# The Circuit Analysis, the Monetary Economy of Production and the Multisectorial Analysis
# Proposals for a S.N.A. built on income-value
# A "Detransformation " of Prices in Values

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In this paper we will explain how the French circuit analysis, built from the works of Bernard Schmitt, and the Keynesian monetary economy of production (MEP), leads to a new theory of value, the income-value theory.

The expression "monetary economy of production" has been used by Keynes in 1933, but this type of economy is already described in the 1930 Treatise on Money. In two major books published in French, Monnaie, Salaires et Profits (1966) et l'Analyse Macroéonomique des Revenus (1971), and one published in English, Macroeconomic Theory (1972), Bernard Schmitt has given revival to the MEP and let the basis of the French circuit analysis.

We will first describe a very simplified model of MEP (§ 1), we will qualify canonical. This model represents the developed economies of the present time. It considers the macroeconomic circulation of money and goods between three groups of agents, those creating money, the banks, those transforming raw materials and labour in goods, the firms, and the households. Money measures raw materials, labour and goods and assures their circulation. Firms and banks use a common language, called business accounting, which is very necessary for performing these functions. The canonical MEP sets at least two problems. The first one is the possibility of the retained profits for the firms taken as a whole. The second one is the adequacy of the model to more complex economies. Nowadays the economies are described by national accounts. So testing the adequacy of a model to an economy comes to test its adequacy to a set of national accounts.

We will see (§ 2) that the first problem is a false one. It comes from the wrong idea of the economists that profit is either a flow or a stock of money, when business or national accountants define it only as the result of a calculus. So the true problem is the economic meaning of that calculus. We will show that profit comes from the existence of two successive systems of prices, the prices in income-value (that Bernard Schmitt calls macroeconomic prices, and that may be compared to the Marxian value) and the selling prices (that Bernard Schmitt calls the microeconomic prices, and that may be compared to the Marxian prices of production). These two systems involve the profit, which is a transfer of purchasing power from households to firms. Its measure in income-value is given, on the macroeconomic level, by the equations of ebb and flow derived from the Fundamental Equations of The Treatise on Money.

The second problem, the adequacy of a model to a set of national accounts should be tested in numerous ways, since the economy is complex. For instance, can the model explain that there exist several banks, and not only a group of them, that banks perceive interests, that there exist financial flows between households and firms (that we call secondary monetary flows to distinguish them from the principal ones)? that banks lend to other agents besides firms, that government may play an important part in collecting taxes and distributing "collective goods"... All facts that are not described by the canonical MEP, but, we think, do not modify its core.

In this paper we will concentrate on a problem linked to the first one : the existence of several firms involves that the transfer of purchasing power constituted by the profit be shared out between the firms.

To apprehend the existence of several firms and to be able to make a concrete application, we will consider Input-output table (§3). We shall see that in the published the profit is no more measured in income-value but in prices. So to be coherent with the results of the §2, we propose a method to build an IOT in income-value and to calculate the profit of each activity of an IOT in income-value.

Then (§4) we study the formation of the profit. The profit being the balance of the balance-sheet is calculated only at the end the year, but it is fomed all the year round through successive transfers. First, the firms selling consumption goods gain the whole profit; then they transfer a part ot it to the other firms through monetary flows when purchasing fixed and circulating capital. This study allow us to see why the profit in income-value differs frome the accountin profit.

In a 5th paragraph, we present an application of this model to the French econmy of the year 2007.

But in the §6 we show that a true analysis can only be applied to the world economy.

In conclusion we expose that our model in a model of "detransformation"; which can be applied to any kind of value and not only to the income-value,

## 1. Description of a canonical MEP

The MEP considers collective exchanges between groups of agents, each group being taken as a whole. There are three groups : the banks, which are the institutions creating money ; the firms, which are the institutions producing commodities (including services) from other commodities and from natural ressources (including work) ; and the households.

Between these groups of agents take place : 4 flows of debt : (1), (1'), (4) and (4') ; 4 principal flows of money or principal monetary flows: (1), (2), (3) and (4) and 2 flows of non-monetary objects (natural ressources and commodities) or real flows : (2') and (3').

Flows (1) and (4) are both flows of debt and principal monetary flows.

These eight flows append successively during four phases, the two flows of each phase making a collective exchange between two groups of agents, each flow being the counterpart of the other. The phases 2 and 3, which append only between the firms and the households, are called the central phases. By convention the two flows, parts of the same collective exchange of the phase i, are called (i) and (i').

The four principal monetary flows of the four successive phases make a monetary circuit that we shall call the principal monetary circuit. This circuit starts from the banks, by the creation of money, and ends at the banks, by the destruction of money.

The expressions principal monetary flows and principal monetary circuit mean that it would be possible to consider secondary monetary flows forming a secondary monetary circuit. These flows would be the financial flows between the households and the firms, whose function would be to compensate the insufficiency of flow (3) due to savings (Vallageas 1988).

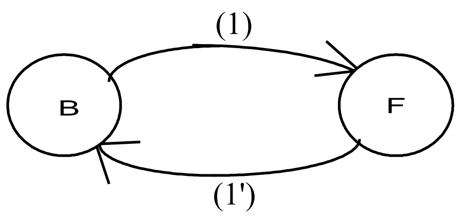
The two real flows of the two successive central phases make a real circuit beginning from the households by the contribution of the natural ressources and ending at the households by the consumption of the produced goods.

We shall first expose the four successive phases constituting four collective exchanges of two flows each, then we shall expose the circuits. Finally we will suggest an interpretation fo the Treatise on Money in term of circuits.

