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Keywords: Profit sharing; evolutionary dynamics; income distribution; capacity utilization; economic growth.

JEL Codes: E12; E25; J33; O40.

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1. Introduction

Employee profit sharing has experienced an increasing (even if fluctuating) popularity in several advanced economies in the last few decades (D'Art and Turner, 2004, Kalmi et al., 2005, and Kruse et al., 2010). Meanwhile, from a longer-term perspective, there have been rising and falling waves of interest in employee profit-sharing schemes since their inception in the 19th century (Mitchell et al.,1990, D'Art and Turner, 2006, Blasi, et al., 2013).

The main motivation behind a firm's offering of profit sharing to workers is that connecting workers' earnings to the profit performance of the firm is believed in theory to induce workers to increase commitment, effort and other attitudes leading to their higher productivity. Indeed, attitude surveys find that employers and employees usually believe that profit sharing helps improve firm performance in several dimensions (Weitzman and Kruse, 1990, and Blasi et al., 2010).

Some other compensation modes through which workers' earnings depend on the performance of the firm (or work group) are gain sharing, employee ownership and stock options. In fact, profit-sharing plans themselves vary considerably, and some major ways in which they differ concern what is shared (e.g., total profits or profits above a certain target or threshold level), how and when sharing is made (e.g., in cash or company stocks, in a deferred or non-deferred way) and with whom sharing is made (e.g., directly to workers or to some workers' retirement or pension plan). However, there is survey evidence that non-deferred and in cash profit sharing is ranked first by employees as a motivation device (Blasi et al., 2010).

There is a large empirical literature that confirms that profit sharing raises labor productivity in the firm. Although the estimated magnitude of the productivity gain varies (sometimes sizably) from study to study, it is often non-negligible. Weitzman and Kruse

(1990) apply meta-analysis to sixteen studies to find that the productivity gain of profit sharing is positive at infinitesimal significance levels. Doucouliagos (1995) applies meta-analysis to a set of forty-three studies to find that profit sharing is positively associated with productivity. Cahuc and Dormont (1997) use French data to find that profit sharing firms outperform other firms in productivity and profitability. Conyon and Freeman (2004), using UK data, find that firms that adopt profit-related pay tend to outperform other firms in productivity and financial performance. D'Art and Turner (2004) employ data for 11 European countries to find that profit sharing is positively associated with productivity and profitability. However, Kim (1998), using U.S. data, finds that albeit profit sharing raises labor productivity, profits do not rise, as gains from profit sharing are cancelled out by increased labor costs. Meanwhile, a field, quasi-experimental investigation conducted by Peterson and Luthans (2006) randomly assigned profit-sharing plans at three of twenty-one establishments within a U.S. firm, finding that labor productivity and profits increased in the profit-sharing establishments relative to the control group.

Motivated by this compelling empirical evidence, this paper sets forth a dynamic model of capacity utilization and economic growth, in which income distribution features a profit-sharing scheme. Unlike in a related approach developed in Lima (2010), firms behave heterogeneously as regards employee compensation strategy. Firms can choose to compensate workers with either only a base wage or a share of profits on top of a base wage. In accordance with the empirical evidence reported above, workers hired by profit-sharing firms have a higher productivity than their counterparts in base-wage firms. Meanwhile, unlike in Lima (2012), the heterogeneity in employee compensation strategies and the productivity gain which accrues to profit-sharing firms are not parametric constants, but instead co-evolve endogenously through an evolutionary process. Therefore, our model is well fitted to explore possible explanations for the evidence using U.K. data that firms switch modes of employee compensation frequently, with the gross changes in modes being far more numerous than the net changes, which suggests that firms have trouble optimizing and that the transaction costs of switching are relatively low (Bryson and Freeman, 2010). More broadly, our model is in position to explore possible accounts for the evidence recalled above that historically there have been rising and falling waves of interest in profit sharing. Furthermore, we explore the implications of the evolutionary dynamics of the distribution of employee compensation strategies and the productivity gain which accrues to profit-sharing firms for aggregate effective demand, and thereby for capacity utilization and economic growth. Indeed, given the crucial role of effective demand and income distribution in the Cambridge (U.K.) Post-Keynesian tradition, it is only fitting to explore the macroeconomic implications of profit sharing in a framework that conforms to essential tenets of this tradition.[1]

The remainder of this paper is organized in the following way. Section 2 describes the structure of the model and investigates its behavior in the ultra-short run. Section 3 explores the behavior of the model in the short run, while Section 4 investigates the evolutionary coupling dynamics of the distribution of compensation strategies across firms and the productivity gain which accrues to profit-sharing firms. This section also explores the implications of this evolutionary dynamics for income distribution and therefore for aggregate effective demand, capacity utilization and economic growth. The closing section summarizes the main conclusions reached along the way.

[1] Using, instead, a neoclassical framework, Weitzman (1985) claims that profit sharing can produce full employment with low inflation. If part of workers' total compensation is shared profits, so that the base wage is lower, the marginal cost of labor is lower and firms will be willing to hire more workers. As the marked up price is lower, a real balance effect creates a higher aggregate demand and hence a higher desired output. Thus, Weitzman (1985) ignores effective demand problems and implicitly sees involuntary unemployment as due to downward wage inflexibility (Davidson, 1986-87, Rothschild, 1986-87). In our model, instead, the fraction of profit-sharing firms and the average productivity are endogenously time-varying. Besides, we explore the implications of profit sharing for macroeconomic performance driven by effective demand.

2. Structure of the model and its behavior in the ultra-short run

The economy is closed and without government activities, producing only one (homogeneous) good for both investment and consumption. Production is carried out by a fixed (and large) population of imperfectly-competitive firms, which combine two (physically homogeneous) factors of production, capital and labor, through a fixed-coefficient technology. Each firm is owned by a single capitalist and has some leverage on its price, although it is small with respect to the overall market. Firms produce (and hire labor) according to effective demand, which is assumed to be insufficient for any of them to produce at full capacity at prevailing prices. Firms are homogeneous except as to the employee compensation strategy they choose to play, which in turn determines the average productivity of their hired workers and hence their markup.

An individual firm can choose between two employee compensation strategies: it can either pay workers only a real base wage v (*non-sharing strategy*) or pay them a real base wage and a share of real profits δ (*profit-sharing strategy*). As a result, in a given period there is a proportion $\lambda \in [0,1] \subset \mathbb{R}$ of sharing (or type *s*) firms, while the remaining proportion, $1-\lambda$, is composed by non-sharing (or type *n*) firms. In accordance with the representative empirical literature on profit sharing reviewed in the preceding section, a profit-sharing firm is willing to play such an employee compensation strategy because the resulting labor productivity is strictly higher than otherwise. Yet labor productivity is homogeneous across workers hired by firms playing a given strategy. Meanwhile, since the real base wage is assumed to be the same under both compensation strategies, a worker hired by a sharing firm will receive a higher total real compensation than a worker hired by a non-sharing firm along the economically meaningful domain given by strictly positive profits for sharing firms.[2]

Having chosen a given employee compensation strategy, a firm makes a take-it-or-leave-it offer to available workers to hire as many workers it needs to produce its demand-determined level of output. These workers, who are always in excess supply, not only take the received offer, but also perform the corresponding tasks with a higher productivity if the hiring firm pay them a base wage and a share of profits, which is the entire surplus over the base wage income. Meanwhile, the frequency distribution of worker compensation strategies across firms is not parametric, but rather co-evolves over time with the labor productivity differential between the two alternative compensation strategies, as described in Section 4.

To keep focus on the dynamics of the distribution of employee compensation strategies and its implications for income distribution, capacity utilization and economic growth, we simplify matters by assuming that the real base wage, v, and the profit-sharing coefficient, δ, remain constant over time. The distribution of employee compensation strategies across firms, $(\lambda, 1-\lambda)$, which is given in both the ultra-short run and the short run as a result from previous dynamics, changes beyond the short run according to an evolutionary dynamics (the so-called replicator dynamics). In the ultra-short run, for given values of real base wage, profit-sharing coefficient, labor productivity differential and frequency distribution of employee compensation strategies, individual markups vary so as to ensure that individual prices are equalized. Over time, therefore, the co-evolution of the distribution of employee compensation strategies and the labor productivity differential, by

[2] There is evidence that profit sharing has a meaningful effect on worker total compensation (Kruse et al., 2010). In fact, Capelli and Neumark (2004) find that total labor costs exclusive of the sharing component do not fall significantly in pre/post comparisons of firms that adopt profit sharing. This suggests that profit sharing tends to come on top of, rather than in place of, a base wage.

leading to changes in the average markup and hence in the wage share in income, generates changes in aggregate effective demand and hence in the short-run equilibrium values of capacity utilization and economic growth.

Formally, we define the productivity differential as:

(1) $$\alpha \equiv \frac{a_s}{a_n},$$

where $a_i = X_i / L_i$ denotes labor productivity in firms of type $i = s, n$, X_i is total output of firms of type $i = s, n$, and L_i is total employment of firms type $i = s, n$. As regards pricing behavior, we assume that established firms face a binding limit-price constraint, $\bar{P}$, arising from the purpose of forestalling entry by potential competitors.[3] This exogenously given constant price, which is herein normalized to one, is set as a markup over nominal unit labor costs. Hence, the following condition holds in the ultra-short run:

(2) $$1 = (1 + z_s)\frac{v}{a_s} = (1 + z_n)\frac{v}{a_n},$$

where $z_s \in \mathbb{R}_{++}$ and $z_n \in \mathbb{R}_{++}$ are, respectively, the markups applied by sharing and non-sharing firms, and $v \in \mathbb{R}_{++}$ denotes, therefore, both the nominal and the real base wage. For further simplicity, we also normalize labor productivity in non-sharing firms, a_n, to one, so that $a_s = \alpha > 1$ in (1), which requires further assuming that $v < a_n = 1 < a_s$.

Having chosen to follow a profit-sharing strategy, a firm has therefore to further decide how it will use the corresponding productivity differential. The specification in (2) assumes that a sharing firm uses its productivity differential to apply a higher markup than non-sharing firms, while charging the same price.[4] We could assume that not all sharing

[3] The prolific approach to entry and limiting pricing was pioneered by Bain (1949) and Harrod (1952), although the possibility of such behavior was raised much earlier by Kaldor (1935).

