Unproductive Activity and Endogenous Technological Change in a Marxian Model of Economic Growth

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In *Capital* Marx develops many of his concepts and ideas diachronically across all three volumes. Consider, for example, his analysis of the relations among different classes. In volume one Marx [1] focuses primarily on the relations between 'workers' and 'capitalists'. By volume three[3], however, he has introduced numerous additional types of economic classes, including different types of workers (i.e. productive and unproductive laborers) and different types of capitalists (e.g. industrial capitalists, financial capitalists, merchant capitalists, etc.), and he presents a much more nuanced analysis of relationships among these classes. Meanwhile, in the development from volume one to volume three, he presents his analysis of economic reproduction in volume two [2], which seems to have little to do with his theory of classes and has typically been considered independently of it. But Marx gives ample indication that he considers his class theory and his theory of reproduction to be intimately connected, and indeed they were explicitly connected in at least one of the earlier drafts of *Capital*.[1]

In contrast to those approaches that present Marx's class theory and his theory of reproduction as easily separable, the purpose of this paper is to develop a simple two-sector model of economic growth based on Marx's theory of

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[1]See Marx's multiple *tableau economiques* at the conclusion to his *Manuscripts of 1861-63[?]*. In his tableaus Marx wrestles with how to present the process of aggregate economic reproduction explicitly in terms of the relations among different classes. Only in later drafts did he treat reproduction in the more abstract way that it is presented in the published version volume two.

reproduction from volume two of *Capital* that also makes the relations among different economic class positions involved in this process explicit. The model is organized around a social accounting matrix (SAM) that identifies the flows of value among the various class positions, and makes it clear what the relevant variables are.[2] Section 1, below, briefly discusses the SAM, and section 2 does a brief class analysis of Marx's reproduction schemes using the SAM. Section 3 generalizes Marx's reproduction scheme with a simple model, and also simulates Marx's numerical examples. This first simulation provides a baseline to compare later iterations of the model. The first version of the model includes only productive laborers, industrial capitalists, and interest, dividend and rent (IDR) recipients.

Presenting the theory of reproduction in the form of a model has the advantage of making it far easier to incorporate additional class positions into the analysis of reproduction, and thus to introduce a class structure that more closely approximates the one Marx presents in volume three of *Capital*. A second iteration of the model incorporates unproductive laborers into the model. One incentive for this is provided by empirical studies, notably the work of Shaikh and Tonak [8], that have found a profound growth in the size of unproductive labor as a share of U.S. employment. After incorporating unproductive labor into the model of extended reproduction a second simulation is presented that illustrates some of the potential effects of this growth in uproductive activity. Notable among these is the potential for secular downturns in the rate of growth in an economy resulting from an unproductive "profit squeeze".

But the effect of unproductive labor on growth is complex. One of the potential effects of unproductive labor is to increase productivity either through increased supervision of productive laborers, or, perhaps more importantly, by creating productivity enhancing technological change. This technological change is 'endogenoous' in the sense that it results from industrial capitalists distributing surplus to laborers who do not produce the output of the the enterprise, but rather reduce the constant and variable capital inputs necessary to produce this output. These laborers are unproductive in the sense that they produce no actual surplus for the enterprise, but rather they find ways to increase the relative surplus produced by the firms productive laborers. A third iteration of the model and simulation considers this situation, and makes clear that the unproductive laborers can have a contradictory effects of the rate of growth, both reducing and enhancing it.

1 The Social Accounting Matrix

In *Capital*, volume three in particular, Marx elaborates his class theory by looking at the types and sources of revenues for various groups in a capitalist economy, and then he relates these revenues to the initial exploitation of labor

[2] See Pyatt [11] for a more extensive discussion of SAM-based models. Taylor [10]also discusses the role of the SAM in macroeconomic modeling.

in the production sphere. Individuals are distinguised, or classified, according to whether they produce surplus value, appropriate surplus value produced by others, or receive distributed shares of surplus value appropriated by others. Productive laborers produce surplus; industrial capitalists appropriate the surplus produced by productive laborers in their employment; other types of capitalists (merchant, moneylending, etc.) receive distributed shares of surplus appropriated by industrial capitalists, as do unproductive laborers. Resnick and Wolff [7, Chapter 3] refer to productive laborers and industrial capitalists as "fundamental class positions" and the recipients of distributed shares of surplus as "subsumed classes". I also use these terms extensively throughout this article, with further distinctions made where necessary.

Each of the various fundamental and subsumed class positions also function in the process of economic reproduction. They are variously producers, appropriatiors, consumers, investors, etc., and so in these ways economic reproduction ties all of these various class postions together into a system of mutual interelation. The quantitative aspect of the relations among these class postions is the flow of value among them. Economic reproduction is accomplished through these value flows, and it is possible to illustrate this using a SAM.

A SAM can be understood as similar to a input-output transactions table, but is extended to include current and capital transactions in addition to the transactions between industries. Table 1 presents a simple qualitative Marxian SAM. It is 'Marxian' because it is constructed to depict the production, appropriation, distribution, and circulation of surplus value in a capitalist economy. The SAM assures that all of relevant value flows are accounted for in the model, and it imposes consistency on the subordinate elements. Before developing the formal model it is useful to demonstrate how the quantities that form the basis of Marx's reproduction schemes can be disaggregated into their class components and then these value flows depicted in the SAM.

2 Class Analysis of Marx's Reproduction Schemes

2.1 Simple Reproduction

Marx's reproduction schemes divide productive enterprises into dept. I, which produces means of production, and dept. II, which produces means of consumption. The value of the output of one of the departments (W) is measured by the quantity of constant capital (C), variable capital (V), and surplus value (S) embodied in the product. Letting the subscript i ($= 1, 2$) designate the department, this can be written as,

$$W_i = C_i + V_i + S_i \tag{1}$$

In order to describe the distribution of surplus it is necessary to differentiate incomes out of surplus value according to the three potential types. At this stage I ignore unproductive labor (managers, etc.) and consider only the following three distributions:

SC_{ic} = Interest payments from the ith department to lenders of money capital (Marx's "Monied Men")

SC_{io} = Dividend payments from the ith department to owners of the enterprises (Marx's "Entrepreneurs")

SC_{il} = Rent payments from the ith department to property owners (Marx's "Landlords")

The distribution of surplus in dept. i is therefore given by,

$$S_i = SC_{io} + SC_{ic} + SC_{il} \tag{2}$$

Collectively I refer to these as "IDR" payments. By assumption all of the surplus value produced in the enterprises is distributed (i.e. no retained earnings), and all of these payments are received by the occupants of subsumed class positions located outside of the enterprise in households. Surplus value produced by productive laborers is the source of these subsumed class payments, and the enterprise transfers this surplus value to the IDR recipient households who receive it as revenue. Using these definitions Marx's scheme of simple reproduction can be described in a SAM, and this is done in Table 2.

For a SAM to be consistent the sum for each corresponding row and column must be equal. The conditions for simple reproduction that Marx derives for his reproduction schemes are exactly the same as the consistency conditions for the Marxian SAM. In order for successful reproduction the demand for means of production from both department must be equal to the output of means of production by dept. I. Similarly for the SAM to be consistent the revenues of dept. I (row A) must equal the outflows from the production activities of the dept. I (column A). These conditions can be written as:

$$C_1 + C_2 = C_1 + V_1 + S_1 \tag{3a}$$

Equation 2a reduces to Marx's condition for simple reproduction (Marx, 1967b, p. 402),

$$C_2 = V_1 + S_1 \tag{3b}$$

Similarly the demand for consumer goods by workers and subsumed classes (row B) must be sufficient to absorb the value of the output of dept. II (column B):

$$V_1 + V_2 + SC_{1o} + SC_{2o} + SC_{1c} + SC_{2c} + SC_{1l} + SC_{2l} = C_2 + V_2 + S_2 \tag{4}$$

Since by definition $S_i = SC_{io} + SC_{ic} + SC_{il}$ equation (4) also establishes Marx's condition for simple reproduction. The difference here is that the class decomposition of S emphasizes that this condition can be established in terms of nonclass (wages) and subsumed class payments.

2.2 Extended Reproduction in a Marxian SAM

Marx's schemes of expanded reproduction can also be depicted in the Marxian SAM. In equation (2) the distribution of surplus value was defined according

to three potential recipients: owners, moneylenders, and landlords. In simple reproduction IDR recipients recieve the entirety of the surplus value as income and consume it all. Extended reproduction assumes that these households do not consume their entire income and instead save and accumulate a portion of it. This requires a description of what the recipient households might subsequently do with these subsumed class revenues. Because Marx abstracts here from the potential for changes to the stock of money (hoarding), the potential uses are limited to either consumption (S_{icon}) or investment ($S_{i\Delta}$). The sum of the subsumed class payments must therefore be equal to the consumption or saving/investment from surplus value:

$$SC_{io} + SC_{ic} + SC_{il} = \Sigma SC_i = S_{icon} + S_{i\Delta} \tag{5}$$

Throughout *Capital* Marx assumes that constant and variable capital are augmented by the accumulation of a portion of surplus value. Therefore only those households that are recipients of distributions of surplus value—the occupants of subsumed class positions—can be said to save in this model. Their savings must be allocated to both departments in form of additional constant capital ($S_{i\Delta C}$) and variable capital ($S_{i\Delta V}$):

$$S_{i\Delta} = S_{i\Delta C} + S_{i\Delta V} \tag{6}$$

Table 3 expands Table 2 by including an Accumulation account to account for these additional uses of surplus. Surplus appropriated in the two departments becomes the source of accumulation via the subsumed class process and the subsequent saving/investment decisions of the households that receive these incomes. In order to conserve space Table 3 combines the payments of interest, dividends and rents into one category labeled as "IDR". As with simple reproduction, Marx's condition for successful extended reproduction [2, p. 520] is also the condition for consistency in the SAM, and this can be stated in terms of either department. The demand for means of production from both departments must be equal to the output of means of production by dept. I:

$$C_1 + C_2 + S_{1\Delta C} + S_{2\Delta C} = C_1 + V_1 + S_1 \tag{7}$$

Similarly the demand for consumer goods by workers and subsumed classes (recipients of IDR) must be sufficient to absorb the output of dept. II:

$$V_1 + V_2 + S_{1\Delta V} + S_{2\Delta V} + S_{1con} + S_{2con} = C_2 + V_2 + S_2 \tag{8}$$

Substituting (2) into either (7) or (8) results in Marx's condition for extended reproduction,

$$C_2 + S_{2\Delta C} = V_1 + S_{1con} + S_{1\Delta V} \tag{9}$$

For each department to successfully realize the value of its output in the marketplace, the inter-department transactions must be balanced for both capital goods and means of consumption.

3 Class Analysis of Economic Reproduction and Growth[3]

Marx's analysis of extended reproduction in volume two of *Capital* [2, ch. XXI] is temporal and considers six consecutive periods. Marx provides no formal model of reproduction, rather the reproduction schemes represent numerical examples of what must result if reproduction is to successfully occur. He chooses some hypothetical initial conditions, assumes some positive rate of capital accumulation, and then describes the resulting process of accumulation and growth. In this section I use the SAM to develop a generalized model of Marx's reproduction schemes, and then first use it to reproduce Marx's results. This confirms that the model accurately represents Marx's approach to extended reproduction. Modeling economic reproduction in this way allows for a consideration of more than just generic 'capitalists' and 'workers'. It allows for the rich and complex set of class positions discussed by Marx throughout *Capital*, and developed in class analytic Marxian theory, to be included in the analysis of the reproduction process, and it makes it possible to theorize the effects of the reproduction process on different types of households.

I have chosen to follow Marx's approach to reproduction closely in order to show that the results achieved here are consistent with Marx's own work. This decision invites criticism because Marx's model of extended reproduction has itself been criticized by modern readers. Howard and King [6, p. 191], for example, give three main criticisms: (i) rates of profit do not equalize across the two departments; (ii) the savings-investment behavior of capitalists in the two department differs; (iii) the reproduction schemes ignores the role of excess capacity and inventories in facilitating reproduction.

A few words in defense of Marx's model are in order here. Criticism (i) is valid but it is not a fatal indictment of Marx's approach. Marx does not fully introduce competition among capitals, and the consequent equalization of profit rates, until volume three of *Capital*. His failure to do so in volume two, where the reproduction schemes are presented, is consistent with the overall structure of *Capital*, though it does reduce the generality of his approach. I make no effort to resolve this problem here because resolving it either brings additional problems and would distract attention from what I intend to illustrate with this model. This type of problem is also not unique to Marx's model. It was found in neoclassical two-sector growth models well into the 1960's.