### 1.1. The four phases of four collective exchanges of two flows each.

#### 1.1.1. Phase 1. Creation of money. The exchange of flows of debts between banks and firms.

The banks and the firms exchange debts, the debt (1) of the banks to the firms (equally called the claim of the firms on the banks) and the debt (1') of the firms to the banks (equally called the claim of the banks on the firms) ; the debt (1) is called money by definition and is the first flow of money that we meet. This exchange can be represented by the following graph :




graph 1

It constitutes the following operation in the double entry bookkeeping system of the bank :

current account of firms

| | |
|---|---|
| | (1) $x |

liabilities on firms

| | |
|---|---|
| (1') $x | |

accounts 1

It is also an operation in the double entry bookkeeping of the firms :

banks

| | |
|---|---|
| (1) $x | |

debts to banks

| | |
|---|---|
| | (1') $x |

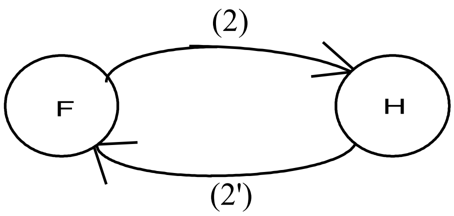
accounts 2

The two debts are by construction of equal value ($ X), this value being counted in monetary units, the unit ($) being itself defined by this operation. The debt of the banks, the money, is in theory payable on presentation, i.e. immediately reimbursable. In fact, as we will see below (cf. phase 3) it is never paid. The debt of the firm may have a time-limit or may not. In the latter case, it may be payable on demand or on advance notice depending of contract.

For the moment this first phase appears utterly unuseful, because it creates only two debts of equal amounts that quash one another. One can say that the money held by the firms at the end of phase 1 has no value, first of all, because it it is not a true wealth for the firms, since they have debts of an equal amount, and secondly because it cannot buy commodities, that do not exist yet. Bernard Schmitt says it is a nominal money.The interest of this purely intellectual phase and of the money appear in the two following phases.

1.1.2. Phase 2. Production and distribution of income. The exchange of flows of natural resources and incomes between households and firms.

The households bear their work and other natural resources (flow (2')) to the firms that will use them to produce commodities. In exchange the firms give the households the money income (2). So, we see now the utility of the banks' debt (1) : it allows the distribution of (2). Moreover (2) gives a monetary measure of (2') ("the national money income measures the national product"). That can be represented by the graph :




graph 2

The situation of the banks accounts at the end of phase 2 is :

current account of households

| | |
|---|---|
| | $x |

liabilities on households

| | |
|---|---|
| $x | |

accounts 3

and the firms' accounts are at the end of phase 2 :

banks

| | |
|---|---|
| null | null |

debts to banks

| | |
|---|---|
| | $x |

accounts 4

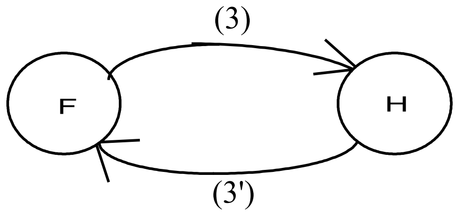
At the end of phase 2 the households hold money and contrary to the situation at the end of phase 1, when money was held by the firms and has no value, the money now held by the households has got a value : first of all, it is always a debt of the banks, but contrary to the firms, the households have no debt, so they own a true wealth ; and secondly because it can buy goods that now have been produced (Bernard Schmitt says this money is real).

This purchase constitutes the phase 3.

1.1.3. Phase 3. Spending and distribution ot the commodities. The exchange of the spending of income and of the consumption goods between households and firms.

In law the banks shoud pay their debt to the households. But in fact (i.e. economically) it is absolutely impossible : the legal way to pay a debt is to pay it in money. Now the debt of the banks is already money, so, by construction, the banks cannot pay their debts[1]. The banks have nothing to give to the households in exchange of their money, the only things that the latter can get are goods from the firms. So they buy the goods to the firms. That can be represented by the graph :

[1]If we consider several banks, then a bank can pay its debt with a money-debt issued by another bank : e.g. a commercial bank can pay its debt (materialised by an account) by banknotes, which are a debt of the central bank. This operation is not actually a payment but a change in the form of the money and from a macroeconomic point of view it does not matter at all.




graph 3

At the end of this purchase we have come back to the same situation that at the end of phase 1 : the firms have got the money back, but they have always a debt of equal amount to the banks, so they do not own any real wealth, the money has no more value (Bernard Schmitt says it is nominal) ; the households have nothing, since they have no more money, and they have consumed the goods they have bought. The accounts of the banks and of the firms are exactly the accounts they have got at the end of phase 1 (see accounts 1 and 2).

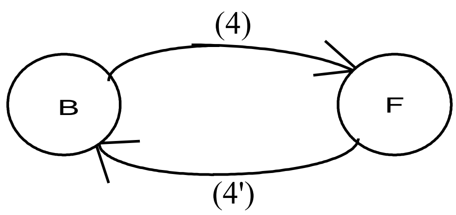
This purchase operation may legally be analysed as the circulation of a bill of exchange : the firms have a debt to the banks, and the latter have a debt to the former : so to pay off these two debts it is easier that the firms pay directly the households. This single payment cannot be made in money (and the households do not wish it), so it is made in commodities measured in money.

Then the money held by the households allows them to get the commodities, and it has got, the value of the commodities that it can purchase ("the national money income measures the national product").

Since at the end of this phase we have come back to the same account situation as at the end of phase 1, the firms can reimburse their debts to the banks.

#### 1.1.4. Phase 4. Destruction of money. The exchange of the money and of the banks' debt between the firms and the banks.

This phase can actually take place after the end the phase 3. It is represented by the graph :




graph 4

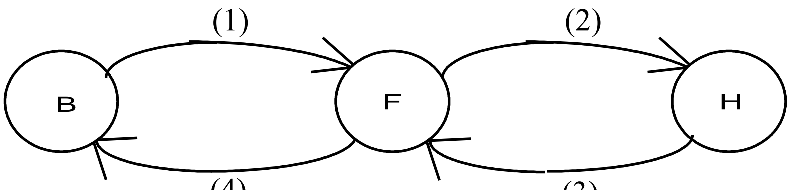
Then a new circuit can occur, beginning at the phase 1 and independant of the precedent.

### 1.2. The circuits

If we follow the trajectory of a monetary unit during the four phases, we get the principal monetary circuit, this unit going through the principal flows (1), (2), (3) and (4), between the banks, the firms, and the households on one way, and between the households, the firms and the banks on the way back. The circuit limited to the flows (2) an (3) is the central monetary circuit.

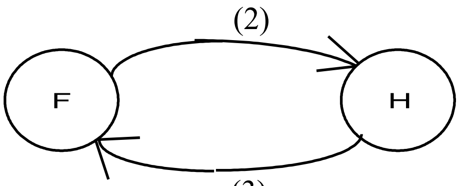
If we follow the trajectory of a non-monetary object going through the real flows (2') and (3'), we get the real circuit.