[4] Another alternative would be for sharing firms to use their productivity differential to charge a lower price than non-sharing firms while applying the same markup, which would allow sharing

firms make the same decision with respect to how to use their (common) productivity differential given by $\alpha > 1$, but we simplify matters by assuming that all sharing firms behave alike in that respect. Moreover, given that such productivity differential varies over time with the frequency distribution of employee compensation strategies across firms, as described in the next section, the average markup given by $z \equiv \lambda z_s + (1-\lambda) z_n$ varies over time as well. Therefore, the ultra-short-run equilibrium values of the individual markups can be obtained by combining (1) and (2):

$$z_s^* = \frac{\alpha}{v} - 1 \tag{3}$$

and:

$$z_n^* = \frac{1}{v} - 1, \tag{4}$$

from which it follows that $z_s^* > z_n^*$ for any $\alpha \in (1, \infty) \subset \mathbb{R}$. Therefore, a firm playing the profit-sharing strategy can be intuitively described as a firm willing to bet on the possibility of obtaining a productivity differential, α, which is high enough to allow it to set a markup, z_s, which is sufficiently higher than the (exogenously given) markup set by a non-sharing firm, z_n, while charging the same price (and therefore without harming its ability to sell as much output as a non-sharing firm). Moreover, a sharing firm expects this markup differential to be high enough to allow it to make at least as much (and preferably more) profits (net of shared profits) than a non-sharing firm. As explored in Section 4, however, the resulting labor productivity differential may fall short of the level required for the profit-sharing bet to prove successful.

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firms to have a higher market share than non-sharing firms and eventually drive the latter out of the market. Yet another possibility would be for sharing firms to use their productivity differential to both raise their markup and improve their price competitiveness. We abstract from these other possibilities by assuming that an individual firm faces a kinked demand curve (Hall and Hitch (1939), Sweezy (1939)), the market price for which is stable. In fact, this price is sustained over time by each firm's fear that, if it undercuts, the other firms will do the same. Meanwhile, a firm has no incentive to charge more than such price as it fears that the other firms will not do the same.

Using (1), the total real profits of sharing and non-sharing firms, respectively, are given by:

(5) $$R_s \equiv X_s - vL_s = \left(1 - \frac{v}{\alpha}\right) X_s$$

and:

(6) $$R_n \equiv X_n - vL_n = (1-v) X_n \,.$$

Using (5) and (6), the shares of real profit in real output of sharing and non-sharing firms in the ultra-short-run equilibrium are given, respectively, by:

(7) $$\pi_s^* \equiv \frac{R_s}{X_s} = 1 - \frac{v}{\alpha}$$

and:

(8) $$\pi_n^* \equiv \frac{R_n}{X_n} = 1 - v \,.$$

Therefore, since the markup set by non-sharing firms is an exogenously given constant, the share of real profits of non-sharing firms in their total real output is an exogenously given constant as well. Meanwhile, the profit share expression in (7) denotes the proportion of gross profits in the real output of sharing firms, since an exogenously given fraction of such profits, given by $\delta \in (0,1) \subset \mathbb{R}$, is shared with workers. As a result, using (7), the net unit return of sharing firms in the ultra-short-run equilibrium is given by:

(9) $$\pi_s^{c*} \equiv \frac{(1-\delta) R_s}{X_s} = (1-\delta)\left(1 - \frac{v}{\alpha}\right),$$

Using (8) and (9), the ultra-short-run equilibrium value of the average net profit share can be expressed as:

(10) $$\pi^* = \lambda \pi_s^{c*} + (1-\lambda)\pi_n^* = \lambda(1-\delta)\left(1 - \frac{v}{\alpha}\right) + (1-\lambda)(1-v) \,.$$

As explained formally in the next section, firms accumulate capital at the same rate, which implies that the aggregate capital stock, K, remains uniformly distributed across firms. It then follows that:[5]

$$(11) \qquad \frac{K_s}{\lambda} = \frac{K_n}{1-\lambda} = K,$$

where K_i is the total capital stock of firms of type $i = s, n$. Given (10), it follows that the proportion of the aggregate capital stock that is available to the firms of each type is proportional to the share of each type in the population of firms, that is, $K_s / K = \lambda$ and $K_n / K = 1-\lambda$.

Meanwhile, given that prices are equalized across firms, aggregate effective demand is uniformly distributed not only across firms, but across compensation strategies as well. Thus, capacity utilization is also equalized across compensation strategies:

$$(12) \qquad u_s = u_n = u = \frac{X}{K},$$

where $u_i \equiv X_i / K_i$ is a proxy for the degree of capacity utilization of type $i = s, n$ firms, while u denotes average capacity utilization and X average output.[6]

Using (5), (6), (7), (8), (11) and (12), the (gross) profit rates of sharing and non-sharing firms in the ultra-short-run equilibrium can then be expressed as follows:

$$(13) \qquad r_s^* \equiv \frac{R_s}{K_s} = \left(1 - \frac{v}{\alpha}\right)\frac{X_s}{K_s} = \pi_s^* u,$$

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[5] The meaning of the implied assumption (11) can be explained as follows. Let F be the total measure of firms in the economy and F_s the measure of sharing firms. As the aggregate capital stock is uniformly distributed across firms, it follows that $\frac{K_s}{F_s} = \frac{K_n}{F - F_s} = \frac{K}{F}$. By definition, $\lambda = \frac{F_s}{F}$, so we obtain (11) by multiplying both sides of these equalities by F.

[6] Therefore, we are assuming that firms are also homogeneous as regards the ratio of capital to full-capacity output, which is an exogenously given constant.

and:

(14) $$r_n^* \equiv \frac{R_n}{K_n} = (1-v)\frac{X_n}{K_n} = \pi_n^* u\,.$$

Using (9) and (13), the net profit rate of sharing firms in the ultra-short-run equilibrium is then given by:

(15) $$r_s^{c*} \equiv \frac{(1-\delta)R_s}{K_s} = (1-\delta)\left(1-\frac{v}{\alpha}\right)\frac{X_s}{K_s} = \pi_s^{c*} u\,.$$

Therefore, using (14) and (15), the ultra-short-run equilibrium value of the average net profit rate can be expressed as:

(16) $$r^{c*} = \lambda r_s^{c*} + (1-\lambda) r_n^* = \left[\lambda(1-\delta)\left(1-\frac{v}{\alpha}\right) + (1-\lambda)(1-v)\right]u\,.$$

3. Behavior of the model in the short run

We have assumed earlier that the population of firms, F, the (limit-)price level, $\bar{P}$, the real base wage, v, the labor productivity and the markup in non-sharing firms, a_n and z_n, respectively, and the profit-sharing coefficient, δ, all remain constant over time. The short run period t is defined as a time frame in which the aggregate capital stock, K_t, the labor supply, N_t, the labor productivity differential of sharing firms, $a_{s,t} = \alpha_t$ (and hence their corresponding markup differential, $z_{s,t} / z_n$), the frequency distribution of employee compensation strategies across firms, λ_t, and therefore income distribution measured by the average profit share, π_t, can all be taken as predetermined by the previous dynamics of the economy.[7] The existence of excess aggregate (and individual) capacity ensures that aggregate (and individual) output will adjust to remove any excess aggregate (and

[7] Since in the next section we explore the behavior of the economy in the transition from the short to the long run, thereafter we attach a subscript t to all the short-run variables (be they endogenous or predetermined). Therefore, in the short run we assume that the equilibrium values of the ultra-short-run variables are always attained.

individual) demand or supply in the economy, so that in short-run equilibrium, net aggregate savings, S_t, are equal to aggregate desired investment, I_t^d.

The economy is inhabited by two classes, capitalists who own the firms and workers. Following the Cambridge (U.K.) tradition, we assume that these classes have different consumption and saving behaviors. Workers provide labor and earn a base wage income, if they work for non-sharing firms. Meanwhile, workers hired by sharing firms also receive a share of the latter's profit income, which is the entire surplus over the base wage bill. In terms of the alternative profit-sharing schemes alluded to in the Introduction, we assume that what is shared is total profits, and that compensation is made in cash, in a non-deferred way and directly to workers. We further assume that workers' total compensation is all spent on consumption, while capitalists have a homogeneous saving behavior, and save a fraction, $\gamma \in (0,1) \subset \mathbb{R}$, of their net profit income irrespective of the employee compensation strategy they play. Therefore, using (11)-(15), net aggregate savings as a proportion of the capital stock at period t can be expressed as follows:

$$(17) \qquad \frac{S_t}{K_t} = \gamma \left[\frac{(1-\delta)R_{s,t} + R_{n,t}}{K_t} \right] = \gamma \left[\lambda_t \pi_{s,t}^{c*} u_{s,t} + (1-\lambda_t)\pi_{n,t}^{*} u_{n,t} \right] = \gamma \left[\lambda_t \pi_{s,t}^{c*} + (1-\lambda_t)\pi_{n,t}^{*} \right] u_t .$$

Let us now turn to the derivation of firms' aggregate and average investment plans. For simplicity, we have assumed that workers always consume all of their earnings no matter for what firm they work, whereas firm-owner capitalists have the same saving rate regardless of what employee compensation strategy they play. Also for simplicity, we assume that firms behave alike as regards investment plans. Reasonably, we assume that firms' desired investment depends on their expected profits (due to profitability-type effects) and demand-driven output production (due to accelerator-type effects). But when an individual firm makes and implements (at the same period, for simplicity) investment plans, it is irrevocably uncertain as to what employee compensation strategy it (or, for that matter, any other firm) will be playing at each period of the relevant future. In fact, the replicator dynamics driving the frequency distribution of employee compensation strategies across firms (to be introduced in the next section) takes individual firms as having limited and localized knowledge as to the system as a whole. Consequently, it is reasonable to assume that the desired capital accumulation of an individual firm varies positively with its expectation of the average levels of the profit share and capacity utilization.

To specify it formally, average desired capital accumulation at period t, which is aggregate desired investment as a proportion of the aggregate capital stock, both at period t, is given by:

$$\frac{I_t^d}{K_t} = \beta_1 \pi_t^E + \beta_2 u_t^E, \tag{18}$$

where π_t^E and u_t^E denote, respectively, the expected average levels of the profit share and capacity utilization by any individual firm, whereas $\beta_1 \in \mathbb{R}_{++}$ and $\beta_2 \in \mathbb{R}_{++}$ are parametric constants. The time index of π_t^E and u_t^E refers to the period at which the expectation about the relevant future is formed. Therefore, the specification in (18) is an expectations-augmented version of the desired capital accumulation function put forward in Marglin and Bhaduri (1990), the latter from which it is known that the resulting output growth rate can vary either positively or negatively with the (average) profit share depending on the relative strength of the causal effects at play. We could assume that firms (even when playing the same employee compensation strategy) have heterogeneous expectations concerning the average values of the profit share and capacity utilization in the relevant future. However, we postulate that, facing an uncertain future, firms uniformly proxy these expected average levels by their corresponding current average levels. In this model a given share of the aggregate profit income accrues to workers as profit sharing, though. Hence, it is reasonable to assume that firms proxy the expected average profit share by the current average net profit share, which is given by (10).

