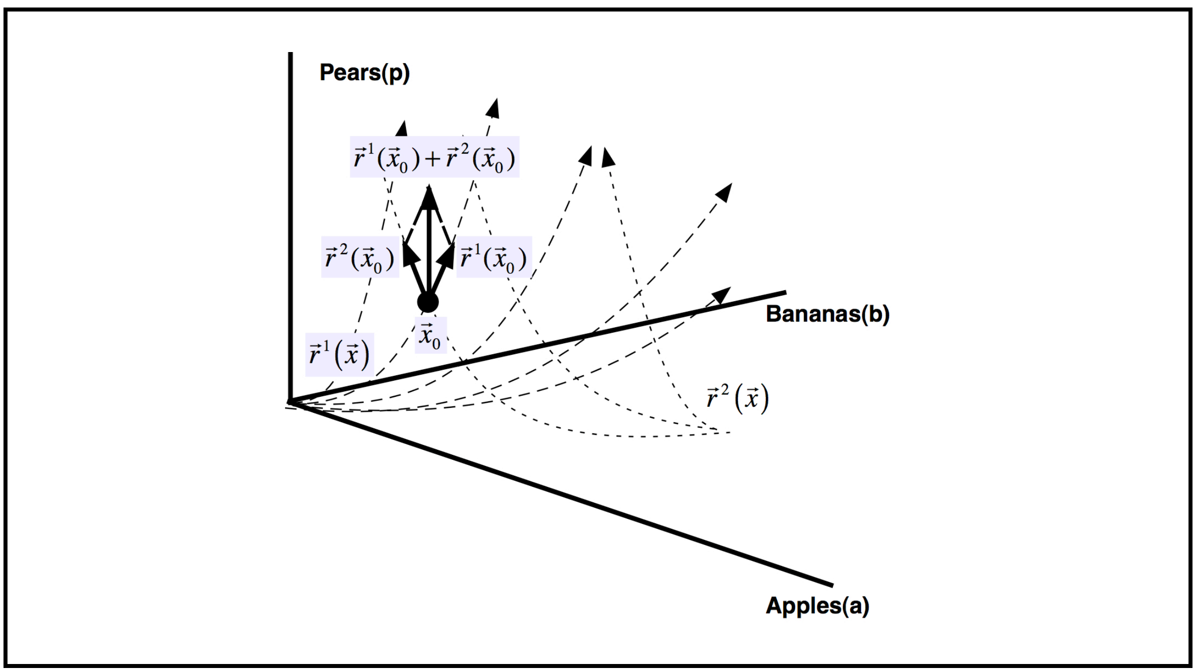

Figure 2.3-1 Addition of Marginal Value Fields

For each possible *situation*, the average response is found for all agents. Taken together, the average responses for all situations form the community's consensus as to how one should act under any circumstance. It is only after all participants have agreed to abide by the consensual standard that it is revealed to them what their individual circumstances will be.

The thought experiment embodied in Equation 2.3-1 is similar to that of Rawls, to the extent that the individual consumers are asked to reveal their marginal prices for any bundle they might hold. It is important to remember that the bundles they hold represent the economic and social circumstances upon which their decisions are conditioned. We would expect an individual's choices to be influenced both by his circumstances and by his tastes. If tastes are truly individual, and vary unpredictably from person to person, their influence should be zero in the average. Thus, the choices made by a typical agent would thus depend entirely on his circumstances, which are both observable and subject to influence by policy.

3) THE TYPICAL AGENT AND MASLOW'S HEIRARCHICAL NEEDS

In reality, the characterization of the typical agent will require a far more practical approach than that used in Section 2.3. Researchers in the marketing profession have made considerable progress along those lines. For decades they have sought to predict the goods and services one would buy, based on their demographic profile. Young, single professionals tend to spend a significant portion of their incomes on luxury goods that facilitate "the mating game". Once married, these same individuals spend heavily on the appliances needed by a growing family[47]. Alternatively, a person of low income would be expected to spend what he is able to on food, clothing and shelter. If he lives in an American inner city, security (safety) becomes a concern, increasingly so as the consumer becomes older.