This notion of circuit corresponds very exactly to that one of the mathematical graph theory (MGT) (Andrásfai) (Diestel). This theory considers arcs going from one pole to another. A group of economic agents may be considered, as  as a pole. So the arc (1) starts from the pole banks and goes to the pole irms. The graph theory calls circuit a succession of arcs starting from a pole going through different poles and ending back at the starting pole. It can be easily verified that all the circuits  we have considered (i.e. the principal and central monetary circuits and the real circuit) are circuits in the meaning of the graph theory. More peculiarly let us give the graph representation of the principal monetary circuit :




graph 5

The central monetary circuit is a part of that graph :




graph 6

When considering either the principal monetary circuit or only the central part of it, it can be  easily seen  that no group of agents can accumulate money, i.e., at the end of a circuit, when all  the money has flowed back to the banks, there is no money left in the circuit. This simple fact is the source of the profit problem. But before exposing it, we will give  an interpretation of The Treatise on Money in circuit terms.

### 1.4. An interpretation of The Treatise on Money in circuit terms.

The volume I of this book gives a description of the economy which corresponds approximatively to our description of paragraph 1.1. It makes a distinction between the banks, the firms and the households. Keynes calls community's money income a) "everything that is paid to employees including any  payment made to unemployed or partially employed or pensionned employees" (We would say today any direct or indirect wages) ; "b) the normal remuneration of entrepreneurs; c) interest on capital ; d) regular monopoly gains, rents and the like." So he excludes of this enumeration the part of profit that he judges not normal, and he will call windfall profits.

We suggest a reinterpretation of Keynes' thought. It seems difficult, without an *a priori* theory of distribution,  to say what is a normal profit, and correlatively what is a regular monopoly gain. So we suggest to adopt an objective classification by considering on one hand what is paid to the households, i.e. a) and c) (including dividends), and that constitutes flow (2), and on the other hand what is not paid to the households, i.e. retained profits, we will call more simply profits.

This is an essential condition for interpreting the Treatise in circuit terms, because it makes a distinction between what circulates and what does not, whereas the General Theory cannot be interpreted in circuit terms, because, in considering a complete profit, it cannot distinguish the flow (2).

But this leads to a new problem : what are retained profits? They are not a flow, since they do not circulate, so what are they?

## 2. The possibility and the nature of retained profits

The existence of the dividends does not make any problem : the dividends paid by firms to other firms are monetary flows internal to the group of firrms, and the dividends paid to the households are by construction part of the flow (2). Nevertheless in the circuit literature (see Marc Lavoie), the problem of the existence of a monetary profit, taken as a whole, is very often questionned. This problem follows the Marxian problem of the monetary realization of the surplus-value.

The problem concerns only the retained profits which, by construction, are not paid. So they cannot be a monetary flow. And they are either a stock of money, either something else which is evaluated in money (like goods or real flows have got a monetary evaluation but are not money).

It can be easily shown that the retained profits cannot be money stocks, for two reasons, the first one being purely a quesion of logic, the second coming from the double entry bookkeeping system.

### 2.1. The retained profits cannot be money stocks

2.1.1. In a canonical MEP firms cannot accumulate money stocks.

If retained profits are money stocks, the profit of all the firms taken in their whole is null. This in an evidence when considering the central monetary circuit compounded of the flows (2) and (3) circulating between the firms and the households. As all the money coming into the households comes from the firms by (2) , the households cannot give back to the firms by flow (3) more money than they have received by (2).

Also in a canonical MEP firms cannot accumulate money stocks.

There are two ways to consider a non canonical MEP The first one is to consider foreign ex-changes. We will see that these exchanges cannot lead to money stocks. The other one is to consider that other agents besides firms can borrow to the banks. In that case the firms taken in their whole can accumulate money stocks, but these ones cannot be profits.

2.1.2. The foreign exchanges cannot lead to money stocks

The distinction of these exchanges implies there exist at least two different monetary zones, one internal with a pole of home banks creating monetary units called \$, e. g., and lending to home firms, and an other one with a pole of foreign banks creating other monetary units called £, e. g., and lending them to foreign firms. So in that answer, the foreign firms or the foreign households would buy to the home firms and would allow them extra money coming from the foreign banks, and allowing money stocks. But one can expect the profit (and the money stocks) of the home firms be in \$, and then they must come from the home banks (if a part of a firm makes profit in £ and not in \$, it belongs, by definition, to the the foreign firms and not to the home ones).

Also when a foreign agent, either firm or household, wants to buy to a home firm, it must exchange its £ for \$ by the home banks first. Of course the \$ created then are not borrowed, and apparently they can assure a money-stock for the home firms, who have not to repay them. But when deposing its £ by a home bank, a foreign agent withdraws them from the foreign circulation and occurs a problem of reimbursment for the foreign firms. So, when we consider all the firms, home and foreign, in their whole, we cannot have any money stock.

#### 2.1.3. The lending of money to other agents besides firms allows the firms, in their whole, to have money stocks, but these stocks are not retained profits

Suppose that the households, or an other new agent, e.g. the government, borrows to the banks. With this money this agent can spend by the firms and all the spendings received by the latter exceed the value of flow (2). Now the firms own in full property money stocks that they have not to repay back. This situation can go on so long as the banks do not demand the repayment of their loans to the households or to the government, or so long as they accept to renew the loans.

This situation cannot occur when all the loans are done to the firms through the flow (1), because in that case, although the firms can have money stocks so long as the repayment is not demanded, these money stocks are not their full property, since they have debts to the banks for an equal amount.

So many economists, considering profit is monetary, think profits are these money stocks in full property, and, as this full property remains in the firms, they say they are the retained profits.

Their conclusion is that the existence of profit leads to the impossibility of a canonical MEP : it is necessary for that existence that other agents besides firms borrow to banks, and what allows the existence of profit is essentialy the budget deficit.

It is evident that other agents besides firms can borrow and that money stocks in full property can exist in that case. But these money stocks are not profits. If we look to the balance-sheet of a firm, we see that the money stock is a current asset (in fact it is composed of two current assets, the cash and the current account at the bank) appearing in the assets column, while the retained profit appears elsewhere. In the same manner in the national accounts, the money stocks of the firms appear in their financial account, under the name currency and deposits, while the retained profit appears elsewhere too.