Criticisms (ii) and (iii), on the other hand, have less merit. They are valid only if one assumes that the issue that Marx is trying to address with his reproduction schemes is simply coordination among the two departments and the potential for crises emerging from imbalances between them. This,

[3] In developing the basic model of Marx's schemes of extended reproduction presented below I found the work of Howard and King [6, chapter 11] and Morishima [9, Part IV] most useful. Howard and King (pp. 183 and 185) list a number of assumptions that either explicitly on implicitly underlie Marx's reproduction schemes. I have omitted them here for the sake of brevity, but the reader may wish to refer to that source for an itemized list. It should be obvious when I drop some of these assumptions in this chapter.

I believe, is based on a misreading of Marx's overall problematic that leads to confusion over how the reproduction schemes fit in his theorizing of this problematic. While Marx does derive the conditions under which reproduction can take place, this is not his sole, or even primary, purpose. As will become clear in what follows, Marx *assumes* that reproduction will take place. He states this explicitly in a number of places. For example: "In the following pages we shall assume that capital circulates in its normal way" [1, p.564], and "If things are to proceed normally, accumulation in II must take place more rapidly than in I . . . [2, p. 512]. So it is difficult to support the interpretation of the reproduction schemes as a theory of crisis.

But if the reproduction schemes are not intended to explore the potential success or failure of reproduction, then what purpose do they serve? They are part of Marx's more general theory of economic reproduction, which includes his class theory, and they provide a system of macroeconomic aggregates. These aggregates were necessary to establish the basic value accounting Marx employs, and they also resolve a problem with Adam Smith's theory of value. They also begin the transition from considering individual capitals to a consideration of ". . . the individual capitals as part of the aggregate social capital" [2, p. 354], or from the individual site to aggregations of sites. Marx clearly describes this aspect of the structure of *Capital* on pp. 352-354 of volume two [2]. But they should not simply be taken in isolation from the larger theoretical arguments that Marx develops across the three volumes of *Capital*. The model presented in this section is an effort to make the connections between the various parts, and thereby to illustrate the larger narrative and argument in *Capital*.

3.1 The Basic Model of Extended Reproduction

The class analytic SAM provides a simplified but reasonably comprehensive description of the value flows in a simple economy consisting of two types of sites (enterprises and households), two different types of enterprises (Dept. I, and Dept. II), and two different types of households (workers and subsumed classes or IDR recipients). The basic model of extended reproduction must explain how each of the variables in the class analytic SAM is determined in each of the t periods ($t = 1, ..., 6$).

3.1.1 Definitions

The quantity of aggregate social capital in either department at any given period t is the sum of the capital in the previous period plus the quantity of surplus used to augment the capital stock in the previous period:

$$C_{i(t)} = C_{i(t-1)} + S_{i\Delta C(t-1)} \tag{10}$$

$$V_{i(t)} = V_{i(t-1)} + S_{i\Delta V(t-1)} \tag{11}$$

The surplus used to augment the capital stock is the savings of households that receive income in the form of subsumed class payments.

The rate of exploitation (ϵ) is assumed to be equal in both departments and in the absence of technological change—an assumption that is dropped later in this article—it remains constant,

$$\epsilon = S/V$$

The quantity of surplus value produced in any period can be derived directly from the definition of the rate of exploitation:

$$S_{i(t)} = \epsilon V_{i(t)} \tag{12}$$

Marx's model of expanded reproduction assumes constant production coefficients and no technical change during the time periods under consideration:

Constant capital input coefficient: $c_i = C_i/W_i$
Variable capital or labor input coefficient: $v_i = V_i/W_i$
Rate of surplus per value unit output: $s_i = S_i/W_i$
And from equation (1) (definition of $V_{i(t)}$)

$$c_i + v_i + s_i = 1$$

In the absence of technological change the surplus value to be accumulated must be divided into constant and variable capital so that the organic composition of capital remains constant. The organic composition of capital can be defined as a simple ratio of constant to variable capital or as a percentage of the total capital, and both measurements are useful in what follows, so define the two different measurements as,

$$q_i = \frac{C_i}{V_i} \qquad \text{and} \qquad \kappa_i = \frac{C_i}{C_i + V_i}$$

3.1.2 Accumulation and Growth in Department I

In order to assure successful reproduction Marx makes the rate of accumulation in dept. II variable, and requires it to adjust to assure that the demand for inputs from dept. I is such that the output of that sector is consumed—no more and no less. In other words, Marx assumes that accumulation in dept. II will adjust to assure smooth reproduction for system as a whole. By choosing dept. I rather than dept. II as the regulator of accumulation in the reproduction schemes Marx gives precedence to supply considerations rather than demand conditions, but this appears to be done merely for convenience. Marx also assumes that the recipients of surplus value from each department invest only in that department (which prevents the profit rates from converging).

Letting $a_1 \in [0, 1]$ stand for the savings rate, the rate of capital accumulation α_1 can be expressed as a percentage of the subsumed class payments made by that department:

$$\alpha_1 = \frac{a_1(SC_{1o} + SC_{1c} + SC_{1l})}{\Sigma SC_1} = \frac{S_{1\Delta}}{\Sigma SC_1} \tag{13}$$

It in only for later convenience that I distinguish between the rate of capital accumulation α and the savings rate at this stage. Since $SC_{1o} + SC_{1c} + SC_{1l} = \Sigma SC_1$ then $\alpha_1 = a_1$. In other words, the rate of accumulation in dept. I is equal to savings rate of dept. I subsumed classes. The distinction between the two becomes relevant later when unproductive labor is included in the model.

The total quantity of surplus accumulated in dept. I in time period t is,

$$S_{1\Delta(t)} = \alpha_1 s_1 W_{1(t)} \tag{14}$$

The accumulation of constant capital by department 1 in period t is,

$$S_{1\Delta C(t)} = \kappa_1 \alpha_1 s_1 W_{1(t)}$$

The accumulation of variable capital by department 1 in period t is ,

$$S_{1\Delta V(t)} = S_{1\Delta C(t)} \frac{1}{q_1} = \kappa_1 \alpha_1 s_1 W_{1(t)} \frac{1}{q_1} \tag{15}$$

The parameters q_1 , κ_1 and the coefficient s_1, are given by Marx's "Initial Scheme for Accumulation on an Extended Scale" [2, p. 510], as is the initial values for total output $W_{1(t)}$. The growth rate of the economy is defined by the rates of accumulation in the two sectors.

3.1.3 Accumulation and Growth in Department II

In the same way as dept. I, the total quantity of surplus accumulated in dept. II is equal to,

$$S_{2\Delta(t)} = \alpha_{2(t)} s_2 W_{2(t)}$$

But unlike dept I the rate of accumulation of surplus in dept II, $\alpha_{2(t)}$ varies to assure smooth reproduction of both departments, and hence has a time subscript associated with it. To assure extended reproduction the rate of surplus accumulation in dept II must be sufficient to absorb the means of production produced by dept I in excess of its own needs. In other words, dept II reacts to the behavior of dept I in order to assure that the conditions for successful extended reproduction (9) are met. The accumulation of constant capital by dept II in period t must be equal to the gross output of dept I net of dept I's own demand for inputs and accumulation of means of production in the period, and net of dept II's demand for inputs of means of production in the period. The accumulation of constant capital by dept II must therefore be,

$$S_{2\Delta C(t)} = \kappa_2 \alpha_{2(t)} s_2 W_{2(t)} = W_{1(t)} - c_1 W_{1(t)} - \kappa_1 \alpha_1 s_1 W_{1(t)} - c_2 W_{2(t)} \tag{16a}$$

or

$$S_{_{2\Delta C(t)}} = (1 - c_1 - \kappa_1 \alpha_1 s_1) W_{1(t)} - c_2 W_{2(t)} \tag{16b}$$

The accumulation of variable capital by dept. II is, as it was for dept. 1, a function of the quantity of constant capital accumulated in that department (16b) and the organic composition of capital q_2 :

$$S_{_{2\Delta V(t)}} = \left[(1 - c_1 - \kappa_1\alpha_1 s_1)W_{1(t)} - c_2 W_{2(t)}\right] \frac{1}{q_2} \tag{17}$$

Equation (17) gives the accumulation of variable capital in terms of two variables, $W_{1(t)}$ and $W_{2(t)}$, the coefficients c_1, c_2, s_1, and the parameters κ_1 and q_1. The variable rate of accumulation in dept II ($\alpha_{2(t)}$) can be solved directly from (16a)[4], but is not necessary and unnecessarily complicates matters here.

The definitions given in the previous section and equations (13) – (17) determine the variables in this SAM-based model of Marx's basic extended reproduction. Table 4 presents the basic model of expanded reproduction in SAM form.

3.1.4 Simulation Results: Basic Model of Extended Reproduction

Simulating Marx's model economy requires initial values for C_1, C_2, V_1, V_2, and a_1 the savings rate in dept I. These values for the capital stocks are given by Marx in his "Initial Scheme for Reproduction on an Extended Scale"[2, p. 510]:

$$\begin{array}{ll} C_1 = 4,000 & C_2 = 1,500 \\ V_1 = 1,000 & V_2 = 750 \end{array}$$

In *Capital*, volume I, Ch. 24 Marx argues that the recipients of surplus—limited there to IDR recipients—have multiple contradictory impulses when confronted with the question of consuming or saving. He refers to this as the "Faustian conflict between the passion for accumulation, and the desire for enjoyment" [1, p. 594]. But in his reproduction schemes he resolved this conflict, at least for dept I capitalists, by setting their propensity to save and accumulate (a_i) equal to 0.5.

Using these initial values and computing values for five additional periods using the model described in sections 2.4.1 - 2.4.3 yields the same results that Marx derives in *Capital*. Appendix 1 presents the complete results for the simulation over all six periods (Marx's results are found on pp. 510-513 of volume two of *Capital* [2]; Howard and King (1988) [6] also provide a useful summary table on p. 189, as does Luxemburg (2003)[5, pp. 86-91]. These results demonstrate that the model outlined above accurately represents Marx's approach to extended reproduction, and will also serve as a baseline for comparison of subsequent elaborations of the model.

Enterprises Figure 1 shows the growth of output in dept I, dept II, and overall output over six consecutive periods. Consistent with Marx's expanded reproduction scheme both departments reproduce smoothly over the time period

[4] Solving **17** for $\alpha_{2(t)}$ yields $\alpha_{2(t)} = \frac{(1-c_1-\kappa_1\alpha_1 s_1)W_{1(t)}}{\kappa_2 s_2 W_{2(t)}} - \frac{c_2}{\kappa_2 s_2}$

and overall output grows steadily. The rate of growth in output for both departments and overall converges on 10% after an initial period of adjustment.

Figure 2 shows the rate of capital accumulation (fixed and variable) over these periods. As noted above, investment decisions by the subsumed class recipients of surplus from dept I establish the growth rate of the economy and the dept II recipients respond in such a way as to assure the reproduction of both departments. So while the rate of capital accumulation in dept I remains a constant 10%, capital accumulation in dept II is initially 6.67% in the first period but then converges on the 10% rate established by dept I. Consistent with Marx's assumption of successful reproduction, the output of both departments grows without interruption and capital accumulates at a steady 10% rate after the first period.

Households Figures 3 and 4 show the growth of household consumption over the sequence of periods, both overall and disaggregated by income source. Again, consistent with Marx's assumption of smooth reproduction, household consumption rises consistently over the periods. Also after the first period IDR recipients and wage earner's incomes increases at a constant 10% rate. This convergence and constant rate of growth in household income is a consequence of the treatment of income distribution, investment, and the rate of exploitation in the model. The incomes of both types of households are directly proportional to the rate of growth of output in the two departments. The incomes of the households overall grow at the same rate that the output of the economy grows. Since there is no change in the relative shares of income over the period, and incomes overall are rising at a constant rate of 10%, then the consumption of both groups rises at this rate as well.

3.1.5 Underconsumption?

Given the set-up of this model reproduction would occur without interruption. Given that Marx assumed successful reproduction at the outset this conclusion is not surprising. But the seemingly infinite reproducibility of the reproduction schemes leads Rosa Luxemburg [5, p. 315] to criticize them as an "untiring merry-go-round in thin air ". She argues that since output inevitably grows larger, while the number of capitalists and workers to consume it or industries to invest in does not, an economy like the one described by Marx's reproduction schemes would necessarily be constrained by a tendency towards underconsumption.

Luxemburg's underconsumptionist arguments are premised on a particular reading of Marx that not only fails to link his theory of reproduction to his class theory, but actually rejects class differences within capitalist economic activity as irrelevant:

> Within the limits of Marx's diagram there are in fact only the two sources of income in a society: the labourers' wages and the surplus value. All the strata of the population we have mentioned

> as apart from the capitalists and the workers, are thus to be taken only for joint consumers of these two kinds of income. Marx himself rejects any suggestion that these 'third persons' are more than a subterfuge . . . (Luxemburg, [5, p. 107]).