Used extensively in market research is the theory of hierarchical needs developed by Abraham Maslow[48]. Put simply, Maslow believed in the existence of needs that are common to all humans, and that we seek to meet these needs in a hierarchical order. This hierarchy begins with physiological needs such as food, sleep, and shelter, and then moves on to personal security, which constitutes the second order. Third comes the need for kinship and intimacy. The highest order needs consist of those related to esteem and the ability to use ones talents in a creative and self-expressive manner[49]. The extent to which a consumer is able to satisfy his needs is a function of his circumstances. Maslow would never have claimed that people are largely alike, far from it. Maslow would have argued that man is a complex and creative creature, using decision-

[47] Kotler (1994)
[48] Kotler (1994)
[49] Maslow (1943)

making criteria unique to each individual. On the other hand, he argued, there are conditions that must be met for any individual to lead a healthy productive life. These conditions or *needs* are common to all members of the species. The inability of any individual to meet the most basic of these needs will likely lead to his or her behaving in a socially dysfunctional manner.[50] Needs, as Maslow conceived them are broad categories, somewhat resembling Sen's Capabilities[51], upon which the United Nation's Human Development Index (HDI) is built

3.1) The Typical agent

To construct an economic model of a typical agent, we must first assume that we can identify a set of goods corresponding to the needs the agent seeks to meet, one good for each need in the hierarchy. To simplify the discussion we shall consider only two goods, one (x_1) satisfying a lower order need ant the other (x_2) a higher need. A more technical discussion that considers an arbitrary number of goods is differed to another work[52]. The components of the marginal value function for the two goods are $r_1(\vec{x})$ and $r_2(\vec{x})$ respectively. The vector $\vec{x}$ represents the consumer's bundle consisting both goods. We start by proposing a margin value function $\tilde{\vec{r}}(\vec{x}) = \tilde{r}_1(\vec{x})\hat{\varphi}_1 + \tilde{r}_2(\vec{x})\hat{\varphi}_2$ based purely on economic intuition. We shall find however that the proposed components $\tilde{r}_1(\vec{x})$ and $\tilde{r}_2(\vec{x})$ will require some adjusting in order to satisfy the zero-curl condition for integrability. The adjustment process will be discussed as we move along.

We know that the value she places on a marginal increment of either good must diminish in some consistent manner. The simplest manner by which such diminution can take place is by proportion. That is, the marginal value diminishes by a fixed proportion, with each increment of the good consumed. To extend Jevons' famous example, we would say that the marginal value obtained from each increment of food, diminishes by, say, one-fifth with each increment. This relationship is expressed by the differential equation:

$$\frac{dr}{dx} = \frac{1}{\lambda} r(x) \tag{3.1-1}$$

Equation 3.1-1 says that the *rate* (dr/dx) at which the margin value diminishes is a fraction ($1/\lambda$) of its current value. Equation 3.1-1 is solved by:

$$r^0 e^{-x/\lambda} \tag{3.1-2}$$

The constant r^0 represents the value of $r(x)$ when $x = 0$ (i.e. before the consumer takes his first increment.) The constant λ in this case represents the quantity the consumer must acquire before her margin value drops to $1/e$ (approximately ½) of its initial value. The quantity λ is referred to as the *half-life* of consumption. To stretch Jevons' example even further; If the

[50] Lowry (1973)
[51] Sen (1999)
[52] McLaren (Forthcomming)

consumer's half life for food consumption is five increments, then his marginal value is $r^0/2$ after his first five increments, $r^0/4$ after 10 increments, $r^0/8$ after 15 increments, and so on.

We now apply the general relationship in Equation (2) to the two goods in question using the following thought experiment: Consider the consumer as receiving a constant stream of income I. In other words she receives a flow of a numerare good n in differentially small increments Idn, one increment in each of many small time intervals dt such that $I = dn/dt$. The consumer divides each increment into expenditure on x_1, and / or x_2 according to her marginal value for each. Whatever remains is saved. Since x_1 is the lower order good, we assume she spends all of her first increment on x_1 hence, when $x_1 = x_2 = 0$:

$$Idn = r^0 e^{-x_1/\lambda_1} dx = r^0 dx \quad \Rightarrow \quad r^0 = I$$

With each succeeding increment of x_1 purchased, the amount she is willing to pay for an additional increment of x_1 diminishes as $Ie^{-x_1/\lambda_1} dn$. This leaves $I(1 - e^{-x_1/\lambda_1})dn$ to be spent on x_2, or saved as is shown in Figure 3.1-1