This leads to the questions : where do the profits appear in the accounts? what are their accounting and economic meaning?

### 2.2. The true nature of profits : the increase of firm's value

The definition of the firm's profit, as it is given by the business accounting system, is the increase of revenue reserves during the period, these reserves being the part of owner's equity not brought by the owners (e.g. with new shares) ; so the profit is the increase of equity due to internal reasons. This is why it appears on the balance-sheet, which is a view of the assets and liabilities of the firm. But the double entry bookkeeping system has been constructed in such a way, that it appears as the balance of the profit and loss account (PLA) (called income statement in U.S.) too. The national accounts use a sequence of accounts which is, for the firms, a decomposition of the PLA, so they get the same profit.

### 2.3. The profit, the canonical MEP, the business accounts and the equations of ebb and flow

We come back to the canonical MEP and we consider its principal monetary circuit. Following the business accounting system, we know that the profit is the increase of the firm's value during the period.

To measure it, the accounting system constructs the balance-sheet and put the accumulated value of its assets, in one column, and of its debts in the other column.

First of all, we see that we must delimitate an accounting period. There is no reason, that, at the end of it, the flow back (3) of the money earned by the households by the flow (2) be finished. So at the end of the period, this money not yet flowed back is their savings, $S_H$. Let us call C the part of (3) achieved at the end of the accounting period and R' the value of flow (2)[2], then we have

$$R' = C' + S_H \quad \text{(eq. 1)}$$

A part of $S_H$, that we will call L, may have flowed back through a secondary monetary flow (i.e. a financial flow, a loan, or a subscription of new shares by the households to the firms) and then may have been reimbursed to the banks. The other part, H, is hoarded by the households, so it cannot be reimbursed yet and is always a debt to the banks. We see that :

$$S_H = L + H \quad \text{(eq. 2)}$$

and that $S_H$ is the total variation of the debt of the firms during the period.

As for the increase of the assets, we must consider the part of the production that at the end of the period is always the firms' property, namely the investment. The only measure we have for it, is its price cost since, the firms being taken in their whole, there is no transaction on investment and then no selling price. Here the price cost is limited to salaries and other payments (essentialy interests and dividends) to the households and we will call it I'. If we call Π the profit, we have :

$$\Pi = I' - S_H \quad \text{(eq. 3)}$$

and we get the variation of the balance-sheet during the period :

| Δ assets | Δ liabilities |
|---|---|
| I' | $S_H$<br>Π |

account 5

Let us consider now the PLA. The income of the firms comes from the sale of goods to the households, i.e. the flow (3) limited to the end of the period and that we will call C, like consumption. The expenditures are those engaged for the production of the sales C, which here are limited to salaries and other paiements to the households, and that we will call C'. As these payments are engaged to produce either investment or consumption, C' + I' is the whole of them, that we will call R'. So we have :

$$R' = C' + I' \quad \text{(eq. 4)}$$

and R' is the exact measure of flow (2).

The combination of these equations leads to :

$$\Pi = C - C' \quad \text{(eq. 5)}$$

[2]we put the sign prime on the R to distinguish this revenue (not including the retained profit) of the one of the General Theory (including the whole profit).

As C is the income part of the PLA, C' being the expenditure part, this equation defines exactly the PLA which may be represented by :

| expenses | incomes |
|---|---|
| C' | C |
| Π | |

account 6

We see that the double entry principle is respected, since the equation (5) and the equation (3) of the balance-sheet define the same profit Π.

We must notice that in the bookkeeping practice another presentation is generally used. In it the value R' is accounted instead of C' in the expenses and I' is added in the incomes (under the name of production for oneself). Thus the PLA becomes :

| expenses | incomes |
|---|---|
| R' | I' |
| Π | C |

account 6'

Now as equations (4) and (5) may be derived from the Fundamental Equations of Chapter 10 of the Treatise (Vallageas 1996), we may say that the Treatise on Money is compatible both with the canonical MEP and the business accounting system.

But, beyond its accounting definition, the profit has got an economic meaning in the framework of the MEP.

**2.4. The meaning of the profit in the MEP : a reinterpretation of the Treatise on Money**

We have seen that the accountants define the profit on an accrual basis, as the increase of the firm's value due to internal reasons and that this basis corresponds to equations (3) and (5) of the MEP.

But what is the meaning of the increase of the firm's value for the MEP?

Its evaluation is $I' - S_H$. A central idea of the MEP is that the households (including the workers and the rentiers as well, or people being both workers and small rentiers) have the right to get the whole production, since this latter has been evaluated by the flow (2) in phase 2. But in fact the households get only C' for the price C. So they accept to leave in the firms goods of value $\Pi = C - C'$, these goods becoming the property of the firms. In fact they leave in the firms, at the end of the accounting period, more goods than C - C', since they leave I'. But as $I' = \Pi + S_H$, they have always a claim of value $S_H$ on the firms. For the part H (hoarding) of $S_H$, they can claim immediately, but for the part which corresponds to financial loans, they must wait for the repayment of them, and for the part which corresponds to shares, they own the investment goods indirectly through these shares.

So Π represents a transfer of purchase power from the households to the firms. This power is measured in income-value, since Π represents a part of the goods I' themselves measured in income-value. This kind of evaluation is the most pertinent. Now the whole product has been evaluated by income paid to the households during phase 2. The creation of a price C for the consumption goods different of their income-value C' allows the transfer of purchase power to the firms, but that transfer does not create any new value, so it is logic that it be measured in income-value, and Π is this measure indeed.

We will see that when we will disaggrege the firm in different industries, the profit got by each industry and calculated with the usual rules of the business or national accounting (and that we will call accounting profits for that reason) are not income-valued but selling price-valued. We will propose a method to calculate them in income-value.

## 3. The disaggregation of the firm

To allow this disaggregation, we will adopt the input-output model, constructed on the Leontiev hypothesis, with industries being groups of production units making the same output. This choice will allow us to present an empirical application, since the S.N.A. has integreted these tables.

First we will remember the matrix representation of the input-output tables (I.O.T.). Next we will show that, in respecting accounting rules, the profit we get for every industry from these tables is measured in selling prices, whereas we would like to have profit measured in income-value. So, for that purpose, we will expose a mode of construction of I.O.T.'s in income-value. Then we will be able expose the processus of formation of the accounting profit in combining figures given by both tables in selling prices and income-value. At end we will see what corrections must be done to the accounting profit in selling prices to get the profit in income-value.