Reading Marx's reproduction schemes in isolation from his class theory provides a theoretical justification for these arguments.

Luxemburg's [5] criticism is based on four basic assumptions: (1) the demand from IDR recipients for consumption will be inadequate to absorb increases in surplus (p. 104); (2) investment in the two departments of production will be unable to absorb this increased output (p. 104); (3) rising demand from subsumed classes other than IDR recipients will not be able to absorb the increased output (p. 107); (4) domestic population growth of productive laborers and IDR recipients will be inadequate to increase demand sufficiently to absorb the growing output (p. 105).

Individually each of these arguments might make sense, but together they do not. Increased demand from any one of the sources that Luxemburg considers—IDR recipients, productive laborers, unproductive laborers, future population growth, or increases in constant capital—might be insufficient to realize the growing output, but together there is no reason to assume that these various sources could not provide adequate demand for the growing output. Growing demand for output from all of these sources is evident in the next two extensions of the basic model of extended reproduction.

What these models show is that growth in output can be absorbed to some degree by some or all of these sources. There is no justification for strict limits on the rising living standards of surplus recipients, whether they are IDR recipients or unproductive laborers, or for limits on the increase in the number of these subsumed class recipients from population growth. Furthermore Luxemburg ignores the potential depressing effects on savings, capital accumulation, and output growth that results from the distribution of surplus to unproductive laborers who may save less than IDR recipients. She assumes that the distribution of surplus has no effect on the rate of accumulation and growth in the two departments and the economy overall. But the rate of growth in Marx's reproduction schemes depends critically on the savings rate of surplus recipients; any reduction in their distributive share of surplus reduces this growth rate and would ameliorate any potential underconsumptionist tendencies. This effect is discussed at length in the next section of this chapter.

All of the models presented herein assume that wages (both nominal and real) remain constant, therefore all increases in variable capital assumes that new labor is entering the labor market. The source of this additional labor could be increased participation in the capitalist labor force resulting from laborers leaving non-capitalist workplaces (e.g. households), natural population growth, or both.

The subsequent two iterations of Marx's basic reproduction model presented below introduce additional subsumed class positions (i.e. unproductive labor), changes in the distribution of surplus, technological change, and changes in the

rates of exploitation, and then considers the effects of these things on output, capital accumulation, and growth in household consumption. They provide a new and different class analytic way of understanding the basic issues that Luxemburg struggled with. The results presented below do not support her basic conclusion that a closed capitalist economy cannot grow.

3.2 Extended Reproduction with Unproductive Labor or 'Third Persons'

The previous section assumed that the distribution of surplus was limited to IDR recipients. By considering only the recipients of IDR payments it abstracts from the rich set of other class positions that Marx develops through the three volumes of *Capital*. This is not surprising given that the reproduction schemes are presented at the conclusion of volume two, and Marx develops the concepts of subsumed classes most completely in volume three. But this does not mean that these issues should always be considered in isolation from one another, or that, as Luxemburg argues, that the role of class positions other than 'capitalist' and 'worker' are irrelevant to the reproduction process. Maintaining a separation between reproduction and class theories has inhibited the development of Marxian economics and should no longer be maintained. In order to rectify this problem this section introduces an additional set of subsumed class positions that are referred to here collectively as "unproductive labor" to indicate that they derive their income neither from performing productive laborer nor from the ownership of capital (real or financial) or property.

Included in this category would be payments to subsumed class positions inside industrial capitalist enterprises, such as managers who extract labor from productive laborers, employees who conduct research and development for the firm, sales personnel, the firm's internal bureaucracy, etc. Also included would be payments to the occupants of subsumed class position outside of the industrial capitalist enterprises such as wholesale and retail trade, legal advisors, consultants, insurers, certain agents of government, etc. This type of subsumed class position represents a new type of household income.

To accommodate these groups another term should be added to (2):

$$S_i = SC_{io} + SC_{ic} + SC_{il} + SC_{iu}$$

where SC_{iu} is defined as subsumed class payments from the ith department to unproductive laborers internal and external to the industrial capitalist enterprise.

The addition of this term complicates the matter of the accumulation of capital because it raises the question of how the households receiving these types of subsumed class revenues might subsequently expend them. Resnick and Wolff [7, p. 119] argue that industrial capitalists distribute surplus value to subsumed classes in order to secure the conditions of existence of the capitalist fundamental class process. It can similarly be argued that the occupants of subsumed class positions will make expenditures in order to secure the conditions of exis-

tence of their subsumed class revenues. It follows from this that the different types of subsumed class payments should give rise to different saving-investment decisions by their recipients.

Equity holders, moneylenders, and landlords receive subsumed class payments for providing the enterprise with access to capital and property. Therefore they have an incentive to increase their capital and property by saving and accumulating rather than consuming. In this way they seek to preserve and expand their subsumed class positions and the socially-recognized claims to the surplus generated in the capitalist enterprises. Unproductive laborers, on the other hand, receive subsumed class payments for providing labor power or other services to the enterprise, and hence their income stream is not a consequence of the stock of real or financial capital they may or may not possess. This difference implies that the propensity of households to save out of IDR payments will be significantly larger than the propensity of households receiving wages and salaries for performing unproductive labor. Letting $b_i \in [0, 1]$ stand for the propensity for households to save wages and salary income for performing unproductive labor in enterprises, equation (13) should now be revised to account for the addition of unproductive laborers:

$$\alpha_i = \frac{a_i(SC_{io} + SC_{ic} + SC_{il})}{\Sigma SC_i} + \frac{b_i(SC_{iu})}{\Sigma SC_i} = \frac{S_{i\Delta}}{\Sigma SC_i} \tag{18}$$

But in order to avoid unnecessary complication in what follows, let the propensity of households to save from IDR (a_i) remain at one-half, and assume that the savings propensity for households out of wages and salaries for unproductive laborer (b_i) to be zero. In this case (18) reduces to,

$$\alpha_i = \frac{a_i(SC_{io} + SC_{ic} + SC_{il})}{\Sigma SC_i} = \frac{S_{i\Delta}}{\Sigma SC_i} \tag{19}$$

But (19) is no longer equivalent to (13). The reasoning here is simple. Previously the distribution of surplus was limited to the payment of IDR, in which case,

$$\frac{(SC_{io} + SC_{ic} + SC_{il})}{\Sigma SC_i} = 1 \Rightarrow \alpha_i = \frac{a_i(SC_{io} + SC_{ic} + SC_{il})}{\Sigma SC_i} = a_i$$

Formerly the rate of accumulation of capital was equal to the savings propensity of the owners of enterprise, moneylenders, and landlords. Now that the payments of IDR no longer exhaust the subsumed class payments, this is no longer true. Since unproductive laborers are assumed not to save, increased distributions of surplus to these subsumed classes will reduce the rate of capital accumulation. The rate of capital accumulation now depends on (i) the savings propensity (a_i) of the owners of industrial capital and property, and (ii) the distribution of surplus between the occupants of these subsumed class positions and the unproductive laborers inside and outside the firm who also derive their income from distributed shares of surplus value in the firm. Letting the share of surplus distributed as IDR be designated as $\gamma \in [0, 1)$, the rate of capital

accumulation (α_i) can be written as,

$$\alpha_i = a_i \frac{(SC_{io} + SC_{ic} + SC_{il})}{\Sigma SC_i} = a_i \gamma_i \tag{20}$$

In this case the rate of capital accumulation responds positively to both the savings propensity and the distributive share of capital and property holders, and negatively to any increase in the incomes of unproductive subsumed class positions. Denoting the share of surplus distributed to unproductive subsumed class positions as υ , it is defined as,

$$\upsilon = 1 - \gamma$$

With the inclusion of unproductive laborers the outcome of the reproduction process is affected by this distribution of surplus between households that receive IDR and who save and invest a portion of their income, and households that receive wages and salary income for providing unproductive labor and who are assumed not to save. An increase in the incomes of the non-savers will reduce the rate of accumulation and thereby the aggregate rate of growth. This effect is obvious from the construction of the model and it is probably redundant to simulate these effects and present them here. What is, however, a more interesting question is how the distribution between these two types of subsumed classes might change over time, say over the course of a business cycle.

This iteration of the model is also motivated by the analysis of capital accumulation at the firm level done by Resnick and Wolff [7, pp. 184-191]. They propose that one effect of capital accumulation is the "industrial expansion effect", and find that distributions to IDR and unproductive activities reduce the enterprises rate of capital accumulation and reduces the industrial expansion effect. The introduction of unproductive labor into the aggregate class structure enables me to consider whether increased distributions to unproductive laborers also reduce expansion at the aggregate level as well. This aggregate class structure also pushes back the boundary of the analysis and allows me to consider the potential effects of this on households as well.

Consider, for example, a situation in which the distribution of surplus becomes progressively more favorable for the occupants of unproductive subsumed class positions relative to IDR recipients. Such a situation might arise if capital accumulation occurs more rapidly than the growth in the laborers with the skills, training and education necessary to perform unproductive labor (college graduates, for example), increasing the bargaining power of individuals with these skills and allowing them to command higher salaries. Widespread co-optation of corporate boards by corporate management in order to increase compensation packages of management, such as was observed in the United States during the 1990's, would have a similar effect. This situation might also arise if additional surplus must be progressively allocated by industrial capitalists to unproductive activities such as new product development, advertising, marketing, wholesale and retail trade, etc. in order to successfully sell their product to consumers, or if firms must increase their expenditures on corporate advisors for such things

as strategic planning, legal counsel, financial advising, and risk management assessment. These types of changes would likely take longer to develop than would the cyclical labor market effects, but cumulatively they could be dramatic. Shaikh and Tonak [8, Table F.1], for example, estimate that the share of the U.S. labor force that performed unproductive labor rose from 43% to 64% over the 1948-1989 time period. They also found that unproductive labor accounted for 85% of all new employment over the period. In the absence of an increase in the rate of exploitation, or if demand by unproductive laborers for surplus value outpaced any increases resulting in the rate of exploitation, the increased demands would come at the expense of IDR payments and would result in a 'profit squeeze' if profit is interpreted as the return on capital (gross revenues minus operating costs).

Formally such a change in distributive shares could be described as,

$$\gamma_{i(t)} = 1 - (1 + d)\upsilon_{i(t-1)} \tag{21}$$

where $d \in [0, 1]$ is an exogenously specified rate of increase in the share of surplus going to unproductive subsumed class positions.

Table 5 presents the model of expanded reproduction with unproductive labor in SAM form. In order to limit this SAM to one page the cells that are unchanged from Table 4 are not shown. Also some of the obvious time subscripts have been suppressed. The most significant changes from the earlier SAM are that (i) unproductive laborers have been included, and (2) the distributional parameters $\gamma_{(t)}$ and $\upsilon_{(t)}$ have been added to capture the changing distribution between the two types of subsumed class positions. The rate at which such change might occur is a complex result of economic, political, cultural, and natural (demographic) factors. It is, however, enlightening to consider what results might emerge if this rate is exogenously set at some arbitrary value. The simulation performed for this model assumes that $d = .25$. Appendix 2 summarizes the results of this simulation. Period 0 assumes the same initial conditions and the same rate of savings from IDR as Marx's original reproduction model. Note that the rate of surplus value and the value rate of profit are unchanged through all six periods, but the share of surplus value accruing to IDR recipients decreases as the share distributed to unproductive laborers progressively rises through the subsequent five periods.

3.2.1 Results of the Model with Unproductive Labor - Unproductive 'Profit Squeeze'

Enterprises Figure 5 shows the consequence of the changing distribution of surplus on the level of output produced in the model of expanded reproduction. Over six periods the effect of the addition of unproductive labor on output is observable but not dramatic. At the end of this time the value of total output is 11.4% lower than the under the basic reproduction model. But as Figures 6 and 7 show the results of the inclusion of these subsumed class positions on the rate of output growth is dramatic.

In contrast to Marx's reproduction model, which converged to a constant 10% rate of growth, the rate of capital accumulation in both departments now falls throughout all of the periods in the sequence, and the rate in dept II exceeds dept I throughout. The downward trend in the rate of capital accumulation is a consequence of the 'profit squeeze' resulting from the shifting distribution of surplus from IDR recipients, who accumulate capital, and other types of subsumed class recipients who do not. In the initial period department II must accumulate enough to absorb the excess means of production produced by department I in the first period. This results in the initial spread between rates of accumulation in the two departments. The two rates do not converge over the remaining five periods because as more surplus is consumed rather than accumulated dept II must expand faster than dept I to accommodate the increasing demand for consumption goods.

Figure 7 compares the rates of accumulation between Marx's baseline reproduction model and the model with unproductive activities and an unproductive 'profit squeeze'. The two rates are initially similar (8.97% vs. 9.31%) but then consistently diverge. If the sequence of periods was extended the rate of capital accumulation would eventually reach zero, and the accumulation of capital would cease.