As it turns out, assuming in (18) that $\pi_t^E = \pi_t^*$ (along with using (10)) and $u_t^E = u_t$, we obtain the following expression for the average desired rate of capital accumulation:

$$\text{(19)} \qquad \frac{I_t^d}{K_t} = \beta_1 \pi_t^* + \beta_2 u_t .$$

Finally, by substituting (17) and (19) in the goods market equilibrium condition given by $S_t / K_t = I_t^d / K_t$ and using (10), we obtain the short-run equilibrium capacity utilization:

$$\text{(20)} \quad u_t^*(\alpha_t, \lambda_t) = \frac{\beta_1 \left[\lambda_t \pi_s^{c*}(\alpha_t, \lambda_t) + (1-\lambda_t)\pi_n^* \right]}{\gamma \left[\lambda_t \pi_s^{c*}(\alpha_t, \lambda_t) + (1-\lambda_t)\pi_n^* \right] - \beta_2} = \frac{\beta_1 \left[\lambda_t (1-\delta)\left(1 - \frac{v}{\alpha_t}\right) + (1-\lambda_t)(1-v) \right]}{\gamma \left[\lambda_t (1-\delta)\left(1 - \frac{v}{\alpha_t}\right) + (1-\lambda_t)(1-v) \right] - \beta_2} .$$

Note that the short-run equilibrium capacity utilization depends on parametric constants along with the productivity differential, α_t, and the distribution of employee compensation strategies, λ_t, which are predetermined in the short run and co-evolve in the transition from the short to the long run (as described in the next section).[8]

Meanwhile, we can substitute (20) in (17) to obtain the short-run equilibrium output growth rate:

$$\text{(21)} \qquad g_t^*(\alpha_t, \lambda_t) = \gamma \left[\lambda_t \pi_s^{c*}(\alpha_t, \lambda_t) + (1-\lambda_t)\pi_n^* \right] u_t^*(\alpha_t, \lambda_t) .$$

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[8] We assume that $\gamma \left[\lambda_t \pi_s^{c*}(\alpha_t, \lambda_t) + (1-\lambda_t)\pi_n^* \right] - \beta_2 > 0$ for all $\alpha_t \in (1, \infty) \subset \mathbb{R}$ and $\lambda_t \in [0,1] \subset \mathbb{R}$, which is the standard Keynesian stability condition in effective demand-driven models like the one set forth in this paper. This means that $u_t^*(\alpha_t, \lambda_t)$ is positive and stable if average savings are more responsive than average desired investment to changes in average capacity utilization, which in turn requires that the denominator of the expression in (20-a) is positive.

Hence, the short-run equilibrium output growth rate also depends on parametric constants along with the productivity differential, α_t, and the distribution of employee compensation strategies, λ_t.

4. Behavior of the model in the long run

In the long run we assume that the ultra- and short-run equilibrium values of the income distribution, capacity utilization and economic growth are always attained, with the economy moving towards the long run due to changes in the aggregate stock of capital, K, the supply of available labor, N, the productivity differential, α, and the distribution of employee compensation strategies, λ. In order to sharpen focus on the coupling dynamics of the distribution of employee compensation strategies and the productivity differential (and the ensuing implications for income distribution, capacity utilization and economic growth), we assume that the supply of available labor grows endogenously at the same rate as the capital stock.[9]

Let us start by deriving the dynamics of the productivity differential. At a given (short-run) period t there is a fraction $\lambda_t \in [0,1] \subset \mathbb{R}$ of the population of firms, which may vary from one period to the next one, adopting the profit-sharing strategy. The remaining fraction, $1-\lambda_t$, is made up of firms that pay only the base wage (non-sharing strategy). Let $\bar{y}_t \equiv \lambda_t y_{s,t} + (1-\lambda_t) y_{n,t}$ be the average real earnings of workers at period t, where $y_{s,t} \equiv v + \delta R_{s,t}/L_{s,t}$ and $y_{n,t} \equiv v$ are the real earnings of a worker hired by a profit-sharing firm and a worker hired by a non-sharing firm in period t, respectively. Thus, the

[9] Consequently, the constancy of the labor productivity differential and the distribution of employee compensation strategies in the long-run equilibrium guarantee the constancy of both the average labor productivity and the average rate of employment. In fact, using (1) and (11)-(12), the average employment rate in the short-run equilibrium is $e_t^* \equiv \frac{L_{s,t}^* + L_{n,t}^*}{N_t} = \left[\frac{\lambda_t}{\alpha_t} + (1-\lambda_t)\right]\frac{K_t}{N_t} u_t^*$, where the expression in square brackets represents the weighted average of the inverse of the individual labor productivities $a_{s,t} = \alpha_t$ and $a_{n,t} = 1$.

differential between the higher real earnings and the average real earnings can be written as $y_{s,t} - \bar{y}_t = (1-\lambda_t)\delta R_{s,t} / L_{s,t}$ for all $\lambda_t \in [0,1] \subset \mathbb{R}$. In line with the empirical evidence reported in the Introduction, we assume that the extent to which labor productivity in profit-sharing firms is greater than labor productivity in non-sharing firms varies positively with the relative real earnings differential given by $y_{s,t} - \bar{y}_t$. Formally, we consider the following productivity differential function:

(22) $$\alpha_{t+1} = f(y_{s,t} - \bar{y}_t) = f\left((1-\lambda_t)\delta R_{s,t} / L_{s,t}\right),$$

where $f'(\cdot) > 0$ and $f''(\cdot) < 0$ for all differential $(y_{s,t} - \bar{y}_t) \subset \mathbb{R}$. Moreover, we assume that $\lim_{(y_s - \bar{y}) \to \infty} f'(y_s - \bar{y}) = 0$. Therefore, in accordance with the empirical evidence that the productivity gains arising from profit sharing are not unlimited, the productivity differential in (22) not only increases at a decreasing rate, but it also tends to become insignificant for very large values of the relative real earnings differential.[10] We can use (5) to re-write (22) as follows:

(22-a) $$\alpha_{t+1} = f\left(\delta(1-\lambda_t)(\alpha_t - v)\right).$$

As re-written in (22-a), the productivity differential function has an intuitive interpretation. Given that α_t is the output per worker of a sharing firm and v is the respective unit cost of labor, it follows that $\alpha_t - v$ is the profit per worker of a sharing firm and $\delta(\alpha_t - v)$ is the amount of profit per worker of a sharing firm which is shared with its hired workers. Therefore, given α_t and λ_t, the next-period productivity differential varies positively with the profit-sharing coefficient and negatively with the base wage.[11]

[10] In the meta-analyses performed by Weitzman and Kruse (1990) and Doucouliagos (1995), the size of the estimated effect of profit sharing on labor productivity is usually on the order of 3 to 7 percent.

[11] In fact, Kruse (1993) finds that the productivity gains associated with profit sharing increase with the size of the profit sharing bonus as a proportion of profits.

Meanwhile, given α_t, v and δ, the next-period productivity differential increases with $1-\lambda_t$, the proportion of firms playing the non-sharing strategy in a given period, which is an indicator of the prospects of not receiving any shared profits in the next period. Since an increase in $1-\lambda_t$ acts as an incentive on workers hired by sharing firms in the next period to provide a higher productivity differential, it follows that $\delta(1-\lambda_t)(\alpha_t - v)$ reflects how valuable it is to work for a sharing firm.

In fact, note from (22-a) that $\partial\alpha_{t+1}/\partial\lambda_t = -\delta(\alpha_t - v) f'\left(\delta(1-\lambda_t)(\alpha_t - v)\right) < 0$ for all $\lambda_t \in [0,1] \subset \mathbb{R}$ and for any $\alpha_t > v$ (recall that the latter condition was assumed earlier to ensure a strictly positive markup for sharing firms in (3)). The greater the proportion of sharing firms in a given period, the smaller the labor productivity differential between sharing and non-sharing firms in the next period. One firm's decision to play the profit-sharing strategy in a given period, by reducing $\delta(1-\lambda_t)(\alpha_t - v)$ for a given α_t, makes it less valuable to workers to be employed by a sharing firm in the next period and hence has a negative payoff externality on all other sharing firms. Consequently, there is strategic substitutability in the firms' choice of employee compensation mechanism. Meanwhile, if all firms follow the sharing strategy $(\lambda_t = 1)$, the relative real earnings differential given by $y_{s,t} - \bar{y}_t = \delta(1-\lambda_t)(\alpha_t - v)$ vanishes. In this case, given that the labor productivity is uniform across all firms, and should actually be higher than the average labor productivity when all firms pay only the base wage, which we have normalized to one, we further assume that $f(0) > 1$. Instead, if all firms choose to play the non-sharing strategy $(\lambda_t = 0)$, the potential relative earnings differential given by $y_{s,t} - \bar{y}_t = \delta(\alpha_t - v)$ takes its maximum value. In this case, a non-sharing firm which decides to switch compensation strategy to play the sharing strategy is able to reap the largest possible productivity gain, since

$f\left(\delta(\alpha_t - v)\right) > f\left(\delta(1-\lambda_t)(\alpha_t - v)\right)$ for all $\lambda_t \in (0,1] \subset \mathbb{R}$ and for any $\alpha_t > v$. In order to better convey the substance of all these properties of the productivity differential function (22-a), they are displayed in Figure 1.

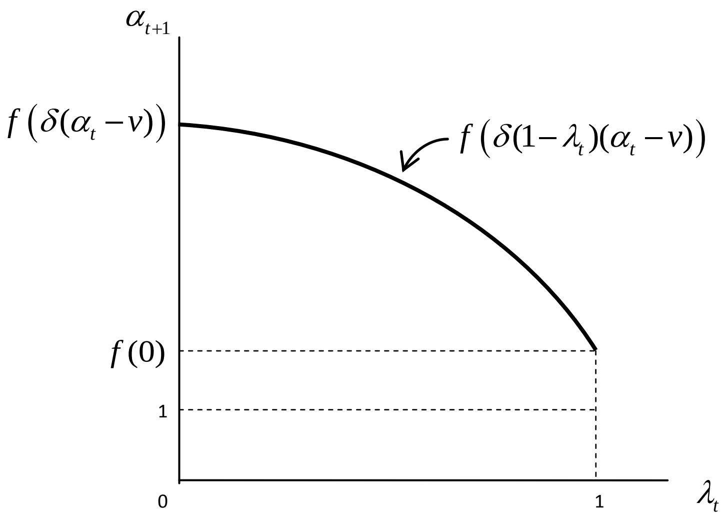


Figure 1. *Productivity differential function*

More broadly, the following intuitive rationales can be proposed for the specification of the labor productivity gain in (22). First, the average earnings can be seen by a worker as a conventional estimate of her outside option or fallback position. As a result, workers who receive a share of profits in addition to a base wage deliver a productivity gain (relatively to the productivity they would deliver if remunerated with only a base wage) which increases with the excess of the higher earnings over their outside option or fallback position. Second, the average earnings can be seen by a worker as the conventional reference point against which a compensation package featuring a base wage and shared profits should be compared when deciding how much above-normal productivity to provide in return. Therefore, above-average earnings are seen by workers as warranting the delivery of above-normal levels of productivity. In fact, Blasi, Kruse and Freeman (2010) propose an interesting rationale for profit sharing based on reciprocity and gift exchange (as these notions are articulated in Akerlof (1982)): a “gift” of higher compensation through profit sharing raises worker morale, and workers reciprocate with a "gift" of greater productivity. More generally, a "gift" of profit sharing on top of a base wage may help to create and reinforce a sense of shared interests and the value of a reciprocal relationship. Alternatively, the conventional reference point provided by the average earnings can be taken as reflecting workers' earnings expectation under uncertainty, so that a compensation package featuring a base wage and shared profits is greeted as a pleasant surprise which warrants the delivery of above-normal levels of productivity.