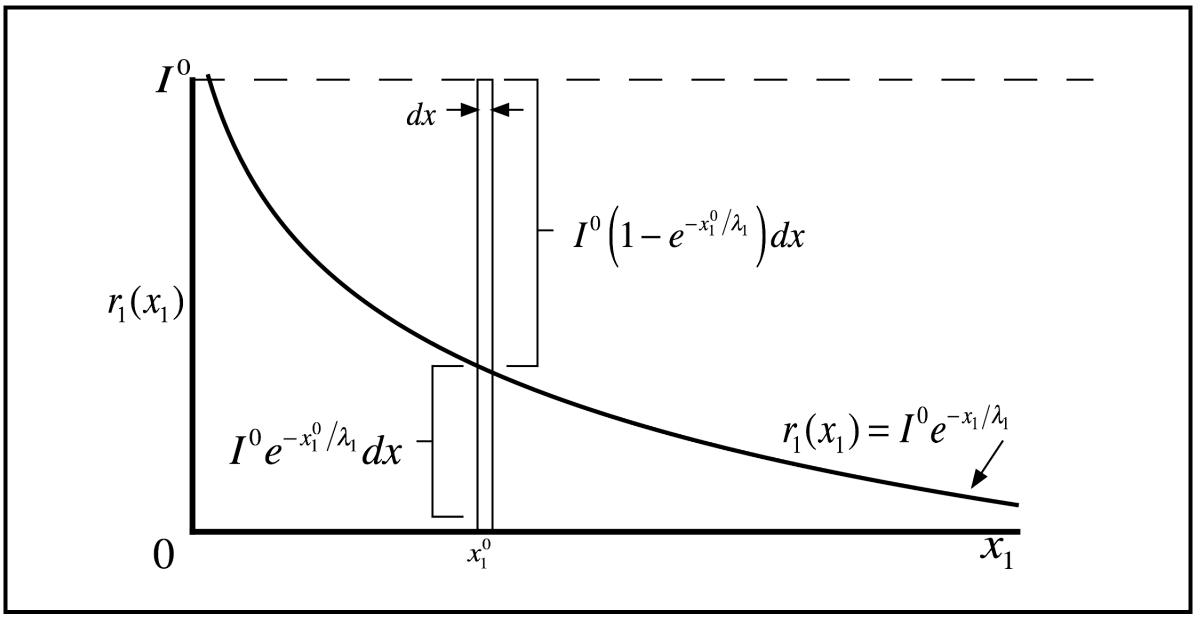

Figure 3.1-1

If the consumer has none of the higher good x_2, we assume, as before that she will spend this remainder in its entirety on x_2. As she begins to consume some x_2, her marginal value for it diminishes exponentially as does her margin price for x_1. The marginal value r_2 she places on an increment of x_2 thus increases with x_1 and diminishes with x_2:

$$r_2(\vec{x}) = I^0(1-e^{-x_1/\lambda_1})e^{-x_2/\lambda_2} \tag{3.1-3}$$

while:

$$r_1(\vec{x}) = I^0\left(1+\frac{\lambda_1}{\lambda_2}\left(1-e^{-x_2/\lambda_2}\right)\right)e^{-x_1/\lambda_1} \tag{3.1-4}$$

The bracketed term in Equation (4) has been added to insure that $\vec{r}(\vec{x}) \triangleq r_1(\vec{x})\hat{\varphi}_1 + r_2(\vec{x})\hat{\varphi}_2$ satisfies the zero curl condition for integrability. This term represents the impact of any x_2 the consumer might possess prior to acquiring any x_1. Plots of r_1 and r_2 are given as Figure 3.1-2 below. Note that the axes are scaled in terms of x_i/λ_i rather than x_i. The coordinates are thus stated in terms of half-lives of consumption rather than in absolute quantities of the goods consumed. As the plots show, exponentially diminishing functions drop to a small fraction of their values within the first two half lives.

Since the constant I^0 is arbitrary, it can be set to one without loss of generality. Note from Figure 3.1-2, that the plot of r_1 corresponds to Figure 3.1-1 if the consumer initially has no x_2. Any x_2 that she might have initially inherited simply acts as additional wealth that can be exchanged for x_1, raising the marginal value placed on x_1. Components r_1 and r_2 are combined in the vector streamline plot of $\vec{r}(\vec{x})$ shown in Figure 3.1-3.