These findings confirm at the aggregate level the results found by Resnick and Wolff at the enterprise level. This is true even though the financing mechanism for investment is the extreme opposite case of what Resnick and Wolff assume; they consider capital accumulation through retained earnings, and I assume that capital accumulation is financed through savings. In both cases the result is the same: increased distributions of surplus for unproductive activities reduce the rate of expansion of industrial capitalist enterprises.

Households Figures 8, 9, and 10 show the impacts of this unproductive 'profit squeeze' on household consumption. Overall household consumption rises throughout the time period, but these increases in consumption are unevenly distributed. Households that supply productive and unproductive labor see their incomes and consumption rise consistently, with unproductive laborers incomes rising more rapidly. The rate of growth of income and consumption for households providing productive labor to the enterprises is also below the rate of growth in their incomes under the basic model of extended reproduction. This can be attributed to the lower overall rate of growth of the economy which depresses the overall rate of income growth and expenditure (as shown in Figure 10). As is to be expected given the parameters of this simulation, the income and expenditure for subsumed class households receiving IDR income falls throughout the time period. Both their absolute level of consumption falls and their rate of income and consumption growth are negative throughout the time period.

3.3 Extended Reproduction with Technological Change

The third and final iteration of this model introduces the issue of technological change in the form of changes to the capital and labor input coefficients. Until this point these have been assumed to be constant. But industrial capitalist enterprises might also distribute surplus specifically in order to bring about reduction in these input requirements. Industrial capitalists support such development through subsumed class payments for research and development to industrial engineers, scientists, *et hoc genus omne*, and to the array of people necessary to support their functions. But this is not the only way that such reductions might occur. A reduction in the cost of constant capital inputs might result, for example, from the relaxation of environmental restrictions, from trade agreements or policies, or from less diplomatic methods such as military interventions. State functionaries, intellectuals, and military personnel may all be employed directly or indirectly by an industrial capitalist to bring about these results and would be remunerated with distributions of surplus. In these ways, as well as innumerable others, the distribution of surplus to the occupants of subsumed class positions may increase the proportion of surplus value per unit output by cheapening input commodities.

Labor saving or capital saving technological change allows a quantity of output to be produced with fewer inputs of raw materials, intermediate commodities, or labor. If the wage rate remains constant then increases in productivity will increase the rate of exploitation of productive laborers.[5] The relationship between the rate of exploitation ϵ and the input coefficients can be derived from the basic equation for the value of a commodity (1). This yields:

$$\epsilon = \frac{1}{v} - \frac{c}{v} - 1$$

Introducing technological change changes the input coefficients from parameters to variables. The rate and direction of technical change is overdetermined by innumerable factors. But in this simulation model the possible consequences of a particular pattern of technological change can be considered. This is one potential course of technological change used as an exemplar here, not an argument that this particular pattern of technological change will be observed from among the many that could be observed in any specific situation. In order to do this I make the constant capital input coefficient variable in the following

[5] **Proof.** The rate of exploitation can be derived from the definition of the value of a commodity: $C + V + S = W \Rightarrow \epsilon = \frac{S}{V} = \frac{W}{V} - \frac{C}{V} - 1$ or after nornalizing W, $\epsilon = \frac{1}{V} - \frac{C}{V} - 1$. Total differentiation yields, $d\epsilon = -\frac{1}{v^2}dv + \frac{c}{v^2}dv - \frac{1}{v}dc$. If both v and c change proportionately then $dv = dc$ and, $d\epsilon = -\frac{1}{v^2}dv + \frac{c}{v^2}dv - \frac{1}{v}dv = \left(\frac{c-v-1}{v^2}\right)dv$. since c, $v \in (0,1)$, $c - v - 1 < 0$, $v^2 > 0$, and $\left(\frac{c-v-1}{v^2}\right) < 0$. So in cases where $dc, dv < 0$ (i.e. increases in productivity) the rate of exploitation ϵ will rise ($d\epsilon > 0$). If technical change is only labor saving (i.e. $dc = 0$), then $d\epsilon = (-\frac{1}{v^2} + \frac{c}{v^2})dv$. Since $\frac{1}{v^2} > \frac{c}{v^2} \Rightarrow (-\frac{1}{v^2} + \frac{c}{v^2}) < 0$. Therefore $\frac{d\epsilon}{dv} > 0 \Leftrightarrow dv < 0$. and labor saving technological change will increase the rate of exploitation. If technical change only reduces the imputs of constant capital (i.e. $dv = 0$) then $d\epsilon = -\frac{1}{v}dc$ and $\frac{d\epsilon}{dc} > 0 \Longleftrightarrow dc < 0$. Therefore when c is falling the rate of exploitation is rising. ■

form,

$$c_{i(t)} = (1 - \delta)c_{(t-1)} \tag{22}$$

where $\delta \in [0, 1]$ is an exogenously specified rate of change. Higher values of δ correlate to greater success by the occupants of various subsumed class positions in reducing the constant capital input requirements of the enterprises, either through changes to the technique of production or by cheapening the input commodities.

Similarly the labor input requirements per unit output might be reduced by the actions of the occupants of subsumed class positions. The direct managers and supervisors of laborers have the responsibility to extract labor from the labor power purchased by the enterprise. Increasing managerial supervision by increasing the distribution of surplus to employ more managers could increase work intensity by productive laborers and thereby reduce the quantity of labor input per unit output. Human resources departments in enterprises also contribute to reducing the cost of labor input by preventing increases in labor compensation in times of rising physical productivity of labor, seeking less costly ways to provide non-wage (medical, retirement, etc.) compensation for laborers, and changing policies to effectively increase the amount of surplus labor performed. The research and development activities of the firm would also be directed to seek changes in the methods of production that could reduce the quantity of labor input per unit output through labor-saving changes in technique, or perhaps by developing ways to increase the effectiveness of monitoring of work effort. Through these and other methods the industrial capitalist could reduce the labor input per unit output. As with the capital input coefficient, it is not theoretically tractable to simply make these changes a direct function of the quantity of subsumed class payments. Instead the labor input coefficient can be made variable by specifying some exogenous rate of change,

$$v_{i(t)} = (1 - \delta)v_{(t-1)} \tag{23}$$

The rate of change δ given here is the same as the rate of the capital input coefficient. It is not necessary for this to be so, and indeed it is unlikely that this would occur, but for simplicity of exposition and computation I have made them equal here.

Because the coefficients c, v, and s sum to unity any decrease in c or v result in an increase in s (the reverse is also true):

$$s_{i(t)} = 1 - c_{i(t)} - v_{i(t)} \tag{24}$$

Table 6 presents the model of extended reproduction with unproductive labor and technological change. There are three primary differences between this simulation and the previous one (the unproductive 'profit squeeze'). First, while this simulation includes productive laborers in the aggregate class structure, it does not assume that the share of surplus value that is distributed to these households rises during the period of the simulations. Therefore the distributional parameters γ and υ are constant and do not have time subscripts attached

to them. The distribution between IDR recipients and unproductive laborers remains constant at .75 for IDR and .25 for unproductive labor (see Appendix 3). Second, it is assumed here that technological change reduces both constant and variable capital inputs at a rate of 3% per period (i.e. $\delta = .03$). Therefore the coefficients $c_{(t)}$ and the $s_{(t)}$ have time subscripts to indicate that they are changing over time ($c_{(t)}$ does not appear in the model). Third, these technological improvements increase the rate of exploitation throughout the simulation, and so $\epsilon_{(t)}$ also has a time subscript. Because the organic composition of capital is different in each department the rate of exploitation does not rise uniformly in both sectors. In Marx's initial scheme for expanded reproduction the producer's goods department (dept I) has a higher organic composition of capital. Consequently technological change raises the rate of exploitation more quickly in this department than in the consumer's goods department (dept II).

3.3.1 Results of the Model of Extended Reproduction with Unproductive Labor and Technological Change

Enterprises Figure 11 presents the growth in output and compares it with the output projected under the first two simulations. Since the organic composition of capital remains constant throughout the effect of increased productivity takes the form of greater output per unit of capital and labor rather than simply an increased stock of constant capital. As expected productivity enhancing technological change results in more rapid increase in total production when compared with either Marx's original reproduction schemes (the baseline) or the unproductive "profit squeeze" model. At the end of the period total output is 32% higher than under the basic model of extended reproduction.

Capital accumulation, as shown in Figure 12, is positive throughout the simulation, and the rate of accumulation in dept two is higher than the rate of accumulation in dept one. The difference in the rates of growth between the two departments is because the technological change is reducing the input requirements from both departments for means of production relative to total output. Since both departments require fewer inputs per unit output, the output of final goods by department two will grow faster then department one.

Figure 13 compares the rates of capital accumulation in the three cases considered in this chapter. What this figure makes clear is that the most notable difference between the three cases is the completely different growth paths that emerge from the three different assumptions about the composition of the aggregate class structure and the effects of unproductive subsumed classes. Marx ignores the effects of such subsumed classes entirely in his reproduction schemes, focusing instead on establishing the simplest possible circular-flow model of production, distribution, consumption and accumulation. With this he presented the first consistent picture of a dynamic economy structured by a capitalist fundamental class process, resolved Smith's "blunder", and generalized the Physiocratic insight into the circulation of surplus that was constrained by their feudal imaginary. Marx assumes successful reproduction in order to demonstrate his solution to these problems.

But what Figure 13 shows is that it is a mistake to conclude that Marx's reproduction schemes are an argument for the progressive self-development of the forces of production in a capitalist economy. Integrating Marx's class theory with his theory of reproduction allows for markedly different conclusions. In these models the difference between constant, progressively increasing, or progressively decreasing rates of growth is the different and contradictory effects of distribution to unproductive laborers (υ) and technological change (δ). But these effects are simply shorthand ways of isolating the potential consequences of complex economic, political, cultural and natural processes on the process of economic growth. Through these multiple factors both distribution and technological change, and therefore growth and development, are overdetermined by the entire social complex. But what these models demonstrate is that Marxian class theory can be used to develop credible arguments about these potential consequences, but only if the effectivity of class processes is not ruled out at the beginning of the analysis.

Households Figure 14 shows the growth in household consumption over this simulation. Overall consumption rises significantly over the period, from 3,000 to 7,547, a 152% gain. This growth in consumption far exceeds the growth in output overall, which only rises by 111% (from 9,000 to 18,981). Consumption growth can outstrip output growth in this case because technological change reduces the inputs per unit output, or, conversely, because more final commodities can be produced per unit input. Not only does overall household consumption increase significantly over the simulation, but it increases much more rapidly than under either the basic or profit squeeze models (see Figure 16). But while households overall benefit significantly from this technological change, these benefits are not distributed equally.

Figure 15 shows the relative rates of consumption growth by the various fundamental and subsumed classes. The households with the highest rates of consumption growth are IDR recipients—the 'capitalists' of classical political economy. But their gains only slightly exceed the gains by unproductive laborers, and the two growth rates converge over the course of the simulations. This contrasts with both the basic model, in which IDR and productive laborers consumption grew at the same rate, and the profit squeeze case, in which the all groups saw their rates of consumption growth decline throughout the simulation but IDR recipients consumption declined much more rapidly than the other two types of households. Clearly in this simulation the households that receive subsumed class payments fare significantly better than they did under either of the alternative scenarios.

The reason that the income and consumption of the households that subsist on subsumed class payments rises throughout the simulation period is because rising productivity and a constant wage rate leads to rising rates of exploitation. Technological change raises the rate of exploitation, and thereby increases the quantity of surplus available to support the households that receive subsumed class payments. Similarly the demand for productive labor per unit output is

falling, and so while the quantity of productive labor inputs rises as output increases, and hence so is the total consumption of productive laborers, it does not grow as rapidly as the output of surplus value. Consequently the receivers and consumers of surplus value see their incomes progressively rising more rapidly than productive laborers.

4 Concluding Comments on Class, Reproduction, and Growth

Marx's original schemes of extended reproduction offer a vision of consistent and steady accumulation and output growth. But to get this result he had to pose the question of reproduction very narrowly. This paper opens up the theory of reproduction by integrating it with Marx's class theory. Introducing multiple different competing class positions, technological change, and changes to the rate of exploitation leads to two alternative visions of growth and development that are significantly different from Marx's original model, as well as from one another. This paper presents two diametrically different examples in order to emphasize the potential variety of effects of class relations on the distribution of surplus on the growth and development of an aggregate social capital. Technological innovation produced by subsumed class recipients of surplus provides the possibility of progressively increasing rates of growth, while the introduction of subsumed class positions such as those needed simply to manage some of the many contradictions in a modern capitalist economy presents the possibility of a 'profit squeeze' by unproductive laborers leading to the progressive decay in the rate of growth. But there is no justification for arguing that these two different effects should exist in isolation from one another. Consequently there is no justification for concluding that an economy in which capitalist production is prevalent has a tendency to grow, decline, or to manifest oscillations between both potential outcomes. There can be no a priori rationale to choose one of these potential outcomes over the other. Indeed in a diverse modern economy it is likely that both tendencies could coexist, with the observed growth path being a complex and historically-specific consequence of the interaction between the various forces involved.