While the frequency distribution of employee compensation strategies is given in both the ultra-short run and the short run, it varies beyond the short run according to an evolutionary dynamics based on strategy payoffs. More precisely, an individual firm revises periodically its employee compensation strategy in a manner described by the following replicator dynamics:[12]

$$\lambda_{t+1} - \lambda_1 = \lambda_t (r_{s,t}^{c*} - r_t^{c*}) = \lambda_t (1-\lambda_t)(\pi_{s,t}^{c*} - \pi_{n,t}^{*}) u_t^{*}, \tag{23}$$

where $r_t^c \equiv \lambda_t r_{s,t}^{c^*} + (1-\lambda_t) r_{n,t}^*$ is the average net profit rate, which is given by (16), with u_t^* being given by (20), and the latter equality is obtained using (14) and (15), so that $\pi_{s,t}^{c^*}$ and $\pi_{n,t}^*$ are given by (9) and (8), respectively. Under the replicator dynamics in (23), therefore, the frequency of the profit-sharing strategy in the population of firms increases (decreases) exactly when it has above-average (below-average) payoff.

Using (8), (9) and (20), the replicator dynamics in (23) then becomes:

$$\lambda_{t+1} = \lambda_t \left\{ 1 + (1-\lambda_t) \left[(1-\delta)\left(1 - \frac{v}{\alpha_t}\right) - (1-v) \right] u_t^* \right\}. \tag{23-a}$$

[12] The replicator dynamics can be derived from a model of (social or individual) learning as in Weibull (1995, sec. 4.4).

Thus, the state transition of the economy is determined by the system of difference equations (22-a) and (23-a), whose state space is $\Theta = \{(\alpha_t, \lambda_t) \in \mathbb{R}_+^2 : 0 \le \lambda_t \le 1, \alpha_t > v\}$, as represented by the shaded area in each panel in Figure 2.

We will show that the dynamic system given by (22-a) and (23-a) has two long-run equilibria, with each of them featuring survival of only one employee compensation strategy. These pure-strategy equilibria are denoted by E_1 and E_2 in the three panels in Figure 2. Moreover, we will show the possible existence of a third long-run equilibrium (denoted by E_3 in panel (b) in Figure 2), now featuring the survival of both employee compensation strategies.

Note that $\lambda_{t+1} = \lambda_t = 0$ for any $t \in \{0,1,2,...\}$ satisfies (23-a) for any state $(\alpha_t, 0) \in \Theta$. Moreover, let $\alpha_{t+1} = \alpha_t = \bar{\alpha} \in (v, \infty) \subset \mathbb{R}$ for any $t \in \{0,1,2,...\}$. In this case, the difference equation (22-a) is satisfied for any $t \in \{0,1,2,...\}$ if the following condition holds:

(24) $$\bar{\alpha} = f\left(\delta(\bar{\alpha} - v)\right).$$

We demonstrate in Appendix 1 that $\bar{\alpha} \in (v, \infty) \subset \mathbb{R}$ exists and is unique. Therefore, one of the two pure-strategy long-run equilibria of the system, E_1 in Figure 2, is given by the state $(\bar{\alpha}, 0) \subset \Theta$, which features the non-sharing employee compensation strategy as the only survivor in the long run.

Meanwhile, if $\lambda_{t+1} = \lambda_t = 1$ for any $t \in \{0,1,2,...\}$, the difference equation (23-a) is satisfied for any state $(\alpha_t, 1) \in \Theta$ and, given (22-a), it follows that $\alpha_{t+1} = \alpha_t = f\left(0\right)$ for all $t \in \{0,1,2,...\}$. Therefore, the other pure-strategy long-run equilibrium of the system, E_2 in Figure 2, is given by the state $\left(f(0), 1\right) \subset \Theta$, which features the profit-sharing employee compensation strategy as the only survivor in the long run.

Finally, if $\lambda_{t+1} = \lambda_t = \lambda^* \in (0,1) \subset \mathbb{R}$ for any $t \in \{0,1,2,...\}$, the difference equation (23-a) is satisfied if the individual profit shares (8) and (9) are equalized. Given that the labor productivity differential (which is equal to the labor productivity in the profit-sharing firms) is the only adjusting variable among the determinants of the individual profit shares (8) and (9), the latter are equalized when the (long-run) equilibrium value of the labor productivity differential is given by:

(25) $$\alpha^* = \frac{(1-\delta)v}{v-\delta},$$

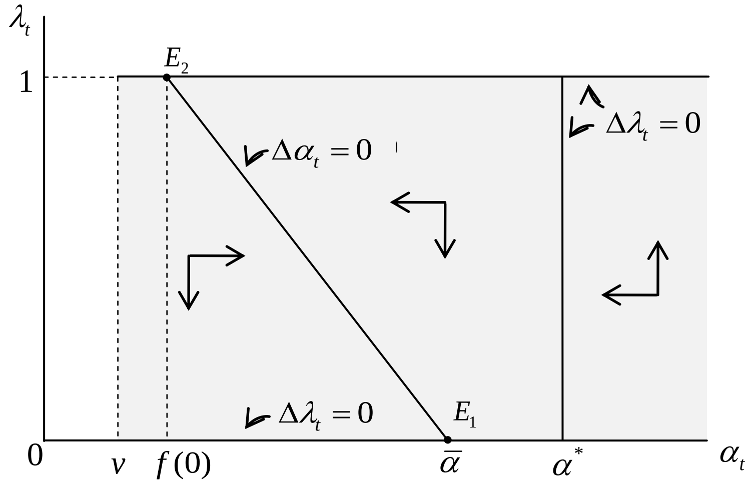
where we assume that $v > \delta$. Meanwhile, given that $\alpha_{t+1} = \alpha_t = \alpha^*$ for all $t \in \{0,1,2,...\}$, the difference equation (22-a) is satisfied if the following condition holds:

(26) $$\alpha^* = f\left(\delta(1-\lambda^*)(\alpha^* - v)\right).$$

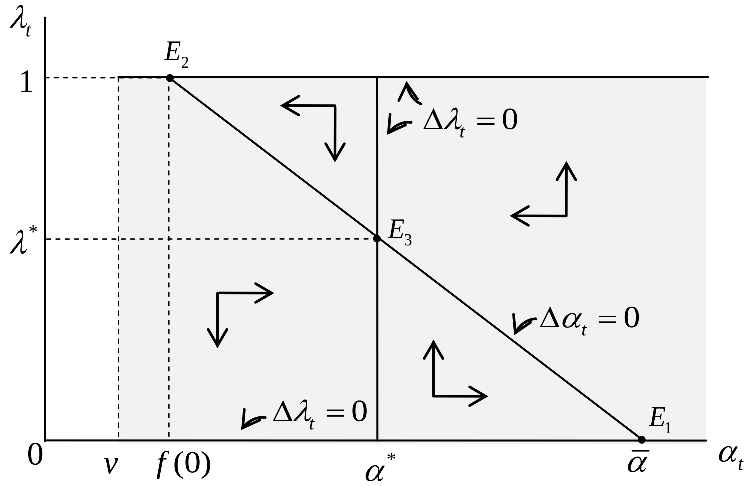
As demonstrated in Appendix 2, there is a unique $\lambda^* \in (f(0), \bar{\alpha}) \subset \mathbb{R}$ which satisfies (26) if the following necessary and sufficient condition is satisfied:

(27) $$f(0) < \alpha^* < \bar{\alpha}.$$

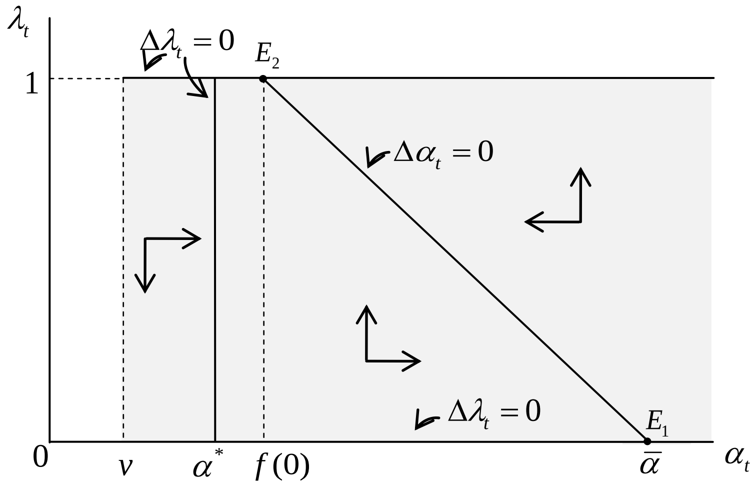
Therefore, if the condition in (27) is satisfied, there exists a third long-run equilibrium given by the state $(\alpha^*, \lambda^*) \subset \Theta$ (and denoted by E_3 in panel (b) in Figure 2), which features the survival of both employee compensation strategies in the long run.



(a) *Relatively weak labor productivity differential:* $f(0) < \bar{\alpha} < \alpha^*$



(b) *Relatively moderate labor productivity differential:* $f(0) < \alpha^* < \bar{\alpha}$



(c) *Relatively strong labor productivity differential:* $\alpha^* < f(0) < \bar{\alpha}$

Figure 2. *Phase diagram for different magnitudes of the labor productivity differential*

The well-defined ordering (27), which implies and is implied by the existence and uniqueness of the mixed-strategy equilibrium, E_3, can be interpreted intuitively with recourse to Figure 3, which plots the next-period productivity differential as a function of its current-period value. Note that, *ceteris paribus*, the function in (22-a) shifts down with an increase in the fraction of profit-sharing firms at period t. More precisely, this function rotates clockwise around the point $\left(v, f(0)\right)$ as λ_t increases, due to the resulting squeeze in the relative real earnings differential given by $y_{s,t} - \bar{y}_t$ for every $\alpha_t > v$.