### 3.1. A matrix representation of the I.O.T.'s.

Let us suppose now that we disaggrege the firm, and that there are n industries (called j with j varying from 1 to n), each of them making a different product, which can be used either as fixed capital, or consumption, or may be unsold, i.e. may be part of an inventory.

The distribution of fixed capital is given by a square matrix **Inv,**[3] whose the element of line i and column j is the quantity of the commodity i (evaluated in selling price) bought by the industry j as fixed capital, so that the sum of the elements of the column j is the quantity of fixed capital bought by the industry j and the sum of the elements of line i is the quantity of product j used as fixed capital by any industry. So the structure of **Inv** is the same as this of a Leontiev matrix. An element of the main diagonal (i.e. for which i = j) does not correspond to any purchase since it is the quantity of fixed capital of the industry j in product j, i.e. in a product manufactured by itself. By convention it is evaluated by the market price of the product j.

Let us call $\vec{u}$, the "vertical' unitary vector of n elements (i.e. of n lines and of n elements equal to 1). The product **Inv** $\vec{u}$, is the "vertical' vector $\vec{I}$ of n elements, each of them being the sum in lines of the matrix **Inv.** So this vector represents the distribution of the investment in fixed capital between the different products. In the same manner we have a "vertical' vector $\vec{\Delta S}$ of n elements giving the distribution of the changes in inventories between the different products, $\vec{C}$ giving the distribution of the consumption and $\vec{X}$ giving the distribution of the exports.

The matrix **IC** is a Leontiev matrix in which the element of line i and column j gives the purchase of the industry j in product i.

[3]In this matrix representation, all the matrices we will use are square with n x n elements and are heavy typed, while the vectors have n elements, are noted with an arrow, and are either "horizontal", and in that case are preceded by the symbol of transposition "$^t$", or "vertical".

Let us call $\vec{\Sigma}$ the vector giving the sums of the uses of each product. We have :

$$\vec{\Sigma} = \mathbf{IC}\,\vec{u} + \vec{C} + \mathbf{Inv}\,\vec{u} + \vec{\Delta S} + \vec{X} \quad \text{(eq. 6)}$$

Let us call ${}^t\vec{VA}$ the "horizontal" vector giving the distribution of the value added between the different industries and ${}^t\vec{M}$ the "horizontal" vector giving the distribution of imports between the different products. We know that we can represent the economy by an I.O.T. of this form :

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **IC** | + | $\vec{C}$ | + | $\vec{I}$ | + | $\vec{\Delta S}$ | + | $\vec{X}$ | = | $\vec{\Sigma}$ |
| + | | | | | | | | | | |
| ${}^t\vec{VA}$ | | | | | | | | | | |
| + | | | | | | | | | | |
| ${}^t\vec{M}$ | | | | | | | | | | |
| = | | | | | | | | | | |
| ${}^t\vec{\Sigma}$ | | | | | | | | | | |

table 1

The equation 6 correponds to the "horizontal" additions begining by **IC** and continuing on the right. We have "vertical" additions too begining by **IC** and going on below. They can be summed up by the matrix equation :

$${}^t\vec{u}\,\mathbf{IC} + {}^t\vec{VA} + {}^t\vec{M} = {}^t\vec{\Sigma} \quad \text{(eq. 7)}$$

It defines the value added of each industry and establishes that the resources are equal to the uses, the vector ${}^t\vec{\Sigma}$ being the transpose of $\vec{\Sigma}$ .

We will now see that the profit we can calculate from this table is measured in selling prices.

### 3.2. The accounting profit calculated from the I.O.T. in selling prices is measured in selling prices.

When there was only one firm, the profit, we calculated with equations 3 or 5, with the account 5 (balance-sheet), and the accounts 6 and 6' (PLA) was measured in income-value. Indeed it measures the increase of firm's worth, constituted of investment goods. Now, as there was only one firm, these investment goods were never selled, so they had no selling prices but only income-values, namely I'. So the profit could only be measured in income-value.

When there are several firms, there is a market for investment goods, which have selling prices then. The business rules prescribe to account assets in cost price. For a firm, the cost price is essentialy the price that it pays to another firm, so essentially a selling price. The only exception is for investment produced by oneself. So profit is essentialy measured in selling prices. Any-way it could not be measured in income-value, because to calculate this value a firm should know costs engaged by other firms, what is impossible.

The SNA prescribes to measure each item of investment by the market (then selling) price even when it is produced by oneself. We have taken this rule in account in the § 3.1. by setting that a product has only one price.

Let us look for the matrix equation giving the retained profit of every industry. We will first notice that we can only calculate a partial profit which we will call profit "on production" to distinguish it from the complete profit of each firm which includes profit on other operations besides production : profit on exceptional operations and profit on financial operations. These other operations are not included in the I.O.T.'s, and they could not, since they concern only firms and cannot concern industries. Any-way they do not matter for our purpose[4]. Moreover the profit which we will consider is gross of depreciation of the fixed capital.

Let us call ${}^t\vec{P}_A$, the vector whose elements are the accounting retained profits of every industry. Following the method of the PLA (which was, for a single firm, given by equation 5 and account 8'), we must make the difference between the resources (${}^t\vec{\Sigma}$ ) and the costs including the incomes paid to the households[5], the purchases of intermediate consumption (${}^t\vec{u}\,\mathbf{IC}$) and the imports (${}^t\vec{M}$)[6]. For reasons that will appear later (cf. 3.3) , we will call ${}^t\vec{VA}\,'$ the vector representing the distribution of the incomes paid to the households by the different industries[7].

So we have, for every industry, the profit on production measured in selling prices :

$${}^t\vec{P}_A = {}^t\vec{\Sigma} - {}^t\vec{VA}\,' - {}^t\vec{u}\,\mathbf{IC} - {}^t\vec{M} \quad \text{(eq. 8)}$$

Following the method of the balance-sheet, this profit can be got as the difference between the increase of the assets of every industry and that one of its liabilities. The increase of the assets for every industry during the accounting period is given by the vectorial sum ${}^t\vec{u}\,\mathbf{Inv} + {}^t\Delta\vec{S}$. ${}^t\vec{u}\,\mathbf{Inv}$ gives the purchases of fixed capital by every industry and ${}^t\Delta\vec{S}$ the changes in inventories recorded at the end of the period. Let us call ${}^t\vec{S}_H$ the vector giving the distribution of $S_H$ between the industries and such as $S_H = {}^t\vec{S}_H\vec{u}$. ${}^t\vec{S}_H$ gives the increase of the liabilities that the industries must subscribe either to the banks or the households because the flow (3) is not achieved at the end of the period.