Marx's class theory is clearly there for the reader who looks for it, and the consequences are profound. This paper elaborates some of them.

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Appendix A.1 Simulation Results: Basic Model of Extended Reproduction

						Uses of Surplus			Rate of	Rate of		Value		Rate	Rate of Growth		
						Cons	Accum		Capital	Ouptut		Rate of	% of SV	of	Consumption		
Period	Dept.	C	V	S	W	IDR	ΔC	ΔV	Accum	Growth	κ	Profit	Invested	SV	V + ΔV	IDR	All
0	I	4,000	1,000	1,000	6,000	500	400	100	10.00%		0.80	0.20	0.5	1			
	II	1,500	750	750	3,000	600	100	50	6.67%		0.67	0.33	0.2	1			
	Σ	5,500	1,750	1,750	9,000	1100	500	150	8.97%		0.76	0.24	0.37	1			
1	I	4,400	1,100	1,100	6,600	550	440	110	10.00%	10.00%	0.80	0.20	0.5	1	10.00%	10.00%	10.00%
	II	1,600	800	800	3,200	560	160	80	10.00%	6.67%	0.67	0.33	0.3	1	10.00%	-6.67%	2.86%
	Σ	6,000	1,900	1,900	9,800	1110	600	190	10.00%	8.89%	0.76	0.24	0.42	1	10.00%	0.91%	6.67%
2	I	4,840	1,210	1,210	7,260	605	484	121	10.00%	10.00%	0.80	0.20	0.5	1	10.00%	10.00%	10.00%
	II	1,760	880	880	3,520	616	176	88	10.00%	10.00%	0.67	0.33	0.3	1	10.00%	10.00%	10.00%
	Σ	6,600	2,090	2,090	10,780	1221	660	209	10.00%	10.00%	0.76	0.24	0.42	1	10.00%	10.00%	10.00%
3	I	5,324	1,331	1,331	7,986	666	532	133	10.00%	10.00%	0.80	0.20	0.5	1	10.00%	10.00%	10.00%
	II	1,936	968	968	3,872	678	194	97	10.00%	10.00%	0.67	0.33	0.3	1	10.00%	10.00%	10.00%
	Σ	7,260	2,299	2,299	11,858	1343	726	230	10.00%	10.00%	0.76	0.24	0.42	1	10.00%	10.00%	10.00%
4	I	5,856	1,464	1,464	8,785	732	586	146	10.00%	10.00%	0.80	0.20	0.5	1	10.00%	10.00%	10.00%
	II	2,130	1,065	1,065	4,259	745	213	106	10.00%	10.00%	0.67	0.33	0.3	1	10.00%	10.00%	10.00%
	Σ	7,986	2,529	2,529	13,044	1477	799	253	10.00%	10.00%	0.76	0.24	0.42	1	10.00%	10.00%	10.00%
5	I	6,442	1,611	1,611	9,663	805	644	161	10.00%	10.00%	0.80	0.20	0.5	1	10.00%	10.00%	10.00%
	II	2,343	1,171	1,171	4,685	820	234	117	10.00%	10.00%	0.67	0.33	0.3	1	10.00%	10.00%	10.00%
	Σ	8,785	2,782	2,782	14,348	1625	878	278	10.00%	10.00%	0.76	0.24	0.42	1	10.00%	10.00%	10.00%

Constant Coefficients

	c	*v*	*s*	*q*	*κ*	α
I	0.67	0.17	0.17	4	0.8	0.5
II	0.50	0.25	0.25	2	0.67	Varies

Appendix A.2 Simulation Results: Extended Reproduction With Unproductive Labor and "Profit Squeeze"

							Uses of Surplus												
							Cons		Accum		Rate of	Rate of		Value		Rate		Rate of Growth	
											Capital	Output		Rate of	% of SV	of		Consumption	
Period	Dept.	C	V	S	W	IDR	UL	ΔC	ΔV	Accum	Growth	κ	Profit	Invested	SV	V + ΔV	IDR	UL	All
0	I	4,000	1,000	1,000	6,000	375	250	300	75	7.50%		0.80	0.20	0.38	1				
	II	1,500	750	750	3,000	263	188	200	100	13.33%		0.67	0.33	0.40	1				
	Σ	5,500	1,750	1,750	9,000	638	438	500	175	9.31%		0.76	0.24	0.39	1				
1	I	4,300	1,075	1,075	6,450	370	336	296	74	6.88%	7.50%	0.80	0.20	0.34	1	6.88%	-1.46%	34.38%	9.08%
	II	1,700	850	850	3,400	353	266	154	77	9.08%	13.33%	0.67	0.33	0.27	1	9.08%	34.40%	41.67%	18.89%
	Σ	6,000	1,925	1,925	9,850	722	602	450	151	7.58%	9.44%	0.76	0.24	0.31	1	7.85%	13.31%	37.50%	13.33%
2	I	4,596	1,149	1,149	6,893	350	449	280	70	6.09%	6.88%	0.80	0.20	0.30	1	6.09%	-5.27%	33.59%	8.81%
	II	1,854	927	927	3,709	320	362	163	82	8.81%	9.08%	0.67	0.33	0.26	1	8.81%	-9.32%	36.35%	9.40%
	Σ	6,450	2,076	2,076	10,602	670	811	443	152	6.98%	7.64%	0.76	0.24	0.29	1	7.31%	-7.25%	34.81%	9.08%
3	I	4,876	1,219	1,219	7,314	312	595	249	62	5.12%	6.09%	0.80	0.20	0.26	1	5.12%	-10.91%	32.62%	8.45%
	II	2,018	1,009	1,009	4,036	260	493	171	85	8.45%	8.81%	0.67	0.33	0.25	1	8.45%	-18.60%	36.01%	9.24%
	Σ	6,893	2,228	2,228	11,349	572	1088	420	148	6.22%	7.04%	0.76	0.24	0.25	1	6.63%	-14.58%	34.13%	8.81%
4	I	5,125	1,281	1,281	7,688	250	782	200	50	3.90%	5.12%	0.80	0.20	0.19	1	3.90%	-19.96%	31.40%	7.98%
	II	2,188	1,094	1,094	4,377	165	668	175	87	7.98%	8.45%	0.67	0.33	0.24	1	7.98%	-36.82%	35.57%	9.02%
	Σ	7,314	2,375	2,375	12,064	414	1450	374	137	5.28%	6.30%	0.75	0.25	0.22	1	5.78%	-27.63%	33.29%	8.45%
5	I	5,325	1,331	1,331	7,987	158	1016	126	32	2.37%	3.90%	0.80	0.20	0.12	1	2.37%	-36.79%	29.87%	7.34%
	II	2,363	1,181	1,181	4,726	20	901	173	87	7.34%	7.98%	0.67	0.33	0.22	1	7.34%	-87.79%	34.97%	8.73%
	Σ	7,688	2,513	2,513	12,713	178	1917	300	118	4.10%	5.38%	0.75	0.25	0.17	1	4.70%	-57.05%	32.22%	7.98%

Constant Coefficients

	c	*v*	*s*	*q*	*κ*
I	0.67	0.17	0.17	4	0.8
II	0.50	0.25	0.25	2	0.67

Distributive Shares of Surplus (% of Surplus)

	0		1		2		3		4		5	
	γ	υ	γ	υ	γ	υ	γ	υ	γ	υ	γ	υ
I	0.75	0.25	0.69	0.31	0.61	0.39	0.51	0.49	0.39	0.61	0.24	0.76
II	0.75	0.25	0.69	0.31	0.61	0.39	0.51	0.49	0.39	0.61	0.24	0.76

Savings Rates

	0	1	2	3	4	5
I	0.50	0.50	0.50	0.50	0.50	0.50
II	0.50	0.50	0.50	0.50	0.50	0.50

Rates of Accumulation

	0	1	2	3	4	5
I	0.38	0.34	0.30	0.26	0.19	0.12
II	Varies	Varies	Varies	Varies	Varies	Varies

Appendix A.3 Simulation Results:Extended Reproduction With Unproductive Labor and Technological Change

						Uses of Surplus				Rate of	Rate of		Value	%	Rate	Rate of Growth			
						Cons		Accum		Capital	Output		Rate of	of SV	of	Consumption			
Period	Dept.	C	V	S	W	IDR	UL	ΔC	ΔV	Accum	Growth	κ	Profit	Invested	SV	V + ΔV	IDR	UL	All
0	I	4,000	1,000	1,000	6,000	375	250	300	75	7.50%		0.80	0.20	0.38	1.00				
	II	1,500	750	750	3,000	263	188	200	100	13.33%		0.67	0.33	0.40	1.00				
	Σ	5,500	1,750	1,750	9,000	638	438	500	175	9.31%		0.76	0.24	0.39	1.00				
1	I	4,300	1,075	1,274	6,649	478	319	382	96	8.89%	10.82%	0.80	0.24	0.38	1.19	8.89%	27.45%	27.45%	15.71%
	II	1,700	850	955	3,505	316	239	267	134	15.71%	16.84%	0.67	0.37	0.42	1.12	15.71%	20.25%	27.35%	18.31%
	Σ	6,000	1,925	2,230	10,155	794	557	649	229	11.09%	12.83%	0.76	0.28	0.39	1.16	11.90%	24.48%	27.41%	16.84%
2	I	4,682	1,171	1,612	7,465	604	403	484	121	10.33%	12.26%	0.80	0.28	0.38	1.38	10.33%	26.46%	26.46%	16.86%
	II	1,967	984	1,231	4,181	426	308	332	166	16.86%	19.29%	0.67	0.42	0.40	1.25	16.86%	34.80%	28.85%	22.40%
	Σ	6,649	2,154	2,842	11,646	1,030	711	815	287	12.52%	14.69%	0.76	0.32	0.39	1.32	13.31%	29.78%	27.48%	19.29%
3	I	5,166	1,291	2,033	8,490	762	508	610	152	11.81%	13.74%	0.80	0.31	0.38	1.57	11.81%	26.13%	26.13%	18.08%
	II	2,299	1,149	1,589	5,038	568	397	416	208	18.08%	20.47%	0.67	0.46	0.39	1.38	18.08%	33.60%	29.14%	23.40%
	Σ	7,465	2,441	3,622	13,528	1,331	906	1026	360	13.99%	16.16%	0.75	0.37	0.38	1.48	14.76%	29.21%	27.43%	20.47%
4	I	5,776	1,444	2,566	9,786	962	642	770	192	13.33%	15.26%	0.80	0.36	0.38	1.78	13.33%	26.25%	26.25%	19.38%
	II	2,714	1,357	2,061	6,132	757	515	526	263	19.38%	21.73%	0.67	0.51	0.38	1.52	19.38%	33.09%	29.66%	24.49%
	Σ	8,490	2,801	4,627	15,919	1,719	1157	1296	455	15.51%	17.67%	0.75	0.41	0.38	1.65	16.26%	29.17%	27.74%	21.73%
5	I	6,546	1,636	3,252	11,434	1,219	813	975	244	14.90%	16.84%	0.80	0.40	0.38	1.99	14.90%	26.70%	26.70%	20.74%
	II	3,240	1,620	2,686	7,547	1,007	672	672	336	20.74%	23.07%	0.67	0.55	0.38	1.66	20.74%	33.06%	30.36%	25.68%
	Σ	9,786	3,257	5,938	18,981	2,226	1484	1648	580	17.08%	19.24%	0.75	0.46	0.38	1.82	17.81%	29.50%	28.33%	23.07%

Constant Coefficients

	q	*κ*	*a*	*α*
I	4	0.8	0.5	0.38
II	2	0.67	Varies	Varies

Distributive Shares of Surplus

	γ	υ
I	0.75	0.25
II	0.75	0.25

Rate of Change (δ) = 0.03

Variable Coefficients

Period	Dept.	*c*	*v*	*s*	ε
0	I	0.667	0.167	0.1667	1.00
	II	0.500	0.250	0.2500	1.00
1	I	0.647	0.162	0.1917	1.19
	II	0.485	0.243	0.2725	1.12
2	I	0.627	0.157	0.2159	1.38
	II	0.470	0.235	0.2943	1.25
3	I	0.608	0.152	0.2394	1.57
	II	0.456	0.228	0.3155	1.38
4	I	0.590	0.148	0.2623	1.78
	II	0.443	0.221	0.3360	1.52
5	I	0.572	0.143	0.2844	1.99
	II	0.429	0.215	0.3559	1.66

Table 1: A Marxian Class Analytic Accounting Matrix (Closed Economy)