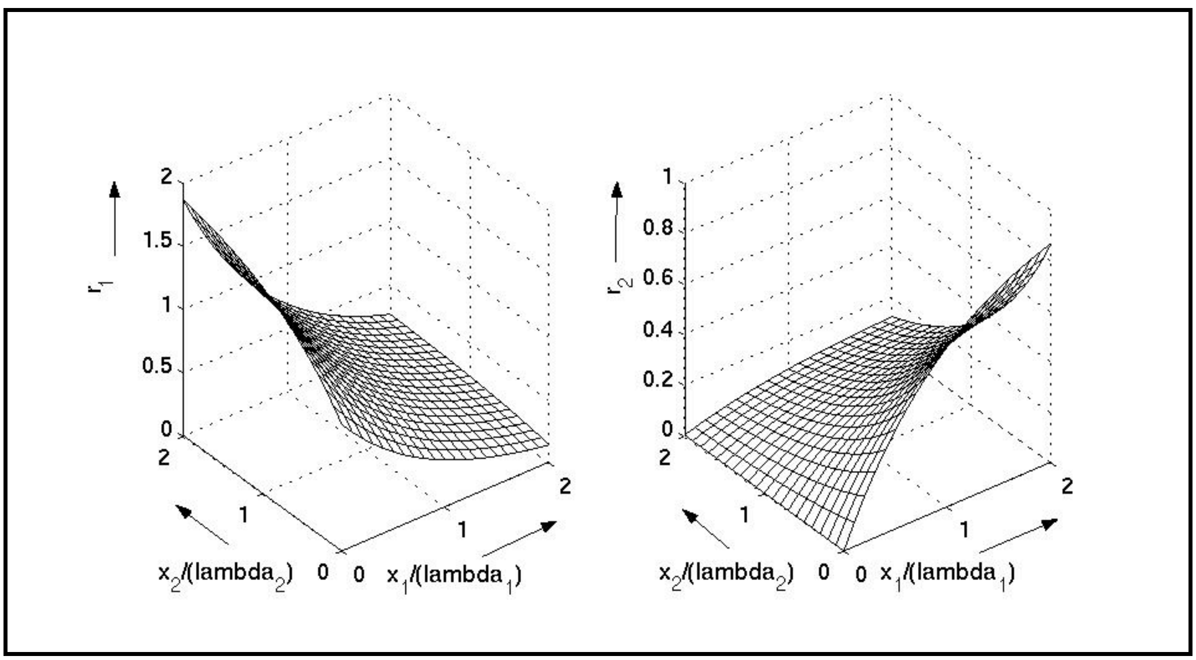

Figure 3.1-2

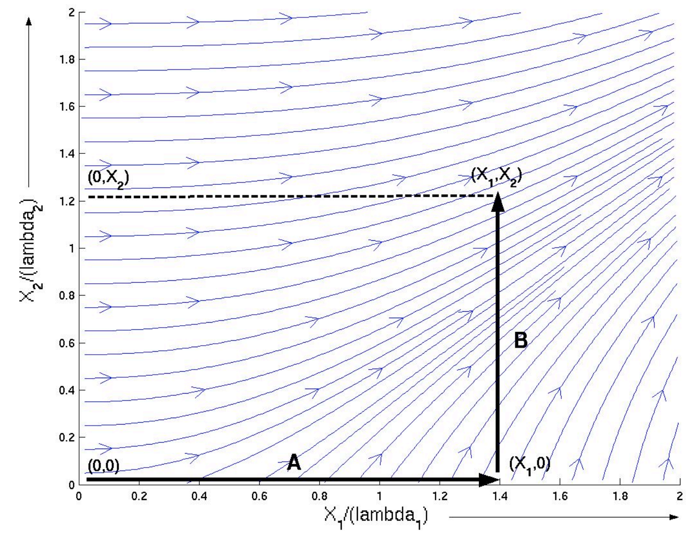

Figure 3.1-3

To find the use value $V(\vec{x})$ we take the integral of $\vec{r}(\vec{x})$ from the origin to x along the path AB as shown in Figure 3.1-3. Along A, $r_2(\vec{x})\hat{\varphi}_2$ is perpendicular to the path and hence its dot product with $d\vec{x}$ is zero. Similarly, $r_1(\vec{x})\hat{\varphi}_1$ is perpendicular along B and also makes no contribution to the integral. The two variable line integral thus reduces to a pair of single variable integrals:

$$
\begin{aligned}
V(\vec{x}) &\triangleq \oint_{AB} \vec{r}(\vec{x}) \bullet d\vec{x} \\
&= \oint_A r_1(x_1, x_2) dx_1 \qquad + \oint_B r_2(x_1, x_2) dx_2 \\
&= \left(1 + \frac{\lambda_2}{\lambda_1}\left(1 - e^{-x_2/\lambda_2}\right)\right) \int_0^{x_1} e^{-x_1/\lambda_1} dx_1 \qquad + (1 - e^{-x_1/\lambda_1}) \int_0^{x_2} e^{-x_2/\lambda_2} dx_2
\end{aligned}
$$

Since $x_2 = 0$ everywhere along path A, the bracketed term preceding the first integral of the last line above reduces to one leaving:

$$
\begin{aligned}
&= \int_0^{x_1} e^{-x_1/\lambda_1} dx_1 \qquad + (1 - e^{-x_1/\lambda_1}) \int_0^{x_2} e^{-x_2/\lambda_2} dx_2 \\
&= \lambda_1\left(1 - e^{-x_1/\lambda_1}\right) \quad + \lambda_2\left(1 - e^{-x_1/\lambda_1}\right)\left(1 - e^{-x_2/\lambda_2}\right) \qquad \text{(3.1-5)} \\
&= \lambda_1\left(1 - e^{-x_1/\lambda_1}\right)\left(1 + \frac{\lambda_2}{\lambda_1}\left(1 - e^{-x_2/\lambda_2}\right)\right) = V(\vec{x})
\end{aligned}
$$