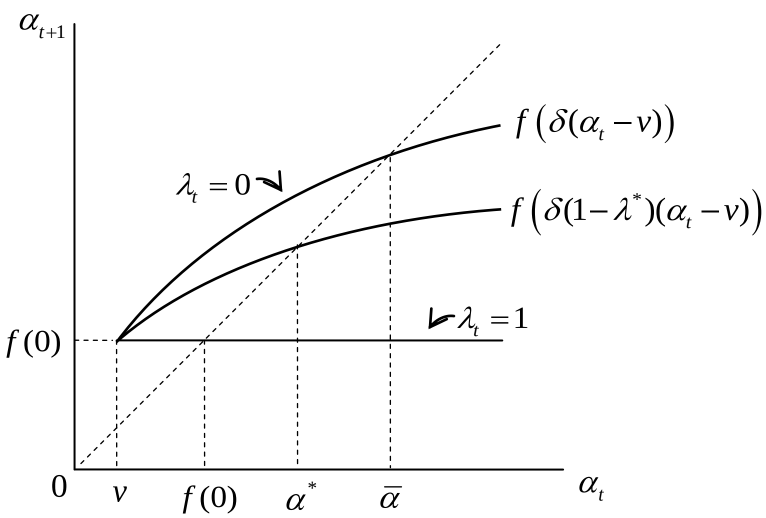


Figure 3. *Ordering of labor productivity differentials when there is a mixed-strategy long-run equilibrium*

Let us now explore the dynamics of the system towards the long run as governed by the motion equations (22-a) and (23-a). The $\Delta\alpha_t \equiv \alpha_{t+1} - \alpha_t = 0$ isocline is the locus of all the states (ou points) of the set given by $\left\{(\alpha_t, \lambda_t) \in \Theta : f\left(\delta(1-\lambda_t)(\alpha_t - v)\right) - \alpha_t = 0\right\}$. Thus, this isocline connects the equilibrium solutions E_1 and E_2, as depicted in Figure 2. In order to know more about the $\Delta\alpha_t = 0$ isocline, we can use (22-a) to compute the following derivative:

(28) $$\left.\frac{d\lambda_t}{d\alpha_t}\right|_{\Delta\alpha_t = 0} = \frac{\delta(1-\lambda_t) f'\left(\delta(1-\lambda_t)(\alpha_t - v)\right) - 1}{\delta(\alpha_t - v) f'\left(\delta(1-\lambda_t)(\alpha_t - v)\right)}.$$

The sign of (28) in the neighborhood of the pure-strategy long-run equilibria ($\lambda^* = 0$ and $\lambda^* = 1$) can be determined by taking the limit of (28) as the state of the system approaches each of these equilibria. These limits are given by:

(29) $$\lim_{(\alpha_t,\lambda_t)\to(f(0),1)} \left.\frac{d\lambda_t}{d\alpha_t}\right|_{\Delta\alpha_t=0} = \frac{-1}{\delta(f(0)-v)f'(0)} < 0 \text{ and } \lim_{(\alpha_t,\lambda_t)\to(\bar{\alpha},0)} \left.\frac{d\lambda_t}{d\alpha_t}\right|_{\Delta\alpha_t=0} = \frac{\delta f'\left(\delta(\bar{\alpha}-v)\right)-1}{\delta(\bar{\alpha}-v)f'\left(\delta(\bar{\alpha}-v)\right)} < 0,$$

with the sign of the second limit in (29) comings from (A-2.2) in Appendix 2. The reason why in the vicinity of the extinction of one worker compensation strategy, the higher is the productivity differential, the lower is the proportion of profit-sharing firms, is that a higher relative earnings differential is necessary to generate a higher productivity differential (recall that the productivity differential in (22) increases at a decreasing rate, as there is strategic substitutability in the firms' choice of employee compensation mechanism).

Meanwhile, the $\Delta\lambda_t \equiv \lambda_{t+1} - \lambda_t = 0$ isocline is the locus of all the states of the set given by $\{(\alpha_t,\lambda_t)\in\Theta : \lambda_t = 0\} \cup \{(\alpha_t,\lambda_t)\in\Theta : \lambda_t = 1\} \cup \left\{(\alpha_t,\lambda_t)\in\Theta : \alpha_t = \alpha^*\right\}$. As depicted in Figure 2, this set is represented by a semi-inverted H-shaped isocline.

From the local stability analysis accomplished in Appendices 3-5, the following results emerge. In the configuration depicted in panel (a) in Figure 2, the productivity gain for profit-sharing firms generated by a given relative real earnings differential is relatively low. In this case, as shown earlier, there are only two pure-strategy long-run equilibria, $E_1 = (\bar{\alpha}, 0)$, with no firm playing the profit-sharing strategy, and $E_2 = (f(0), 1)$, with all firms playing the profit-sharing strategy instead. Besides, as shown in Appendices 3 and 4, $E_2 = (f(0), 1)$ is a repulsor, while $E_1 = (\bar{\alpha}, 0)$ is an attractor.

Meanwhile, in the configuration depicted in panel (c) in Figure 2, the productivity gain for profit-sharing firms generated by a given relative real earnings differential is relatively high. In this case, as shown earlier, there are again only two pure-strategy long-run equilibria, $E_1 = (\bar{\alpha}, 0)$, with all firms playing the non-sharing strategy, and $E_2 = (f(0), 1)$, with all firms playing the profit-sharing strategy instead. Moreover, as shown in Appendices 3 and 4, while $E_2 = (f(0), 1)$ is an attractor, $E_1 = (\bar{\alpha}, 0)$ is a repulsor.

Finally, in the configuration depicted in panel (b) in Figure 2, the productivity gain for profit-sharing firms is relatively moderate. In this case, as shown earlier, there are the same two pure-strategy long-run equilibria, $E_1 = (\bar{\alpha}, 0)$ and $E_2 = (f(0), 1)$, and also one mixed-strategy equilibrium, $E_3 = (\alpha^*, \lambda^*)$, featuring heterogeneity in worker compensation strategies across firms. Besides, as shown in Appendices 2-5, $E_1 = (\bar{\alpha}, 0)$ and $E_2 = (f(0), 1)$ are repulsors, while $E_3 = (\alpha^*, \lambda^*)$ is a (local) attractor if condition (A-5.4) in Appendix 5 is satisfied (which may not be the case, for instance, if the short-run equilibrium capacity utilization is too enough).

However, since the state space of the system is positively invariant (as shown in Appendix 6), in this configuration heterogeneity in employee compensation strategies across firms does persist in the long-run even if the mixed-strategy long-runequilibrium is not an attractor. This positive invariance implies that, if the long-run equilibrium with coexistence of both compensation strategies is not an attractor, the system keeps undergoing endogenous, self-sustaining and persistent fluctuations in the frequency distribution of compensation strategies and the average labor productivity, with the average levels of the income shares, capacity utilization and economic growth persistently fluctuating as well. In fact, Mitchell et al. (1990) and D'Art and Turner (2006) find that the adoption of profit sharing schemes has tended to be cyclical in nature in many advanced countries since the origin of these schemes in the 19th century. Moreover, using data for Ireland, D'Art and Turner (2006) find evidence that the trend in profit sharing is affected by fluctuations in the business cycle.

It is also worth exploring the long-run equilibrium of income distribution, capacity utilization, and economic growth in each one of these three configurations as regards the relative magnitude of the labor productivity gain for profit-sharing firms, as depicted in Figure 2. We can use (10), (20) and (21) to establish:

$$(30)\qquad \pi^*(\bar{\alpha},0)=\pi^*(\alpha^*,\lambda^*)=\pi_n=1-v \quad\text{and}\quad \pi^*\left(f(0),1\right)=\pi_s^c\left(f(0)\right)=(1-\delta)\left[1-\frac{v}{f(0)}\right]$$

and:

$$(31)\qquad u^*(\bar{\alpha},0)=u^*(\alpha^*,\lambda^*)=\frac{\beta_1(1-v)}{\gamma(1-v)-\beta_2}\quad\text{and}\quad u^*(f(0),1)=\frac{\beta_1\pi^*\left(f(0),1\right)}{\gamma\pi^*\left(f(0),1\right)-\beta_2},$$

where the first equality in (30) and (31) follows from the fact that $E_3=(\alpha^*,\lambda^*)$ is defined implicitly by the condition given by $\pi_{s,t}^{c*}(\alpha_t,\lambda_t)-\pi_{n,t}^*=0$ (recall that the profit share in income of non-sharing firms is exogenously given). In panel (a), with $\bar{\alpha}>f(0)$, we get $\pi^*(\bar{\alpha},0)>\pi^*\left(f(0),1\right)$, and therefore $u^*(\bar{\alpha},0)<u^*(f(0),1)$. In panel (b), with $\alpha^*>f(0)$, we get $\pi^*(\bar{\alpha},0)=\pi^*(\alpha^*,\lambda^*)>\pi^*\left(f(0),1\right)$, and hence $u^*(\bar{\alpha},0)=u^*(\alpha^*,\lambda^*)<u^*(f(0),1)$. Meanwhile, in panel (c), with $\alpha^*<f(0)$, it follows that $\pi^*(\bar{\alpha},0)<\pi^*\left(f(0),1\right)$, and hence that $u^*(\bar{\alpha},0)>u^*(f(0),1)$. Therefore, if the economy converges to a long-run equilibrium (which may not occur in panel (b), as shown in Appendix 5), the evolutionary dynamics of profit-sharing adoption and labor productivity takes the economy to a position in which the wage share and capacity utilization are at their lowest possible long-run equilibrium levels.

Using (21), the long-run equilibrium values of the growth rate are given by:

$$(32)\qquad g^*(\bar{\alpha},0)=g^*(\alpha^*,\lambda^*)=\gamma(1-v)u^*(\bar{\alpha},0)\quad\text{and}\quad g^*(f(0),1)=\gamma\pi^*\left(f(0),1\right)u^*\left(f(0),1\right).$$

Using (30)-(32), we can compute the following growth rate differential:

$$(33)\qquad \Delta g^*\equiv g^*(\bar{\alpha},0)-g^*(f(0),1)=\frac{\beta_1\gamma\pi^*(\bar{\alpha},0)\pi^*\left(f(0),1\right)\left[\pi^*(\bar{\alpha},0)-\pi^*\left(f(0),1\right)\right]\theta}{\left[\gamma\pi^*(\bar{\alpha},0)-\beta_2\right]\left[\gamma\pi^*\left(f(0),1\right)-\beta_2\right]},$$

where $\theta \equiv \gamma - \beta_2 \left[\frac{1}{\pi^*(\bar{\alpha},0)} + \frac{1}{\pi^*\left(f(0),1\right)} \right]$. Note that, given the sign of the profit share differential given by $\pi^*(\bar{\alpha},0) - \pi^*\left(f(0),1\right)$, the sign in (33) depends on the sign of θ, which is indeterminate (recall that the Keynesian short-run stability condition assumed in footnote 8 implies that $\gamma\pi^*(\bar{\alpha},0) - \beta_2 > 0$ and $\gamma\pi^*\left(f(0),1\right) - \beta_2 > 0$).