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[4]Especially the financial operations of the firms are outside the canonical MEP which considers only production.

[5]As we want to measure the retained profit, the incomes include, as usual in this paper, the interests and dividends paid to the households.

[6]By writing that purchase of imports is ${}^t\vec{M}$, we write that the industry j imports product j. That means in fact that industry j imports only products j and the totality of the products j imported. These imported products could be redistributed afterwards between the industries by means of the intermediary consumption.

[7]As for the flow (2) these incomes include every thing paid to the households, therefore interests and dividends. But now that we have several firms, firms pay to (and received from) other firms interests and dividends. As so far they concern the production these payments should be accounted to calculate the profit on production. The interests and dividends paid (or received from) abroad should as well.

So the accounting profit can be given by the equation of the balance-sheet :

$${}^t\vec{u}\ \mathbf{Inv} + {}^t\vec{\Delta S} = {}^t\vec{S}_H + {}^t\vec{P}_A \quad \text{(eq. 9)}$$

The assets being accounted in selling prices, the accounting profit measures in selling prices the part of them which is the property of each industry at the end of the period. So we can say that the accounting profit, i.e. the profit calculated with the business rules (which have been transposed in the SNA) is measured in selling prices.

To find a measure of the profit in income-value, one must first built an I.O.T. in income-value.

### 3.3. Construction of the I.O.T.'s in income-value

We want now to build an I.O.T. of the same form as that of table 1, but in which all the values are expressed in income-values instead of selling prices. We will keep exactly the same notations as in table 1 and the equations 6 and 7 (i.e. "horizontal" and "vertical" additions) will remain. To express the changing of evaluation from price to income-value, we will use the symbol prime (e.g. the matrix **IC** in selling prices will become the matrix **IC'** in income-value, or the vector $\vec{C}$ will become the vector $\vec{C}'$).

So the equations 6 and 7 become :

$$\vec{\Sigma}' = \mathbf{IC'}\,\vec{u} + \vec{C}' + \mathbf{Inv'}\,\vec{u} + \vec{\Delta S}' + \vec{X}' \quad \text{(eq. 6')}$$

$${}^t\vec{u}\,\mathbf{IC'} + {}^t\vec{VA}' + {}^t\vec{M}' = {}^t\vec{\Sigma}' \quad \text{(eq. 7')}$$

As each product has got a unique price and a unique cost[8], each element of the line of the product i of the I.O.T. in selling prices is multiplied by the same coefficient $k_i$ to give its value in income-value. $k_i$ is the inverse of the mark-up of the product i. Let us call $\vec{K}$, the "vertical" vector compounded with the n $k_i$ for i varying from 1 to n and let us call **K** the diagonal matrix[9] (n,n) for which the element of the line i and the column i is $k_i$. There we can verify that each term of the equation 6' is equal to the corresponding term of equation 6 pre-multiplied by **K**. So we have :

$$\vec{\Sigma}' = \mathbf{K}\,\vec{\Sigma}$$
$$\mathbf{IC'} = \mathbf{K}\,\mathbf{IC}$$
$$\vec{C}' = \mathbf{K}\vec{C}$$
$$\mathbf{Inv'} = \mathbf{K}\,\mathbf{Inv}$$
$$\vec{\Delta S}' = \mathbf{K}\vec{\Delta S}$$
$$\vec{X}' = \mathbf{K}\vec{X}$$

For building the I.O.T. in income-value, it is necessary either to find n algebric equations in $k_i$, or a matrix equation in **K** or $\vec{K}$.

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[8]we remember that the produce j used by the firm j, either as intermediary consumption or fixed capital is valuated with the same price as if it was effectively purchased, although it is not (cf. 3.1.)

[9]In a diagonal matrix, the elements other than those on the diagonal are null.

We will take the equation 7' with the unknown $\vec{K}$.

In this equation we can replace **IC'** by its value **K IC.** ${}^t\vec{VA}'$ is a datum. Indeed, as we are in a system of income-values, the income paid to the households is the measure of the product, so the value added in income-value is nothing else but this income, and ${}^t\vec{VA}'$ is the vector giving the distribution of the incomes paid by the different industries. To know ${}^t\vec{M}'$, we will suppose that the imported products have got the same mark-up, or the same coefficient $k_i$, as the domestic products. So ${}^t\vec{M}' = {}^t\vec{M}\,\mathbf{K}$.

To know ${}^t\vec{\Sigma}'$, we have to transpose $\vec{\Sigma}' = \mathbf{K}\,\vec{\Sigma}$ · So ${}^t\vec{\Sigma}' = {}^t\vec{\Sigma}\ {}^t\mathbf{K}$, and since **K** is a diagonal matrix, it is equal to its transpose and ${}^t\vec{\Sigma}' = {}^t\vec{\Sigma}\ \mathbf{K}$.

So it results from the equation 7' :

$${}^t\vec{u}\,\mathbf{K\,IC} + {}^t\vec{VA}' + {}^t\vec{M}\,\mathbf{K} = {}^t\vec{\Sigma}\,\mathbf{K} \quad \text{(eq. 10)}$$

As ${}^t\vec{u}\ \mathbf{K} = {}^t\vec{K}$, we have : ${}^t\vec{K}\ \mathbf{IC} + {}^t\vec{VA}' + {}^t\vec{M}\ \mathbf{K} = {}^t\vec{\Sigma}\ \mathbf{K}$

or $$({}^t\vec{\Sigma} - {}^t\vec{M})\,\mathbf{K} - {}^t\vec{K}\ \mathbf{IC} = {}^t\vec{VA}' \quad \text{(eq. 11)}$$

To solve this equation, it is necessary to eliminate the matrix **K** to have only the vector ${}^t\vec{K}$ in position of pre-multiplication as an unknown. For that purpose we will consider the diagonal matrix **Σ** (n,n) of which the element of the line i and the column i is the total resource in product i, and the diagonal matrix **M** (n,n) of which the element of the line i and the column i is the import of product i. Then we can observe that :

$({}^t\vec{\Sigma} - {}^t\vec{M})\,\mathbf{K} = {}^t\vec{K}\ (\mathbf{\Sigma - M})$, so we have :

${}^t\vec{K}\ (\mathbf{\Sigma - M - IC}) = {}^t\vec{VA}'$ and we get the solution : $${}^t\vec{K} = {}^t\vec{VA}'\,(\mathbf{\Sigma - M - IC})^{-1} \quad \text{(eq. 12)}$$

We notice that the diagonal matrix **Σ - M** has got for elements the domestic ressources produced by the different industries.