		PRODUCTION	CURRENT		ACCUMULATION	Σ
		ENTERPRISE	ENTERPRISE	HOUSEHOLD	ENTERPRISE	
PRODUCTION	ENTERPRISE	Inter-Industry Transactions	0	Consumer Goods Consumption	Capital Goods Accumulation	Total Production Output
CURRENT	ENTERPRISE	Surplus Value	-	0	0	Enterprise Current Income
	HOUSEHOLD	Wages of Productive Laborers	Interest, Dividends, Rents	-	Variable Capital Accumulation	Household Current Income
ACCUMULATION	ENTERPRISE	0	0	Household Saving	-	Sources of Accumulation Funds
Σ		Total Production Outlay	Enterprise Current Expenditure	Household Current Expenditure	Uses of Accumulation Funds	

Table 2: Marx's Simple Reproduction Scheme in Class Analytic Form

			PRODUCTION		CURRENT						Σ
			ENTERPRISE		ENTERPRISE		HOUSEHOLDS				
			Dpt. I	Dpt.II	Industrial Cap. (I)	Industrial Cap. (II)	Workers	Entrep-reneurs	Monied Men	Landlords	
PRODUCTION	ENT.	Department I	C_1	C_2							ΣC_i
		Department II					ΣV_i	ΣSC_{io}	ΣSC_{ic}	ΣSC_{il}	$\Sigma V_i + \Sigma SC_{ij}$
CURRENT	ENT.	Ind. Capitalists (I)	S_1								S_1
		Ind. Capitalists (II)		S_2							S_2
	HOUSEHOLD	Workers	V_1	V_2							ΣV_i
		Entrepreneurs			SC_{1o}	SC_{2o}					ΣSC_{io}
		Monied Men			SC_{1c}	SC_{2c}					ΣSC_{ic}
		Landlords			SC_{1l}	SC_{2l}					ΣSC_{il}
Σ			W_1	W_2	S_1	S_2	ΣV_i	ΣSC_{io}	ΣSC_{ic}	ΣSC_{il}	

Table 3: Marx's Extended Reproduction Scheme in Class Analytic Form

			PRODUCTION		CURRENT				ACCUMULATION		Σ
			ENTERPRISE		ENTERPRISE		HOUSEHOLDS		ENTERPRISE		
			Dept. I	Dept.II	Industrial Cap. (I)	Industrial Cap. (II)	Workers	IDR	Dept. I	Dept.II	
PRODUCTION	ENTERPRISE	Dept. I	C_1	C_2					$S_{1\Delta C}$	$S_{2\Delta C}$	$\Sigma C_i + \Sigma S_{i\Delta C}$
		Dept. II					$\Sigma V_i + \Sigma S_{i\Delta V}$	$\Sigma SC_i - \Sigma S_{i\Delta}$			$\Sigma V_i + \Sigma S_{i\Delta V} + \Sigma SC_i - \Sigma S_{i\Delta C}$
CURRENT	ENTERPRISE	Ind. Cap. (I)	S_1								S_1
	ENTERPRISE	Ind. Cap. (II)		S_2							S_2
	HOUSEHOLD	Workers	V_1	V_2					$S_{1\Delta V}$	$S_{2\Delta V}$	$\Sigma V_i + \Sigma S_{i\Delta V}$
	HOUSEHOLD	IDR			ΣSC_1	ΣSC_2					ΣSC_i
ACCUMULATION	ENTERPRISE	Dept. I						$S_{1\Delta}$			$S_{1\Delta}$
	ENTERPRISE	Dept. II						$S_{2\Delta}$			$S_{2\Delta}$
Σ			W_1	W_2	S_1	S_2	$\Sigma V_i + \Sigma S_{i\Delta V}$	ΣSC_i	$S_{1\Delta C} + S_{1\Delta V}$	$S_{2\Delta C} + S_{2\Delta V}$	

Table 4 Basic Model of Extended Reproduction in SAM Form

			PRODUCTION		CURRENT				ACCUMULATION	
			ENTERPRISE		ENTERPRISE		HOUSEHOLDS		ENTERPRISE	
			Dept. I	Dept.II	Cap. (I)	Cap. (II)	Workers	IDR	Dept. I	Dept.II
PRODUCTION	ENTERPRISE	DI	$C_{I(t-1)} + S_{I\Delta C(t-1)}$	$C_{II(t-1)} + S_{II\Delta C(t-1)}$					$\kappa_1\alpha_1 s_1 W_{1(t)}$	$(1-c_1-\kappa_1\alpha_1 s_1)W_{1(t)} - c_2 W_{2(t)}$
		DII					$V_{I(t)} + V_{II(t)} + \kappa_1\alpha_1 s_1 W_{1(t)}\frac{1}{q_1} + \left[(1-c_1-\kappa_1\alpha_1 s_1)W_{1(t)} - c_2 W_{2(t)}\right]\frac{1}{q_2}$	$\varepsilon V_{I(t)} + \varepsilon V_{II(t)} - \alpha_1 s_1 W_{1(t)} - \alpha_{2(t)} s_2 W_{2(t)}$		
CURRENT	ENTERPRISE	C(I)	$\varepsilon V_{I(t)}$							
		C(II)		$\varepsilon V_{II(t)}$						
	HOUSEHOLD	W	$V_{I(t-1)} + S_{I\Delta V(t-1)}$	$V_{II(t-1)} + S_{II\Delta V(t-1)}$					$\kappa_1\alpha_1 s_1 W_{1(t)}\frac{1}{q_1}$	$\left[(1-c_1-\kappa_1\alpha_1 s_1)W_{1(t)} - c_2 W_{2(t)}\right]\frac{1}{q_2}$
		IDR			$\varepsilon V_{I(t)}$	$\varepsilon V_{II(t)}$				
ACCUMULATION	ENTERPRISE	DI						$\alpha_1 s_1 W_{1(t)}$		
		DII						$\alpha_{2(t)} s_2 W_{2(t)}$		

Table 5: Model of Extended Reproduction With Unproductive Labor and 'Profit Squeeze' in SAM Form

			CURRENT					ACCUMULATION	
			ENTERPRISE		HOUSEHOLDS			ENTERPRISE	
			Cap. (I)	Cap. (II)	Productive Laborers	IDR	Unproductive Labor	Dept. I	Dept.II
PRODUCTION	ENTERPRISE	DI						$\kappa_1 a_1 \gamma_{(t)} s_1 W_1$	$(1 - c_1 - \kappa_1 a_1 \gamma_{(t)} s_1) W_1 - c_2 W_2$
		DII			$V_1 + V_2 + \kappa_1 a_1 \gamma_{(t)} s_1 W_1 \frac{1}{q_1} + \left[(1 - c_1 - \kappa_1 a_1 \gamma_{(t)} s_1) W_1 - c_2 W_2\right] \frac{1}{q_2}$	$\gamma_{(t)} e(1\text{-}a)(V_1+V_2)$	$\upsilon_{(t)} e(V_1+V_2)$		
CURRENT	ENTERPRISE	C(I)							
		C(II)							
	HOUSEHOLD	PL						$\kappa_1 a_1 \gamma_{(t)} s_1 W_1 \frac{1}{q_1}$	$\left[(1 - c_1 - \kappa_1 a_1 \gamma_{(t)} s_1) W_1 - c_2 W_2\right] \frac{1}{q_2}$
		IDR	$\gamma_{(t)} e V_1$	$\gamma_{(t)} e V_2$					
		UL	$\upsilon_{(t)} e V_1$	$\upsilon_{(t)} e V_2$					

Table 6: Model of Extended Reproduction With Unproductive Labor and Technological Change in SAM Form

			CURRENT					ACCUMULATION	
			ENTERPRISE		HOUSEHOLDS			ENTERPRISE	
			Cap. (I)	Cap. (II)	Productive Laborers	IDR	Unproductive Labor	Dept. I	Dept.II
PRODUCTION	ENTERPRISE	DI						$\kappa_1 a_1$ $\gamma s_{1(t)} W_1$	$(1 - c_{1(t)} - \kappa_1 a_1 \gamma s_{1(t)}) W_1 -$ $c_{2(t)} W_2$
		DII			$V_1 + V_2 +$ $\kappa_1 a_1 \gamma s_{1(t)} W_1 \frac{1}{q_1} +$ $\left[(1 - c_{1(t)} - \kappa_1 a_1 \gamma s_{1(t)}) W_1 - c_{2(t)} W_2\right] \frac{1}{q_2}$	$\gamma(1\text{-}a_1)(e_{1(t)} V_1 +$ $e_{2(t)} V_2)$	$\upsilon(e_{1(t)} V_1 +$ $e_{2(t)} V_2)$		
CURRENT	ENTERPRISE	C(I)							
		C(II)							
	HOUSEHOLD	PL						$\kappa_1 a_1 \gamma s_{1(t)} W_1 \frac{1}{q_1}$	$\left[(1 - c_{1(t)} - \kappa_1 a_1 \gamma s_{1(t)}) W_1 - c_{2(t)} W_2\right] \frac{1}{q_2}$
		IDR	$\gamma e_{1(t)} V_I$	$\gamma e_{2(t)} V_{II}$					
		UL	$\upsilon e_{1(t)} V_I$	$\upsilon e_{2(t)} V_{II}$					