When there is convergence to $E_1 = (\bar{\alpha},0)$ (panel (a)) or $E_3 = (\alpha^*,\lambda^*)$ (panel (b)), it follows that $\pi^*(\bar{\alpha},0) = \pi^*(\alpha^*,\lambda^*) > \pi^*\left(f(0),1\right)$. In this case, the economy converges to a long-run equilibrium which features the highest (lowest) possible long-run equilibrium growth rate if $\theta > 0$ ($\theta < 0$). Meanwhile, when there is convergence to $E_2 = (f(0),1)$ (panel (c)), it follows that $\pi^*(\bar{\alpha},0) = \pi^*(\alpha^*,\lambda^*) < \pi^*\left(f(0),1\right)$. Thus, if $\theta > 0$ ($\theta < 0$), the economy also converges to the highest (lowest) possible long-run equilibrium growth rate. In sum, when the economy converges a long-run equilibrium, the latter features the highest possible long-run equilibrium growth rate when $\theta > 0$. The intuition for this result is straightforward. Recall from (30)-(31) that, if the economy converges to a long-run equilibrium (which may not happen in panel (b)), the latter features the lowest possible long-run equilibrium values of the wage share in income ($1-\pi^*$) and capacity utilization (u^*). Thus, given $\pi^*(\bar{\alpha},0)$ and $\pi^*\left(f(0),1\right)$, the likelihood of $\theta > 0$ is the higher, given the saving rate γ (accelerator effect β_2), the lower (higher) the accelerator effect (saving rate). Meanwhile, we can use (21) to express the long-run equilibrium growth rate as $g^* = \gamma\pi^* u^*$. Thus, for the lowest possible long-run equilibrium wage share to be accompanied by the highest possible long-run equilibrium growth rate, the also accompanying lowest possible capacity utilization cannot be too lower than in the other long-run equilibria. Intuitively, the likelihood that this latter condition is satisfied, which requires that effective-demand effects are not too strong, varies positively (negatively) with the saving rate (accelerator effect).

5. Conclusions

This paper is motivated by several pieces of empirical evidence. First, historically, there has been a persistent heterogeneity in employee compensation strategies across firms, and employee profit-sharing schemes have experienced a fluctuating popularity. Second, profit sharing raises labor productivity in the firm, and surveys find that both employers and employees usually see profit sharing as helping to improve firm performance in several dimensions. Third, surveys find that non-deferred and in cash profit sharing is ranked first by workers as motivation device. Fourth, firms switch modes of employee compensation frequently, with the gross changes in modes being far more numerous than the net changes. Fifth, profit sharing has a meaningful effect on worker total compensation, which suggests that profit sharing tends to come on top of, rather than in place of, a base wage.

We model firms as periodically choosing to compensate workers with either a base wage or a share of profits on top of a base wage, with the joint frequency distribution of employee compensation strategies and labor productivity across firms being evolutionarily time-varying. Besides, we explore the implications of this coupling evolutionary dynamics for income distribution (and hence aggregate effective demand), and therefore for capacity utilization and economic growth.

When the productivity gain for profit-sharing firms is relatively low, there are two pure-strategy long-run equilibria, one featuring all firms sharing profits and the other with no firm sharing profits. However, the former is a repulsor, while the latter is an attractor. When the productivity gain for profit-sharing is relatively high, the economy has the same two pure-strategy long-run equilibria. Yet the long-run equilibrium with all firms sharing profits is an attractor, whereas the long-run equilibrium with no firm sharing profits is a repulsor. Meanwhile, when the productivity gain for profit-sharing firms is relatively moderate, these two pure-strategy long-run equilibria are joined by one mixed-strategy long-run equilibrium, which features heterogeneity in worker compensation modes across firms. In this case, the pure-strategy long-run equilibria are repulsors, while the mixed-strategy long-run equilibrium is either an attractor or a repulsor. Yet if this long-run equilibrium with coexistence of both employee compensation strategies is not an attractor, the system nonetheless keeps undergoing endogenous, self-sustaining and persistent fluctuations in the frequency distribution of worker compensation strategies and labor productivity, with income distribution, capacity utilization and economic growth also persistently fluctuating.

Meanwhile, when there is convergence to some of the three long-run equilibria, both the total wage share in income (which includes shared profits) and capacity utilization turn stationary at their lowest possible long-run equilibrium values. Yet these lowest possible long-run equilibrium total wage share and capacity utilization may come to be accompanied by the highest possible long-run equilibrium growth rate if effective-demand effects are not too strong.

References

Akerlof, G. (1982) ‘Labor contracts as partial gift exchange’, The Quarterly Journal of Economics, 87, 543-69.

Bain, J. (1949) ‘A note on pricing in monopoly and oligopoly’, American Economic Review, 39, 448–464.

Blasi, J., R. Freeman, C. Mackin and D. Kruse (2010) ‘Creating a bigger pie? The effects of employee ownership, profit sharing, and stock options on workplace performance’, in Douglas Kruse, Richard Freeman and Joseph Blasi (eds), Shared Capitalism at Work, Chicago: University of Chicago Press.

Blasi, J., D. Kruse and R. Freeman (2010) ‘Epilogue (and Prologue)’, in Douglas Kruse, Richard Freeman and Joseph Blasi (eds), Shared Capitalism at Work, Chicago: University of Chicago Press.

Blasi, J., R. Freeman and D. Kruse (2013) The Citizen's Share: Putting Ownership Back into Democracy, New Haven: Yale University Press.

Bryson, A. and R. Freeman (2010) 'How does shared capitalism affect economic performance in the United Kingdom?', in Douglas Kruse, Richard Freeman and Joseph Blasi (eds), Shared Capitalism at Work, Chicago: University of Chicago Press.

Cahuc, P. and B. Dormont (1997) 'Profit-sharing: does it increase productivity and employment? A theoretical model and empirical evidence on French micro data', Labour Economics, 4, 293–319.

Capelli, P. and D. Neumark (2004) 'External churning and internal flexibility: Evidence on the functional flexibility and core-periphery hypotheses', Industrial Relations, 43(1), 148-82.

Conyon, M. and R. Freeman (2004) 'Shared modes of compensation and firm performance: U.K. evidence', in Richard Blundell, David Card and Richard Freeman (eds), Seeking a Premier League Economy, Chicago: University of Chicago Press.

D'Art, D. and T. Turner (2004) 'Profit sharing, firm performance and union influence in selected European countries', Personnel Review, 33(3), 335–350.

D'Art, D. and T. Turner (2006) 'Profit sharing and employee share ownership in Ireland: A new departure?', Economic and Industrial Democracy, 27(4), 543–564.

Davidson, P. (1986–7) 'The simple macroeconomics of a nonergodic monetary economy vs a share economy: is Weitzman's macroeconomics too simple?', Journal of Post Keynesian Economics, 9(2), 212–25.

Doucouliagos, C. (1995) 'Worker participation and productivity in labor-managed and participatory capitalist firms: A meta-analysis', Industrial and Labor Relations Review, 49(1), 58-77.

Farebrother (1973) 'Simplified Samuelson conditions for cubic and quartic equations', The Manchester School, 41(4), 396-400.

Hall, R. and C. Hitch (1939) 'Price theory and business behaviour', Oxford Economic Papers, 2, 12-45.

Harrod, R. F. (1952) 'Theory of imperfect competition revised', in Economic Essays, London: Macmillan, chapter 8, 139–187.

Kaldor, N. (1935) 'Market imperfection and excess capacity', Economica, 2(5), 33-50.

Kalmi, P., A. Pendleton and E. Poutsma (2005) 'Financial participation and performance: New survey evidence from Europe', Human Resource Management Journal, 15(4), 54–67.

Kehoe, J. T. (1998) ‘Uniqueness and stability’, in A. P. Kirman (ed.), Elements of General Equilibrium Analysis, Oxford: Basil Blackwell, pp. 38–87.
Kim, S. (1998) ‘Does profit sharing increase firms’ profits?’, Journal of Labour Research, 19, 351–70.
Kruse, D. (1993) Profit Sharing: Does it Make a Difference? Kalamazoo, MI: W.E. Upjohn Institute for Employment Research.
Kruse, D., J. Blasi and R. Park (2010) ‘Shared capitalism in the U.S. economy: Prevalence, characteristics, and employee views of financial participation in enterprise’, in Douglas Kruse, Richard Freeman and Joseph Blasi (eds), Shared Capitalism at Work, Chicago: University of Chicago Press.
Kruse, D., R. Freeman and J. Blasi (2010) ‘Do workers gain by sharing? Employee outcomes under employee ownership, profit sharing, and broad-based stock options’, in Douglas Kruse, Richard Freeman and Joseph Blasi (eds), Shared Capitalism at Work, Chicago: University of Chicago Press.
Lima, G. T. (2010) ‘Profit sharing, capacity utilization and growth in a Post-Keynesian macromodel’, in Mark Setterfield (org.), Handbook of Alternative Theories of Economic Growth, Cheltenham: Edward Elgar.
Lima, G. T. (2012) ‘A Neo-Kaleckian model of profit sharing, capacity utilization and economic growth’, Metroeconomica, 63(1), 92-108.
Marglin, S. and A. Bhaduri (1990) ‘Profit squeeze and Keynesian theory’, in Stephen A. Marglin and Juliet B. Schor (eds), The Golden Age of Capitalism, Oxford: Clarendon Press, 153–186.
Mitchell, D. B., D. Lewin and E. E. Lawler (1990) ‘Alternative pay systems, firm performance, and productivity’, in Alan S. Blinder (ed.), Paying for Productivity, Washington, D.C.: Brookings Institution, 15–88.
Peterson, S. and F. Luthans (2006) ‘The impact of financial and nonfinancial incentives on business-unit outcomes over time’, Journal of Applied Psychology, 91(1), 156–165.
Rothschild, K. W. (1986-7) ‘Is there a Weitzman miracle?’, Journal of Post Keynesian Economics, 9(2), 198–211.
Sweezy, P. (1939) ‘Demand under conditions of oligopoly’, Journal of Political Economy, 47, 568-573
Weibull, J. W. (1995) Evolutionary Game Theory, Cambridge, MA: The MIT Press.

Weitzman, M. (1985) ‘The simple macroeconomics of profit sharing’, American Economic Review, 95, 937–953.
Weitzman, M. and D. Kruse (1990) ‘Profit sharing and productivity’, in Alan S. Blinder (ed.), Paying for Productivity, Washington, D.C.: Brookings Institution, 95–140.

Appendix 1 - Existence and uniqueness of a long-run equilibrium with all firms playing the non-sharing strategy

Let $x = \alpha - v$ and $h(x) = f(\delta x) - v$. Given these definitions, in order to show the existence and uniqueness of an $\bar{\alpha} \in (v, \infty) \subset \mathbb{R}$ which satisfies (24) we have to show that the function h as a unique strictly positive fixed point, that is, there is a unique $\bar{x} = \bar{\alpha} - v \in \mathbb{R}_{++}$ such that $\bar{x} = h(\bar{x})$.