This solution enables us to build the I.O.T. in income-value, and the comparison between the I.O.T.'s in selling prices and in income-value will enables us to calculate the profit in income-value of every industry and to explain its formation. First of all we must decompose the accounting profit in seven components.

### 3.4. Decomposition of the accounting profit in seven components in order to explain its formation and to calculate the profit measured in income-value.

Let us consider the definition of $\vec{P}_A$ through the PLA, i.e. equation 8 :

$${}^t\vec{P}_A = {}^t\vec{\Sigma} - {}^t\vec{VA}' - {}^t\vec{u}\,\mathbf{IC} - {}^t\vec{M}$$

By replacing ${}^t\vec{VA}'$ with its value from equation 7', we get :

$${}^t\vec{P}_A = {}^t\vec{\Sigma} - {}^t\vec{\Sigma}' - ({}^t\vec{u}\,\mathbf{IC} - {}^t\vec{u}\,\mathbf{CI}' + {}^t\vec{M} - {}^t\vec{M}')$$

or by transposition :

$$\vec{P}_A = \vec{\Sigma} - \vec{\Sigma}' - ({}^t\mathbf{IC}\vec{u} - {}^t\mathbf{IC}'\vec{u} + \vec{M} - \vec{M}')$$

By replacing $\vec{\Sigma}$ and $\vec{\Sigma}'$ with their value given by equations 6 and 6', we get :

$$\vec{P}_A = (\mathbf{IC}\vec{u} + \mathbf{Inv}\vec{u} + \Delta\vec{S} + \vec{C} + \vec{X}) - (\mathbf{IC'}\vec{u} + \mathbf{Inv'}\vec{u} + \Delta\vec{S}' + \vec{C}' + \vec{X}') - ({}^t\mathbf{IC} - {}^t\mathbf{IC}')\vec{u} - (\vec{M} - \vec{M}')$$

In grouping in an other way the different terms of this equation, we get :

$$\vec{P}_A = (\vec{C} - \vec{C}') + (\mathbf{IC} - \mathbf{IC'})\vec{u} - ({}^t\mathbf{IC} - {}^t\mathbf{IC}')\vec{u} + (\mathbf{Inv} - \mathbf{Inv'})\vec{u} + (\Delta\vec{S} - \Delta\vec{S}') - (\vec{M} - \vec{M}') + (\vec{X} - \vec{X}')$$

(eq. 13)

It appears that the acccounting-profit is composed of seven components, that we have put in brackets. Their analysis will enables us to understand the formation of profit and the transition from the accounting profit to the profit in income-value.

## 4. Analysis of the accounting profit. The formation of the profit. The corrections for getting a profit measured in income-value.

Let us analyse the different components of $\vec{P}_A$.

### 4.1. Analysis of the accounting profit.

4.1.1. Analysis of the first component $(\vec{C} - \vec{C}')$ : profit in income-value before any exchanges between industries.

It corresponds to the profit we had when there was only one firm. We noticed at that time that this profit was measured in income-value (since C - C' = I' - $S_m$, it was the part of investment I' measured in income-value and owned by the firm). Now that we have several industries, C and C' are divided between them and have become vectors.The selling prices of the totality of consumption is $C = {}^t\vec{u}\,\vec{C}$, its income-value is $C' = {}^t\vec{u}\,\vec{C}'$ and the value of the global profit is $\Pi = {}^t\vec{u}\,\vec{C} - {}^t\vec{u}\,\vec{C}'$. So the term $(\vec{C} - \vec{C}')$ gives exactly the same profit as the one we have calculated in 2.3 with the equation 5.

But with this first component we have not considered any exchanges between industries. So the distribution of the profit between industries given by $(\vec{C} - \vec{C}')$ is the distribution which exists at the begining of the flow back $\vec{C}$ from the households.

It is evident that further exchanges will modify the distribution.

4.1.2. Analysis of the second component $(\mathrm{IC} - \mathrm{IC'})\vec{u}$ : the values received by industries when selling intermediate consumption.

$\mathbf{IC}\vec{u}$ is the sum in lines of the matrix **IC**. So it represents the sales of the industries measured in selling prices. $\mathbf{IC'}\vec{u}$ represents the same sales but measured in income-value, so the difference $(\mathbf{IC} - \mathbf{IC'})\vec{u}$ represents the values gained by the industries when selling intermediate consumption.

<u>4.1.3. Analysis of the third component $({}^t\mathbf{IC} - {}^t\mathbf{IC'})\vec{u}$ : the values transfered by industries when buying intermediate consumption.</u>

$\vec{u}$ being "vertical", this third component is a "vertical" vector like $\vec{P_A}$. If we transpose it, we get the "horizontal" vector ${}^t\vec{u}\ (\mathbf{IC} - \mathbf{IC'})$. ${}^t\vec{u}\ \mathbf{IC}$ is the sum in columns of the matrix **IC**. So it represents the purchases of the industries measured in selling prices. ${}^t\vec{u}\ \mathbf{IC'}$ represents the same purchases but measured in income-value, then the difference ${}^t\vec{u}\,(\mathbf{IC} - \mathbf{IC'})$ represents the values transfered by industries when buying intermediate consumption. And its transpose, our third component, $({}^t\mathbf{IC} - {}^t\mathbf{IC'})\vec{u}$, represents the same values.

As the total of the purchases is equal to the total of the sales, the sum of the elements of the vector representing the former (second component) is equal to the sum of the elements of the vector representing the latter (third component) (cf §3.2.)

It involves that the addition of the second component of equation 13 and the substraction of the third component of the same equation to the first one $(\vec{C} - \vec{C}')$ does not change the total value $\Pi =$
$= {}^t\vec{u}\ (\vec{C} - \vec{C}')$ of the profit but only its distribution between the industries.