Figure 1
Output in Basic Model of Expanded Reproduction

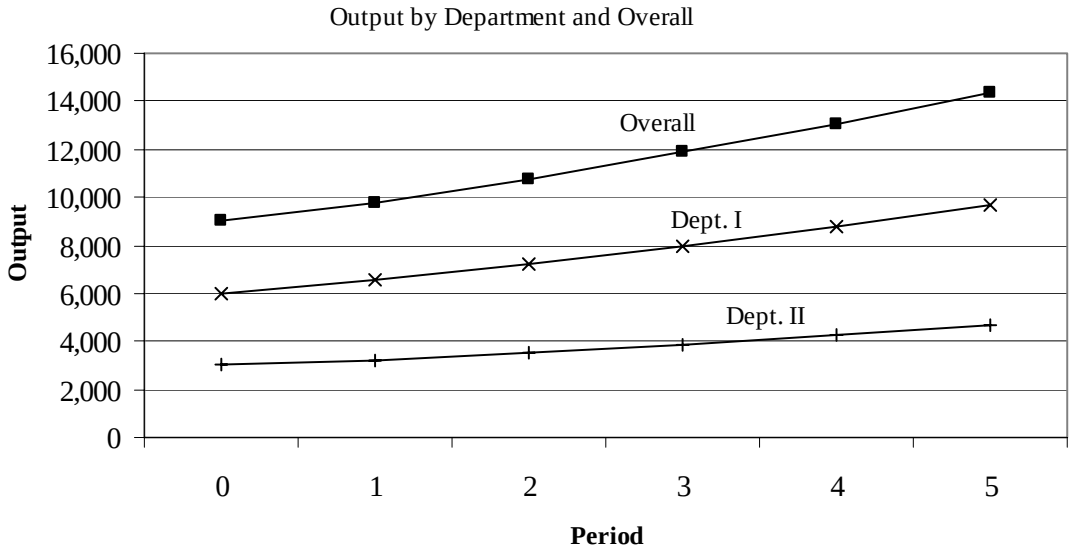

Figure 2
Capital Accumulation in Basic Model of Expanded Reproduction

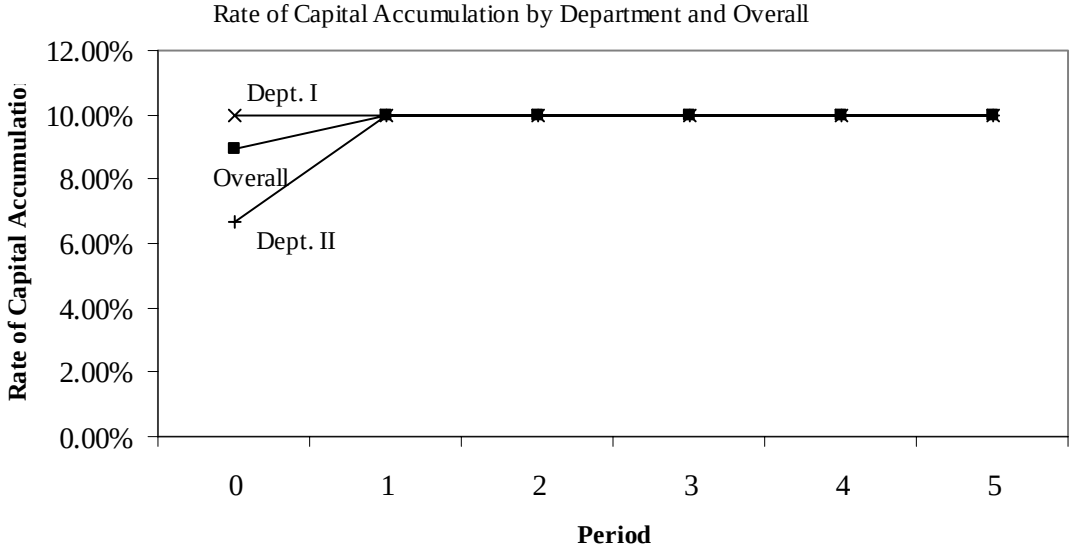

Figure 3
Consumption in Basic Model of Expanded Reproduction

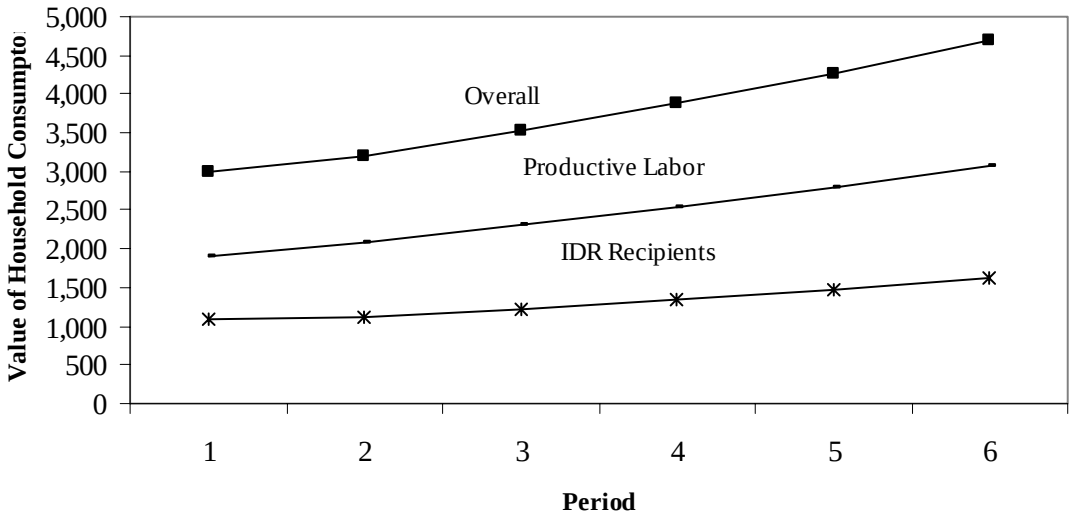

Figure 4
Rate of Consumption Growth in Basic Model of Expanded Reproduction

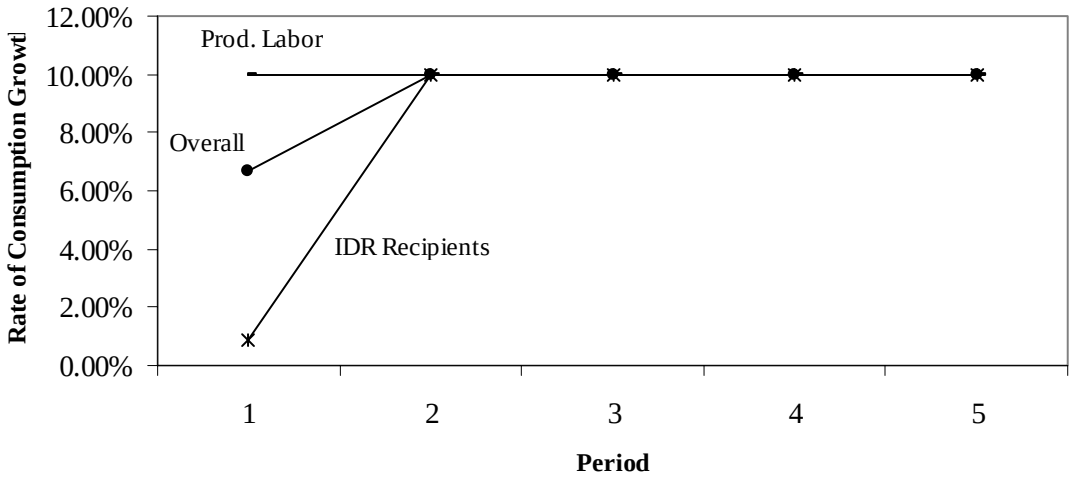

Figure 5
Output in Expanded Reproduction with Unproductive Labor and 'Profit Squeeze'
Level of Output

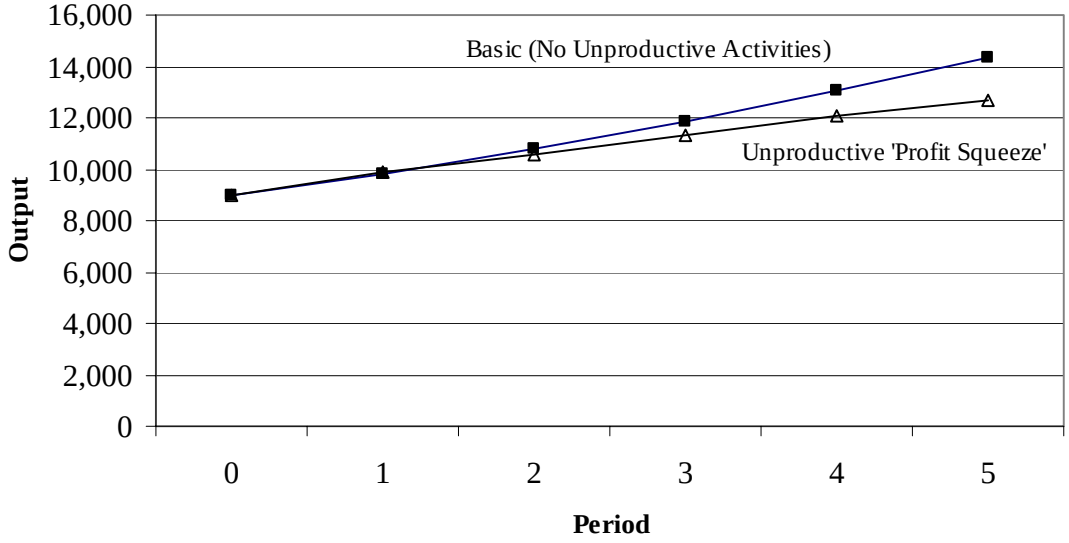

Figure 6
Capital Accumulation in Expanded Reproduction with Unproductive Labor and 'Profit Squeeze'
Rate of Capital Accumulation by Department and Overall

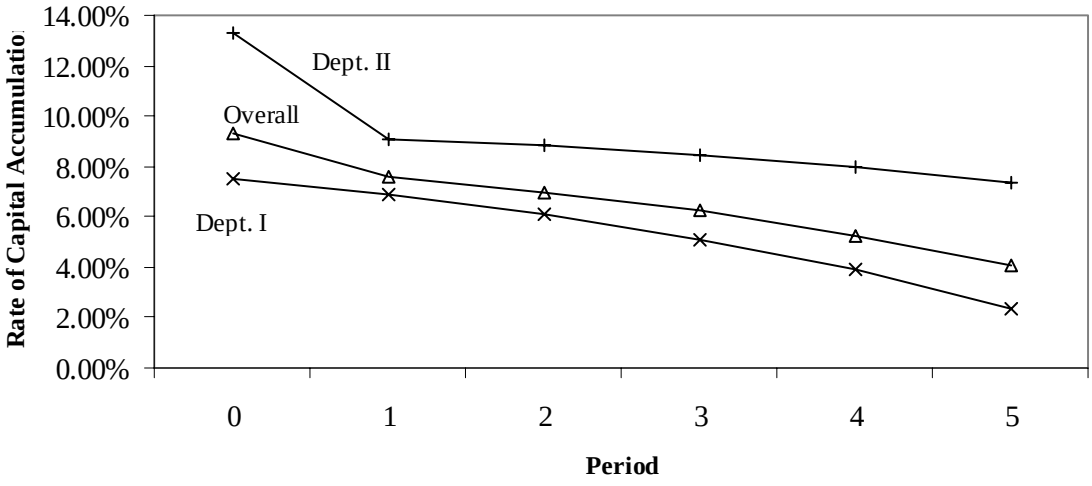

Figure 7
Capital Accumulation in Expanded Reproduction with Unproductive Labor and 'Profit Squeeze'
Comparison of Rates of Growth

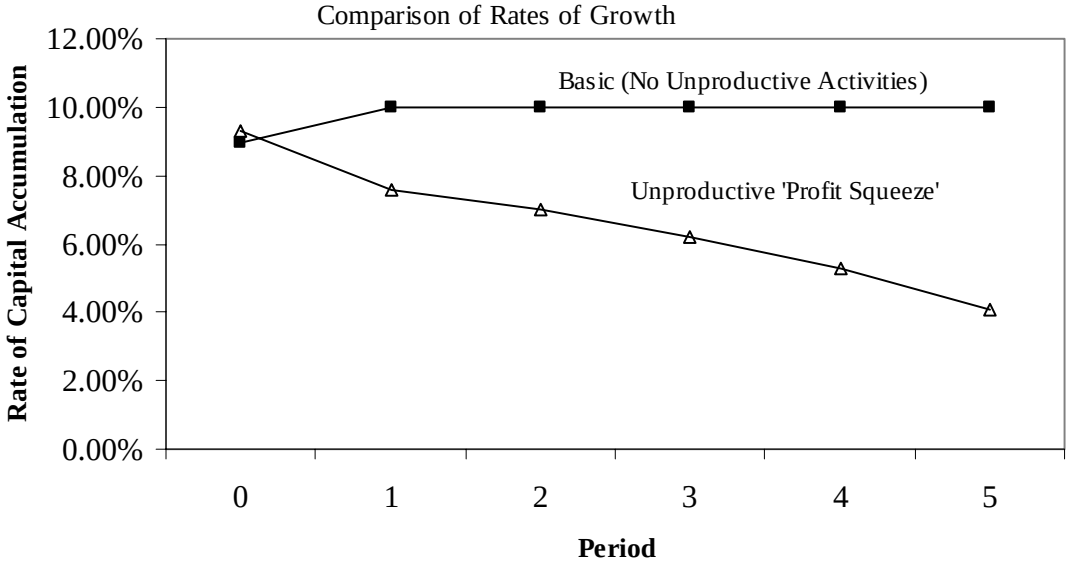

Figure 8
Consumption in Expanded Reproduction With Unproductive Labor and 'Profit Squeeze'
Household Consumption by Income Source

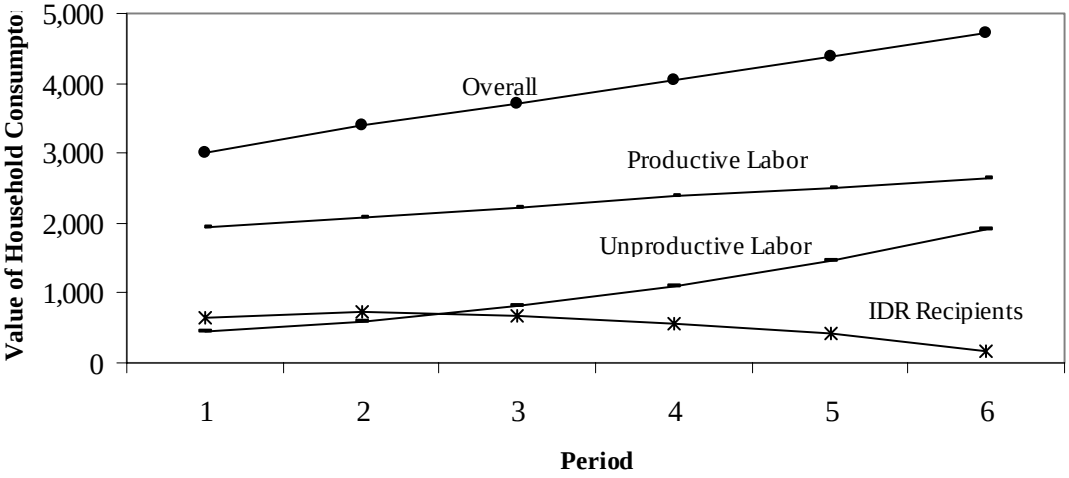

Figure 9
Household Consumption Growth in Expanded Reproduction with Unproductive Labor and 'Profit Squeeze'