A plot of $V(\vec{x})$ appears as Figure 3.1-4

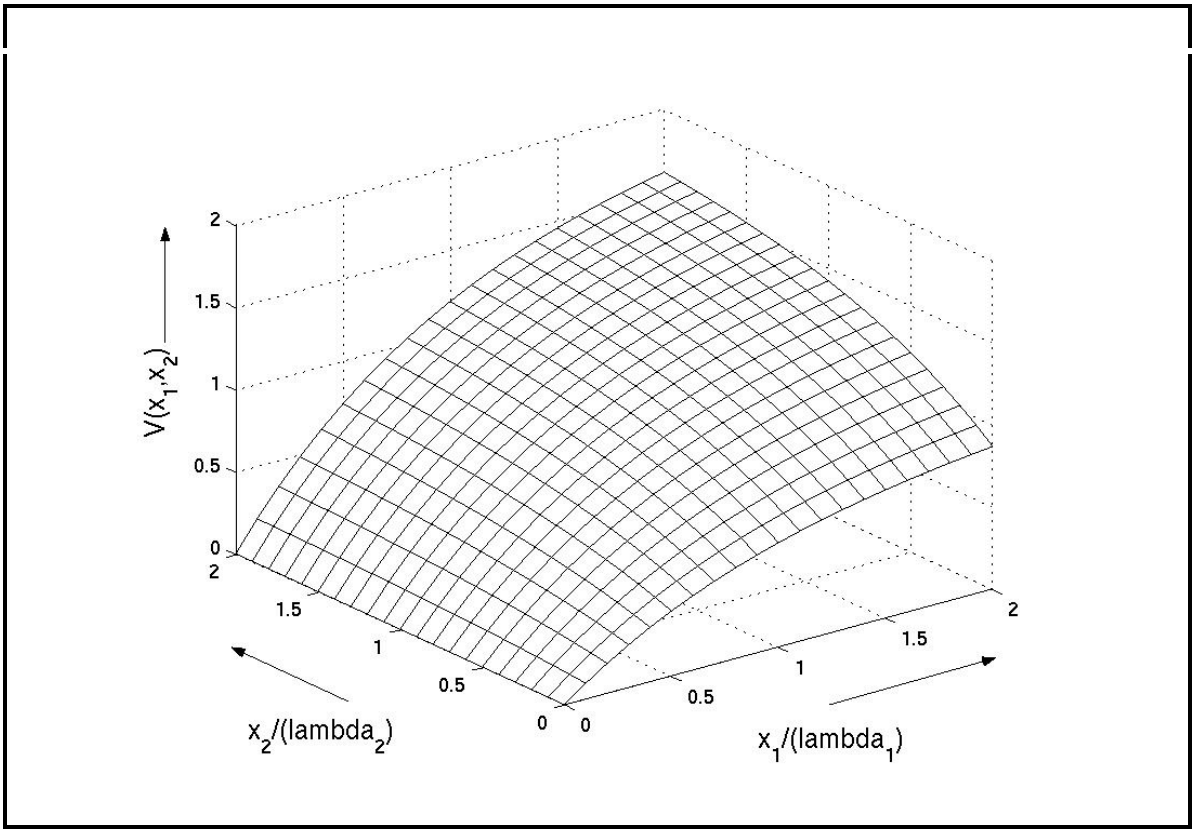

Figure 3.1-4

From Figure 3.1-4, notice that if the consumer began with none of the higher order good (x_2), he would travel up the right edge of the surface from the origin as he acquired x_1. If he began with some initial endowment of x_2 his starting point would be shifted leftward along the x_2. As he consumes x_1 his use value rises faster by virtue of the fact that he is free to trade away his endowment of x_2.

Figure 3.1-5 contains a contour plot of the use value function of Figure 3.1-4. These contours correspond to the indifference curves familiar to neoclassical theory. Each curve of course represents a locus of bundles upon which the consumer places equal value. Superimposed on the contour plot is the marginal value function as shown in Figure 3.1-3. Since $\vec{r}(\vec{x})$ is the gradient of $V(\vec{x})$ the streamlines indicate the direction in which $V(\vec{x})$ increases most rapidly.

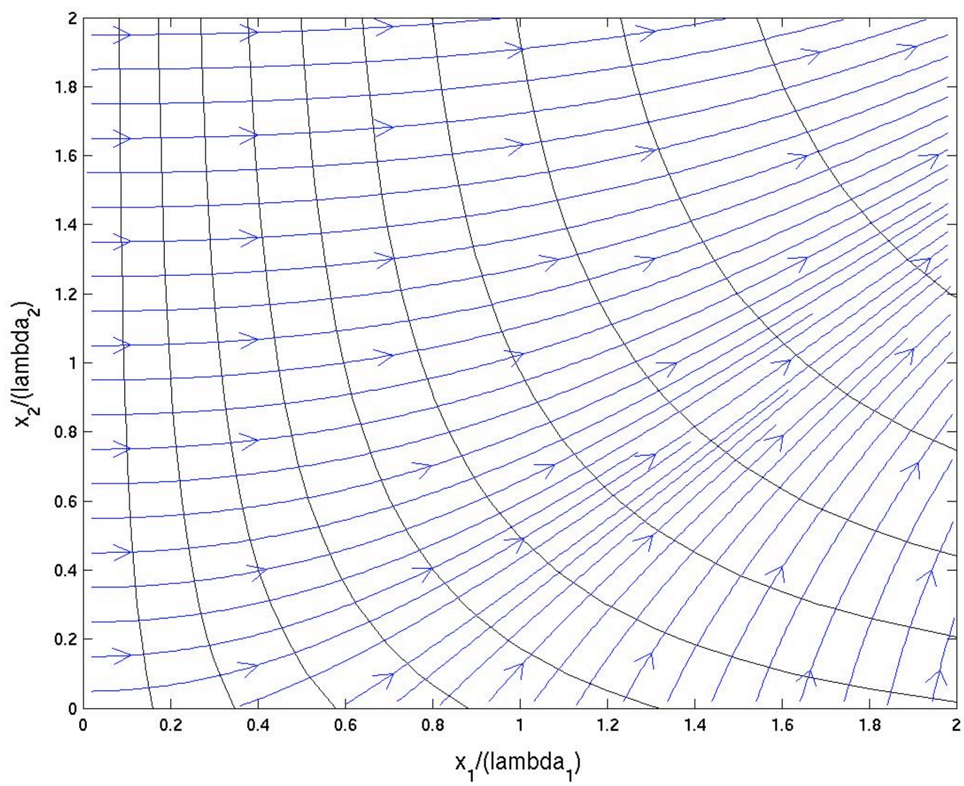

Figure 3.4-5

We now use this information to predict the choices our consumer will make in a market where prices are fixed. As is true in neoclassical theory, the bundles that the consumer will choose are those for which his marginal value (marginal price) equals the market price i.e.:

$$\frac{P_1}{P_2} \equiv \frac{r_1(\vec{x})}{r_2(\vec{x})} = \frac{\left(1 + \frac{\lambda_2}{\lambda_1}\left(1 - e^{-x_2/\lambda_2}\right)\right)e^{-x_1/\lambda_1}}{\left(1 - e^{-x_1/\lambda_1}\right)e^{-x_2/\lambda_2}} \tag{3.1-6}$$