We can accomplish this by using Theorem 3 in Kennan (2001, pp. 895), which makes it possible to ascertain that h has a unique $\bar{x} \in \mathbb{R}_{++}$, if the following conditions are satisfied: (i) h is increasing; (ii) h is strictly concave; (iii) $h(0) \geq 0$; (iv) $h(a) > a$ for some $a > 0$; and (v) $h(b) < b$ for some $b > a$.

Conditions (i) and (ii) are satisfied. Since $f'(\cdot) > 0$ and $f''(\cdot) < 0$ in the entire domain of f, it follows that for all $x \in \mathbb{R}_{+}$ we have:

(A-1.2) $$h'(x) = \delta f'(\delta x) > 0 \text{ and } h''(x) = \delta^2 f''(\delta x) < 0.$$

Therefore, the function h is strictly increasing and strictly concave.

Condition (iii) is also satisfied. Since $f(0) > 1 > v$, it follows that $h(0) = f(0) - v > 0$.

Condition (iv) is likewise satisfied. Given that $0 < v < 1$, it follows that $0 < 1 - v < 1$. Besides, since h is strictly increasing and $f(0) > 1$, there is an $a = 1 - v$ such that $h(1-v) = f\left(\delta(1-v)\right) - v > f(0) - v > 1 - v$.

Condition (v) is also satisfied. Recall that we have assumed that $\lim_{(y_s - \bar{y}) \to \infty} f'(y_s - \bar{y}) = 0$. Hence, since $y_s - \bar{y} = \delta(\alpha - v) = \delta x$ for $\lambda = 0$, it follows that $\lim_{x \to \infty}(y_s - \bar{y}) = \lim_{x \to \infty} \delta x = \infty$. For $\lambda = 0$, therefore, we can deduce that $\lim_{x \to \infty} \frac{h(x)}{x} = \lim_{x \to \infty} \frac{f(\delta x) - v}{x} = \lim_{x \to \infty} \delta f'(\delta x) = 0$, where in the last equality we have used L'Hôpital's rule. Therefore, we can assert that for every $\varepsilon > 0$

there is some $M > 0$ such that for all $x > M$ we have $\left|\frac{h(x)}{x}\right| < \varepsilon$. This last inequality can be re-written as $h(x) < \varepsilon x$, since $h(x) = f(\delta x) - v > f(0) - v > 0$ for all $x > 0$. Particularly, we can set $\varepsilon = 1$. In this case, there is some $M > 0$ such that for all $x > M$ it follows that $h(x) < x$. Thus, it is enough to choose any $x = b > Max\{M, a\}$ to obtain $h(b) < b$ and $b > a = 1 - v$.

Appendix 2 - Existence and uniqueness of a mixed-strategy long-run equilibrium (both employee compensation strategies are played across the population of firms)

Let $\phi(\alpha, \lambda) \equiv \alpha - f\left(\delta(1-\lambda)(\alpha - v)\right)$. Condition (26) is satisfied if, and only if, $\phi(\alpha^*, \lambda^*) = 0$. Let us show that, given α^*, if condition (27) is satisfied, there is a unique $\lambda^* \in (0,1) \subset \mathbb{R}$ such that $\phi(\alpha^*, \lambda^*) = 0$.

Given the existence and uniqueness of the fixed point $\bar{\alpha}$ demonstrated in Appendix 1, we can use the Index Theorem (Kehoe, 1987, p. 52) to write:

(A-2.1) $$index(\bar{\alpha}) = \operatorname{sgn}\left(1 - \frac{\partial f\left(\delta(\bar{\alpha} - v)\right)}{\partial \alpha}\right) = 1,$$

where $\operatorname{sgn}(\cdot)$ stands for the sign function. Based on this function, we can establish that:

(A-2.2) $$\frac{\partial \phi(\bar{\alpha}, \lambda)}{\partial \alpha} = 1 - \delta f'\left(\delta(\bar{\alpha} - v)\right) > 0.$$

Consequently, given the existence and uniqueness of $\bar{\alpha}$, the graph of $\phi(\bar{\alpha}, \lambda)$ in the plane given by $\{(\alpha, \lambda) \in \mathbb{R}^2_+\}$ always crosses the 45-degree line only once and from above, as shown in Figure 3. Since it follows from (24) that $\phi(\bar{\alpha}, 0) = \bar{\alpha} - f\left(\delta(\bar{\alpha} - v)\right) = 0$, we can conclude that $\phi(\alpha, 0) = \alpha - f\left(\delta(\alpha - v)\right) < 0$ for all $\alpha \in (v, \bar{\alpha}) \subset \mathbb{R}$. For all $\alpha^* < \bar{\alpha}$, therefore, we can infer that:

(A-2.3) $$\phi(\alpha^*, 0) = \alpha^* - f\left(\delta(\alpha^* - v)\right) < 0.$$

Moreover, it is straightforward that (27) implies that:

(A-2.4) $$\phi(\alpha^*,1)=\alpha^*-f(0)>0.$$

Since $\phi(\alpha^*,0)<0$, $\phi(\alpha^*,1)>0$ and ϕ is continuous throughout the domain, we can then apply the intermediate value theorem to conclude that there is some $\lambda^*\in(0,1)\subset\mathbb{R}$ such that $\phi(\alpha^*,\lambda^*)=0$. Moreover, given that $f'(\cdot)>0$ for all $\lambda\in[0,1]\subset\mathbb{R}$, we have:

(A-2.5) $$\frac{\partial\phi(\alpha^*,\lambda)}{\partial\lambda}=\delta(\alpha^*-v)\,f'\big(\delta(1-\lambda)(\alpha^*-v)\big)>0,$$

for all $\lambda\in[0,1]\subset\mathbb{R}$. As a result, since the function in (A.2-5) is continuous in the closed interval $[0,1]\subset\mathbb{R}$, there is only one $\lambda^*\in(0,1)\subset\mathbb{R}$ such that $\phi(\alpha^*,\lambda^*)=0$.

Appendix 3 - Local stability of the long-run equilibrium with all firms playing the non-sharing strategy

The Jacobian matrix evaluated around the equilibrium $(\bar{\alpha},0)\subset\Theta$ is given by:

(A-3.1) $$J(\bar{\alpha},0)=\left[\begin{array}{c|c}\delta f'\big(\delta(\bar{\alpha}-v)\big) & -\delta(\bar{\alpha}-v)\,f'\big(\delta(\bar{\alpha}-v)\big)\\ \hline 0 & 1+\left((1-\delta)\left(1-\frac{v}{\bar{\alpha}}\right)-(1-v)\right)\frac{\beta_1(1-v)}{\gamma(1-v)-\beta_2}\end{array}\right].$$

Let ξ be an eigenvalue of the Jacobian matrix (A-3.1). We can then set the following characteristic equation of the linearization around the equilibrium:

(A-3.2) $$|J-\xi I|=\left|\begin{array}{c|c}a-\xi & -b\\ \hline 0 & c-\xi\end{array}\right|=\xi^2-(a+c)\xi+ac=0,$$

where $a\equiv\delta f'\big(\delta(\bar{\alpha}-v)\big)>0$, $b\equiv\delta(\bar{\alpha}-v)\,f'\big(\delta(\bar{\alpha}-v)\big)>0$, and $c\equiv1+\left[(1-\delta)\left(1-\frac{v}{\bar{\alpha}}\right)-(1-v)\right]\frac{\beta_1(1-v)}{\gamma(1-v)-\beta_2}$. The solutions of (A-3.2) are the eigenvalues of the Jacobian matrix (A-3.1), which are given by:

(A-3.3) $$\xi_1=\delta f'\big(\delta(\bar{\alpha}-v)\big) \text{ and } \xi_2=1+[\pi_s^c(\bar{\alpha})-\pi_n]u^*(\bar{\alpha},0),$$

where $\pi_s^c(\bar{\alpha}) = (1-\delta)\left(1-\frac{v}{\bar{\alpha}}\right)$, $\pi_n = 1-v$, and $u^*(\bar{\alpha},0) = \frac{\beta_1(1-v)}{\gamma(1-v)-\beta_2}$.

Let us investigate the absolute value of ξ_1. Given that $\delta > 0$ and $f'(\cdot) > 0$ throughout the domain, it follows that $\xi_1 = \delta f'\left(\delta(\bar{\alpha}-v)\right) > 0$. Moreover, it follows from (A-2.2) that $\xi_1 = \delta f'\left(\delta(\bar{\alpha}-v)\right) < 1$. Therefore, it follows that $|\xi_1| < 1$.

Let us check the absolute value of ξ_2. We want to find out under what condition(s) it follows that $-1 < \xi_2 < 1$. Given (A-3.3), it follows that $-1 < \xi_2 < 1$ obtains if, and only if, $-2 < [\pi_s^c(\bar{\alpha}) - \pi_n]u^*(\bar{\alpha},0) < 0$.

Since $0 < \pi_s^c(\bar{\alpha}) < 1$ and $0 < \pi_n < 1$, we obtain that $-1 < \pi_s^c(\bar{\alpha}) - \pi_n < 1$. Moreover, given that $0 < u^*(\bar{\alpha},0) < 1$, we can establish that $-2 < -u^*(\bar{\alpha},0) < [\pi_s^c(\bar{\alpha}) - \pi_n]u^*(\bar{\alpha},0)$. It is easy to see that $\pi_s^c(\alpha) - \pi_n = (1-\delta)\left(1-\frac{v}{\alpha}\right) - (1-v)$ is increasing in α. Given (25), we know that $\pi_s^c(\alpha^*) - \pi_n = 0$. Thus, if $\bar{\alpha} < \alpha^*$ (panel (a) in Figure 2), we find that $\pi_s^c(\bar{\alpha}) - \pi_n < 0$, and therefore that $[\pi_s^c(\bar{\alpha}) - \pi_n]u^*(\bar{\alpha},0) < 0$, which means that $|\xi_2| < 1$ if $\bar{\alpha} < \alpha^*$. Yet if $\bar{\alpha} > \alpha^*$ (panels (b) and (c) in Figure 2), it follows that $\pi_s^c(\bar{\alpha}) - \pi_n > 0$, and therefore that $[\pi_s^c(\bar{\alpha}) - \pi_n]u^*(\bar{\alpha},0) > 0$, so that $|\xi_2| > 1$ if $\bar{\alpha} > \alpha^*$. This completes the demonstration that the long-run equilibrium with no firm playing the profit-sharing strategy, $(\bar{\alpha},0) \subset \Theta$, is an attractor if $\bar{\alpha} < \alpha^*$ (panel (a) in Figure 2) and a repulsor if $\bar{\alpha} > \alpha^*$ (panels (b) and (c) in Figure 2).