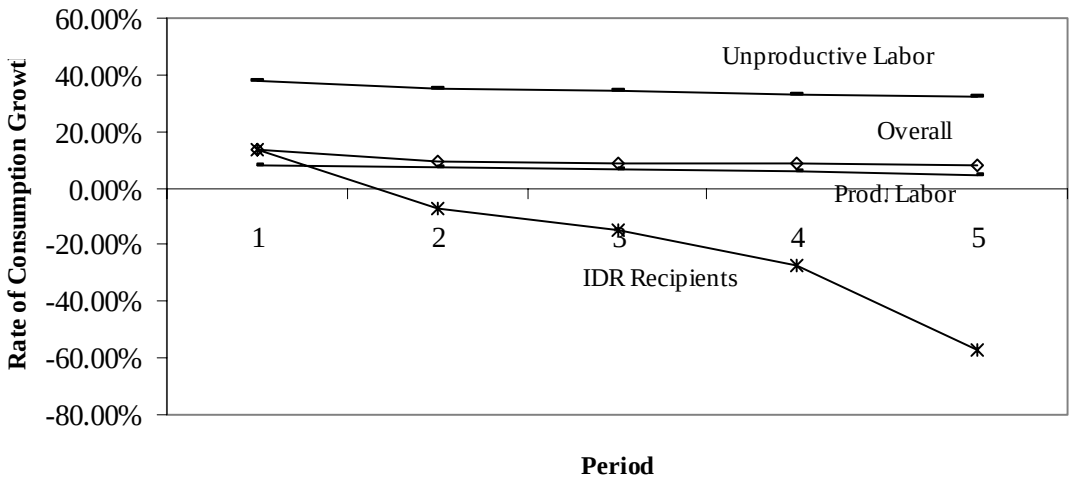

Figure 10
Household Consumption Growth in Expanded Reproduction with Unproductive Labor and 'Profit Squeeze'

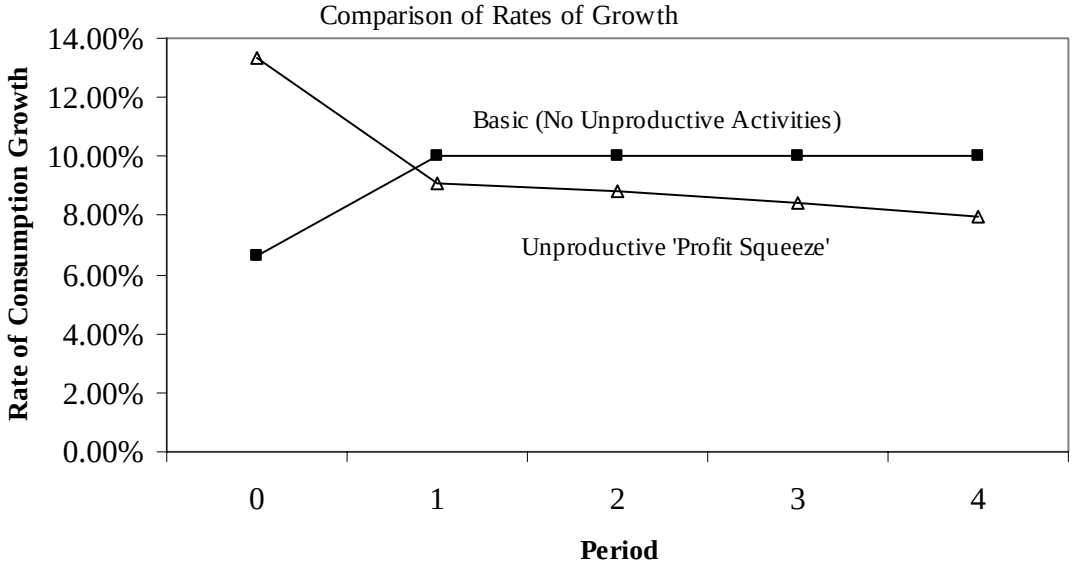

Figure 11
Output in Expanded Reproduction with Unproductive Labor and Technological Change

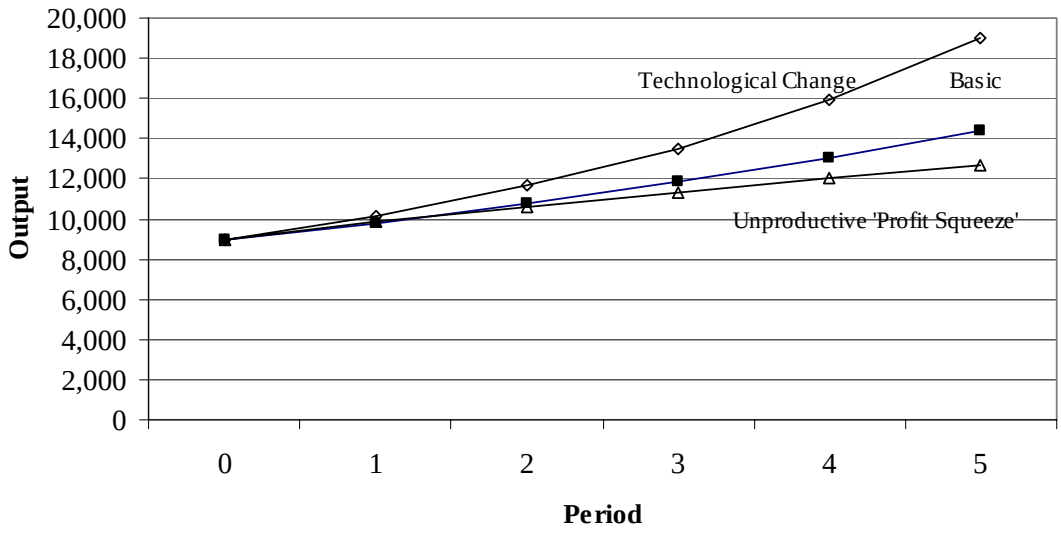

Figure 12
Capital Accumulation in Expanded Reproduction with Unproductive Labor and Technological Change

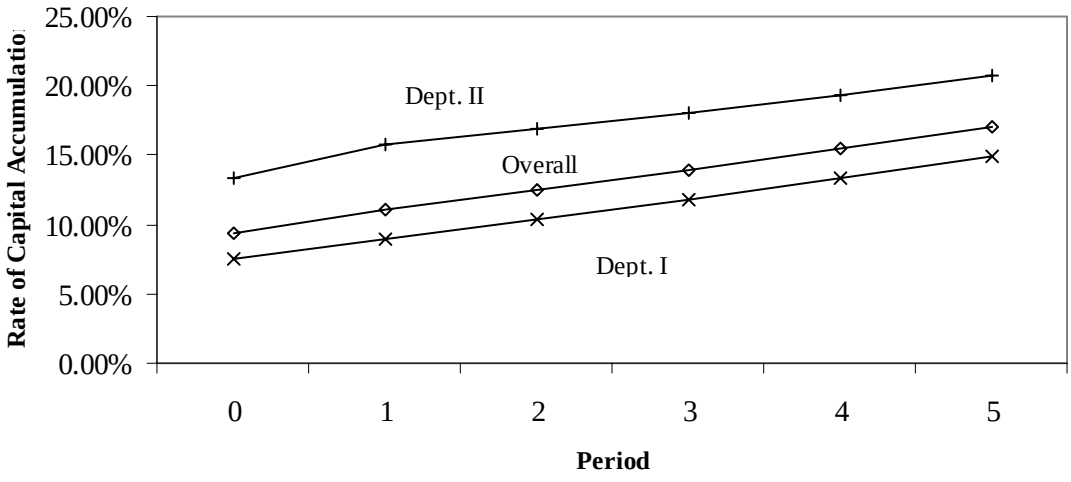

Figure 13
Capital Accumulation in Expanded Reproduction with Unproductive Labor and Technological Change

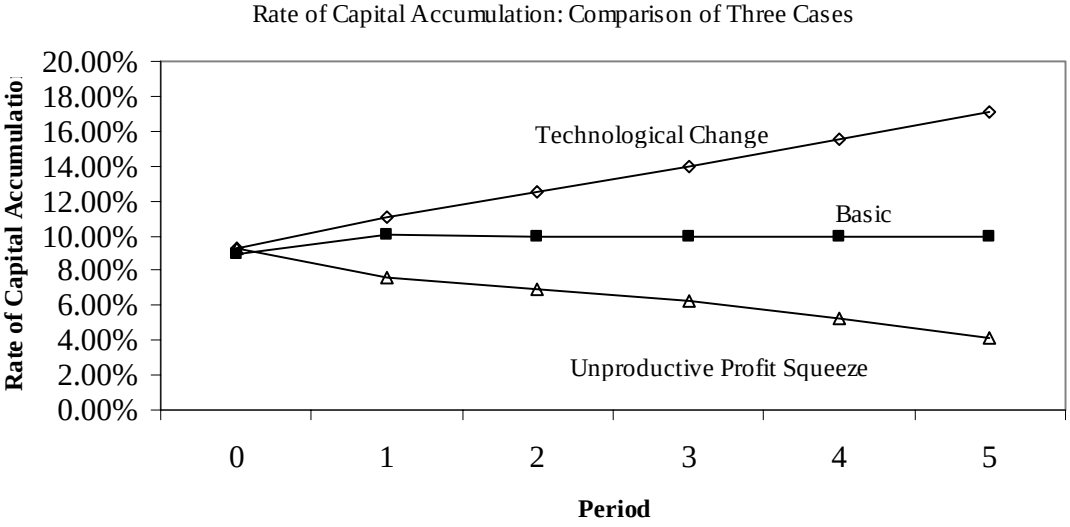

Figure 14
Consumption in Expanded Reproduction With Unproductive Labor and Technological Change

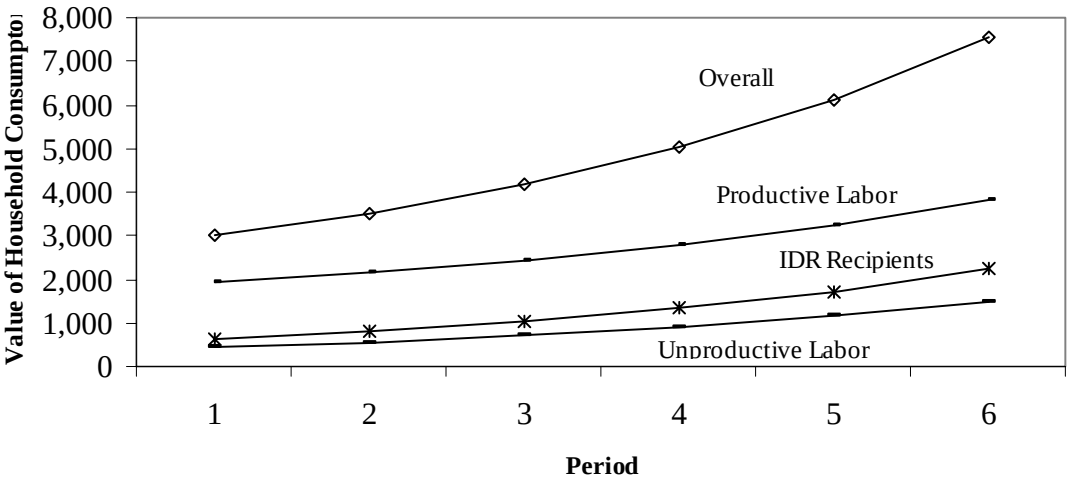

Figure 15
Household Consumption in Expanded Reproduction with Unproductive Labor and Technological Change

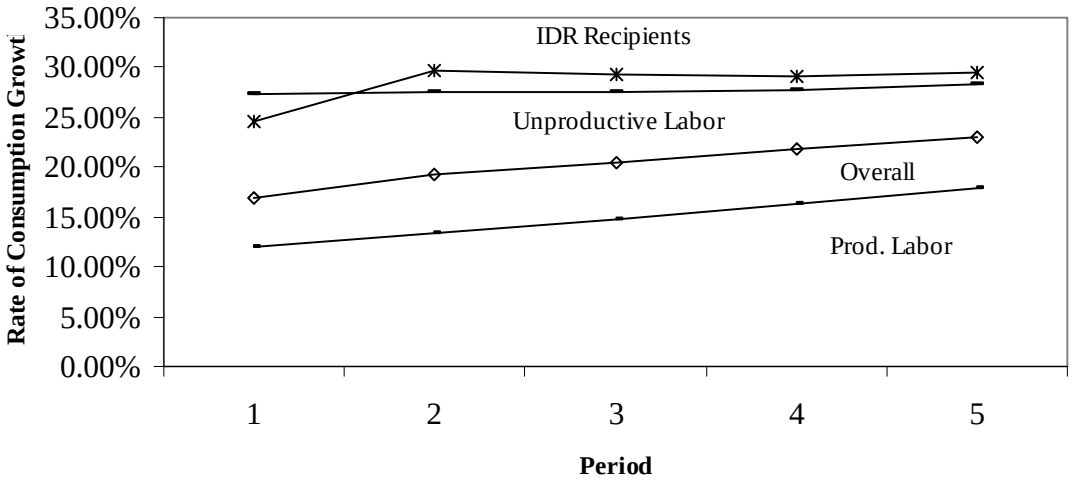

Figure 16
Household Consumption in Expanded Reproduction with Unproductive Labor and Technological Change