By solving Equation 3.1-6 for x_1 we find:

$$x_1 = \frac{\lambda_1}{\lambda} x_2 + \lambda_1 \ln\left(\frac{P_2}{P_1}\right) + \lambda_1 \ln\left(1 + \frac{\lambda_2}{\lambda_1} + \left(1 - \frac{\lambda_2}{\lambda_1}\right)e^{-x_2/\lambda_2}\right) \tag{3.1-7}$$

This is the *income expansion path* familiar from neoclassical microeconomics textbooks. Note that if the half-lives of consumption are nearly the same ($\lambda_1 \approx \lambda_2$) Equation 3.1-7 is approximated by:

$$x_1 = \frac{\lambda_1}{\lambda} x_2 + \lambda_1 \ln\left(\frac{P_2}{P_1}\right) + \lambda_1 \ln(2) \qquad (3.1\text{-}8)$$

This is a straight line with slope determined by the consumption half-lives and intercept determined by relative prices. As shown in Figure 3.4-6

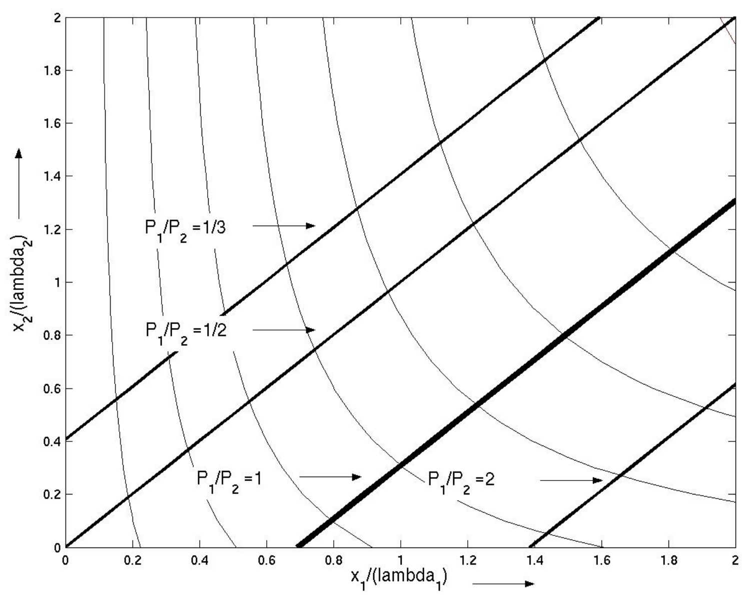

Figure 3.4-6

It is the shape of the income expansion path that resembles the data analyzed by Richard Stone. Stone explained the leftward displacement by assuming that consumers had somehow committed a portion of their income to consumption of one of the goods, and then divided his remaining income between the goods. He offered no explanation for the consumer's commitment. Additionally the model he used did nor work well for income levels close to (or below) the amount committed to the first good.

The Maslow inspired theory offers a somewhat more satisfying explanation: Until the consumer acquires a significant quantity of x_1, there will be no bundles $\vec{x}$ for which his ratio of $r_1(\vec{x})$ to $r_2(\vec{x})$ will not be above the relative market prices. Under these circumstances the consumer will acquire only x_1.

CONCLUSION

Use Value and Marginal Value as presented here represent mathematical expressions of ideas that have been with us since Aristotle at least. Structurally, they differ little from utility and marginal utility as understood by Jevons and Marshall. They are somewhat less heroic than their earlier counterparts with regard to what they say about human nature. They admit no claim of defining human wellbeing or the motivation behind human decision-making. They are simply measures of the value individuals place on the goods and services they consume. That is in essence their strength. Because of the specificity of its definition, Use Value can be considered along with the other factors we find relevant in determining an individual or society's wellbeing. Additionally a model consisting of several typical agents placed in different economic circumstances should open the door of general equilibrium analysis to the study of income inequality and class conflict.

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