Appendix 4 - Local stability of the long-run equilibrium with all firms playing the profit-sharing strategy

The Jacobian matrix evaluated around the equilibrium $\left(f(0),1\right) \subset \Theta$ is given by:

(A-4.1) $$J(f(0),1)=\left[\begin{array}{c|c} 0 & -\delta\left(f(0)-v\right)f'\left(0\right) \\ \hline 0 & 1-\left((1-\delta)\left(1-\frac{v}{f(0)}\right)-(1-v)\right)\frac{\beta_1(1-\delta)\left((1-v/f(0)\right)}{\gamma(1-\delta)\left((1-v/f(0)\right)-\beta_2}\end{array}\right].$$

Let ξ be an eigenvalue of the Jacobian matrix (A-4.1). We can then set the following characteristic equation of the linearization around this equilibrium:

(A-4.2) $$\left|J-\xi I\right|=\left|\begin{array}{c|c} -\xi & -a \\ \hline 0 & b-\xi\end{array}\right|=\xi(\xi-b)=0,$$

with $a\equiv\delta\left[f(0)-v\right]f'\left(0\right)>0$ and $b\equiv1-\left[(1-\delta)\left(1-\frac{v}{f(0)}\right)-(1-v)\right]\frac{\beta_1(1-\delta)\left[(1-v/f(0)\right]}{\gamma(1-\delta)\left[(1-v/f(0)\right]-\beta_2}$.

In this case, the eigenvalues of the Jacobian matrix (A.4-1) are easily computed from (A-4.2):

(A-4.3) $$\xi_1=0 \text{ and } \xi_2=b=1-\left[\pi_s^c\left((f0)\right)-\pi_n\right]u^*\left(f(0),1\right),$$

where $\pi_s^c\left(f(0)\right)=(1-\delta)\left[(1-v/f(0)\right]$, $\pi_n=1-v$, and $u^*\left(f(0),1\right)=\frac{\beta_1(1-\delta)\left[(1-v/f(0)\right]}{\gamma(1-\delta)\left[(1-v/f(0)\right]-\beta_2}$.

Therefore the local stability of $\left(f(0),1\right)\subset\Theta$ depends on ξ_2. Given (A-4.3), it follows that $-1<\xi_2<1$ obtains if, and only if, $0<\left[\pi_s^c\left(f(0)\right)-\pi_n\right]u^*\left(f(0),1\right)<2$.

Since $0<\pi_s^c\left(f(0)\right)<1$ and $0<\pi_n<1$, it follows that $-1<\pi_s^c\left(f(0)\right)-\pi_n<1$. Therefore, given that $0<u^*\left(f(0),1\right)<1$, we can infer that $\left[\pi_s^c\left(f(0)\right)-\pi_n\right]u^*\left(f(0),1\right)<2$.

It is easy to see that $\pi_s^c(\alpha)-\pi_n=(1-\delta)\left(1-\frac{v}{\alpha}\right)-(1-v)$ is increasing in α. Given (25), we know that $\pi_s^c(\alpha^*)-\pi_n=0$. Hence, if $f(0)<\alpha^*$ (panels (a) and (b) in Figure 2), we obtain that $\pi_s^c\left(f(0)\right)-\pi_n<0$, and therefore that $\left[\pi_s^c\left(f(0)\right)-\pi_n\right]u^*\left(f(0),1\right)<0$, which means that $\left|\xi_2\right|>1$ if $f(0)<\alpha^*$. Meanwhile, if $f(0)>\alpha^*$ (panel (c) in Figure 2), we obtain that $\pi_s^c\left(f(0)\right)-\pi_n>0$, and therefore that $\left[\pi_s^c\left(f(0)\right)-\pi_n\right]u^*\left(f(0),1\right)>0$. Thus, it follows that

$|\xi_2| < 1$ if $f(0) > \alpha^*$. This completes the demonstration that the long-run equilibrium with all firms playing the profit-sharing strategy, $\left(f(0),1\right) \subset \Theta$, is a repulsor if $f(0) < \alpha^*$ (panels (a) and (b) in Figure 2) and an attractor if $f(0) > \alpha^*$ (panel (c) in Figure 2).

Appendix 5 - Local stability of the long-run equilibrium with heterogeneity in worker compensation strategies across firms

The Jacobian matrix evaluated around the equilibrium $(\alpha^*, \lambda^*) \subset \Theta$ is given by:

(A-5.1) $$J(\alpha^*, \lambda^*) = \left[\begin{array}{c|c} \delta(1-\lambda^*) f'\left(\delta(1-\lambda^*)(\alpha^* - v)\right) & -\delta(\alpha^* - v) f'\left(\delta(1-\lambda^*)(\alpha^* - v)\right) \\ \hline \lambda^*(1-\lambda^*)(1-\delta)\dfrac{v}{(\alpha^*)^2}\dfrac{\beta_1(1-v)}{\gamma(1-v)-\beta_2} & 1 \end{array} \right].$$

Let ξ be an eigenvalue of the Jacobian matrix (A-5.1). We can then set the following characteristic equation of the linearization around the equilibrium:

(A-5.2) $$|J - \xi I| = \left| \begin{array}{c|c} a-\xi & -b \\ \hline c & 1-\xi \end{array} \right| = \xi^2 - (a+1)\xi + (a+bc) = 0,$$

where $a \equiv \delta(1-\lambda^*) f'\left(\delta(1-\lambda^*)(\alpha^* - v)\right) > 0$, $b \equiv \delta(\alpha^* - v) f'\left(\delta(1-\lambda^*)(\alpha^* - v)\right) > 0$, and $c \equiv \lambda^*(1-\lambda^*)(1-\delta)\dfrac{v}{(\alpha^*)^2}\dfrac{\beta_1(1-v)}{\gamma(1-v)-\beta_2} > 0$.

We can use the Samuelson stability conditions for a second order characteristic equation to determine under what conditions the two eigenvalues are inside the unit circle. Based on Farebrother (1973, p. 396, inequalities 2.4 and 2.5), we can establish the following set of simplified Samuelson conditions for the quadratic polynomial in (A-5.2):

(A-5.3) $$1 + a + bc > |-(a+1)| = a+1 \text{ and } a + bc < 1.$$

Let us prove that these conditions are satisfied if $a < 1$.

Firstly, note that $1 + a + bc > a+1$ simplifies to $bc > 0$, which is trivially satisfied given that $b > 0$ and $c > 0$.

Meanwhile, the second inequality, $a+bc<1$, can be expressed as follows:

(A-5.4) $$a+bc=\delta(1-\lambda^*)f'(\cdot)\left\{1+\frac{v(1-\delta)\lambda^*(\alpha^*-v)}{(\alpha^*)^2}\frac{\beta_1(1-v)}{\gamma(1-v)-\beta_2}\right\}<1.$$

Appendix 6 - Positive invariance of the state space

We want to show that $(\alpha_t,\lambda_t)\in\Theta$ for all $t\in\{1,2...\}$ and initial condition $(\alpha_0,\lambda_0)\in\Theta$.

Let us first demonstrate that $\alpha_{t+1}>v$ for all $t\in\{0,1,2...\}$ and $(\alpha_0,\lambda_0)\in\Theta$. Given (22-a) and the assumptions that $f(0)>1>v$ and $f(.)$ is strictly increasing, we can establish that $\alpha_{t+1}=f\left(\delta(1-\lambda_t)(\alpha_t-v)\right)\geq f(0)>v$ for any $\alpha_t>v$ and $0\leq\lambda_t\leq 1$. By induction, we can the conclude that $\alpha_{t+1}>v$ for all $t\in\{0,1,2...\}$ and $(\alpha_0,\lambda_0)\in\Theta$.

Next, let us prove that $0\leq\lambda_{t+1}\leq 1$ for all $t\in\{0,1,2...\}$ and $(\alpha_0,\lambda_0)\in\Theta$. Let us first show that $\lambda_{t+1}\geq 0$ for all $(\alpha_t,\lambda_t)\in\Theta$. As $\lambda_t\geq 0$ for any $(\alpha_t,\lambda_t)\in\Theta$ and given (23-a), in order to establish that $\lambda_{t+1}\geq 0$ for all $(\alpha_t,\lambda_t)\in\Theta$, we need to demonstrate that for all $(\alpha_t,\lambda_t)\in\Theta$ we have:

(A-6.1) $$(1-\lambda_t)\left[(1-\delta)\left(1-\frac{v}{\alpha_t}\right)-(1-v)\right]u_t^*\geq -1.$$

We can set up the following lower and upper bounds for the profit-sharing differential for any $\alpha_t>v$:

(A-6.2) $$-(1-v)<\pi_{s,t}^c-\pi_{n,t}=(1-\delta)\left(1-\frac{v}{\alpha_t}\right)-(1-v)\leq v-\delta.$$

From (A-6.1)-(A-6.2) and the fact that $0<u_t^*<1$ for all $t\in\{0,1,2...\}$, we can write:

(A-6.3) $$(1-\lambda_t)(\pi_{s,t}^c-\pi_{n,t})u_t^*\geq -(1-\lambda_t)(1-v)u_t^*\geq -(1-\lambda_t)(1-v)=-1+v+(1-v)\lambda_t>-1$$

for any $(\alpha_t,\lambda_t)\in\Theta$. Hence, by induction, $\lambda_{t+1}\geq 0$ for all $t\in\{0,1,2...\}$ and $(\alpha_0,\lambda_0)\in\Theta$.

Finally, let us to demonstrate that $\lambda_{t+1} \leq 1$ for all $(\alpha_t, \lambda_t) \in \Theta$. Given (23-a), in order to establish that $\lambda_{t+1} \leq 1$ for all $(\alpha_t, \lambda_t) \in \Theta$, we need to demonstrate that:

(A-6.4) $$\lambda_t \left[1 + (1 - \lambda_t)(\pi_{s,t}^c - \pi_{n,t}) u_t^* \right] \leq 1.$$

Note that this inequality is trivially satisfied for $\lambda_t = 0$. For any $\lambda_t > 0$, we can re-write (A-6.4) as follows:

(A-6.5) $$(\pi_{s,t}^c - \pi_{n,t}) u_t^* \leq \frac{1}{\lambda_t}.$$

We can again make use of (A-6.2) and conclude that for any $\lambda_t > 0$, we have:

(A-6.6) $$(\pi_{s,t}^c - \pi_{n,t}) u_t^* \leq (\nu - \delta) u_t^* < 1 \leq \frac{1}{\lambda_t}.$$

This completes the proof that the state space Θ is positively invariant, that is, it follows that $(\alpha_t, \lambda_t) \in \Theta$ for all $t \in \{1, 2...\}$ and initial condition $(\alpha_0, \lambda_0) \in \Theta$.