So if we consider only the first component, we have the distribution of the profit in income-value just as it is at the beginning of the flow back ; if we add to it the second and the third components of equation 13, we get always the distribution of the profit measured in income-value, but after the exchanges of intermediate consumption between the industries. The other components will introduce the effect of the sale of fixed capital, of the effect of the valuation of inventories and of foreign exchanges.

<u>4.1.4. Analysis of the fourth component $(\mathbf{Inv} - \mathbf{Inv'})\vec{u}$ : the values received by industries when selling fixed capital</u>

This term is similar to the second one if we replace **IC** by **Inv** and **IC'** by **Inv'.** So it represents the values gained by the industries when selling fixed capital.

We notice that there is no term similar to the third one, i.e. there is no term representing the values transfered by industries when buying fixed capital. That comes from the way these purchases are accounted : for the business accounts, the purchase of fixed capital is not an item of the PLA, but an asset of the balance-sheet, so it is absolutely neutral on the gross profit[10]. The national accounts follow the same way, since the formation of capital appear in a capital account after the saving of the firms has been calculated.

At the opposite the selling of an item of fixed capital is a current operation which increases the profit and this increase is measured by the fourth component of equation 13.

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[10]of course this purchase diminushes the net profit, but this depreciation is charged on several years.

So the sum of the first four components of $\vec{P}_A$ gives a profit larger than the profit measured in income-value. The adjunction of the fifth component will increase the profit still more.

4.1.5. Analysis of the fifth component $\vec{\Delta S} - \vec{\Delta S}'$ : the excess of the accounting measure ot the changes in inventories on their measure in income-value.

At the end of the accounting period the value of the changes in inventories is accounted as if it were an income (in the PLA for the business accounts and in the production account for the SNA). As there is no sale the valuation may be difficult and the valuation methods are different in business accounts and in the SNA[11]. But any-way this valuation is larger than the valuation in income-value and it increases the profit above its income-value once more.

The last two components concern only the foreign exchanges.

4.1.6. Analysis of the sixth and seventh components $(\vec{M} - \vec{M}')$ and $(\vec{X} - \vec{X}')$ : net profits on the foreign exchanges

When selling abroad for $\vec{X}$ goods of which income-value is $\vec{X}'$, the home firms realize an extra profit of $\vec{X} - \vec{X}'$ in income-value. But in buying goods at the prices $\vec{M}$, whereas the income-value is $\vec{M}'$, they transfer a part of profit to foreign firms.

**4.2. The formation of the profit.**

The analysis of the accounting profit enables us to explain the profit formation. If we are in a closed econmy, i.e. if we neglect the sixth and seventh components of equation 13, the profit appears first in income-value $(\vec{C} - \vec{C}')$ with the relations between industries and consumers. This first exchange cannot create any new value, since the production has already occured during the phase 1 of the circuit and that we are now in the phase 2 (distribution). So this first exchange involves only a transfer of value from households to industries selling consumption goods. Then the exchanges between the industries themselves either of fixed or circulating capital can occur. It is evident that these exchanges cannot, like the first ones, create any new value, but modify the distribution of profit capted by the first exchange between the different industries : the buyer-industries transfer profits to the seller-industries. The business and national accounts record that fact very well for the circulating capital since they include the second component of $\vec{P}_A$ (values received by industries when selling intermediate consumption) and the third one (values transfered by industries when buying intermediate consumption). But they include only one component for the fixed capital : the fourth one (values received) and they have no element for the values transfered by purchase of capital. So it appears that the accounts give a measure of the profit above its income-value. The valuation appears further above when we add the fifth component of $\vec{P}_A$ concerning the valuation of the changes inventories beyond any exchanges. As we have already seen in §3.2., the total valuation got is not arbitrary but in selling prices.

This analysis shows us the way to get a vector of profit measured in income-value.

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[11]normally the valuation is done with the cost prices for the former, and with selling prices for the latter (cf. 3.2.)

### 4.3. The corrections to be done to get a profit measured in income-value

The analysis we have just done shows us the two corrections to be managed.

1. The fifth component does not correspond to any flow, to any part of the circulation of money leading to the formation of profit : it is only a way of calculus. In business, changes in inventories are just accounted at the end of the year. So this component must be suppressed.

2. One must put a supplementary component giving the transfer of value supported by firms when buying fixed capital. These tranfers are given by the sums of the lines of the matrix **Inv** - **Inv'**, i.e. the "horizontal" vector : ${}^t\vec{u}$ (**Inv** - **Inv'**) or the "vertical" vector (its transpose) : (${}^t$**Inv** - ${}^t$**Inv'**) $\vec{u}$. We will call this new element "fourth A".

After these two corrections we get a profit :

$$\vec{P}_V = (\vec{C} - \vec{C}') + (\mathbf{IC} - \mathbf{IC'})\vec{u} - ({}^t\mathbf{IC} - {}^t\mathbf{IC'})\vec{u} + (\mathbf{Inv} - \mathbf{Inv'})\vec{u} - ({}^t\mathbf{Inv} - {}^t\mathbf{Inv'})\vec{u} - (\vec{M} - \vec{M}') + (\vec{X} - \vec{X}')$$

(eq. 14)

For the same reasons (economic and mathematic) as for the second and third components (cf. §§ 3.2. and 4.1.3.), the sum of the elements of the fourth component is equal to the sum of those of the fourth A component, thus we get a profit of which the global value is equal to :

$$P_V = {}^t\vec{u}\ \vec{P}_V = {}^t\vec{u}(\vec{C} - \vec{C}') - {}^t\vec{u}\ (\vec{M} - \vec{M}') + {}^t\vec{u}\ (\vec{X} - \vec{X}')$$

Or $P_V = (C - C') - (M - M') + (X - X')$

That confirms us $\vec{P}_V$ is an income-value measure of profit.

An other way to verify it, it to see that the difference between the measures in selling prices (${}^t$**Inv**$\vec{u}$ + $\Delta\vec{S}$) and in income-value (${}^t$**Inv'**$\vec{u}$ + $\Delta\vec{S}'$) of the assets is exactly the sum of the two corrections we suggest, i.e :

$$({}^t\mathbf{Inv}\vec{u} + \Delta\vec{S}) - ({}^t\mathbf{Inv'}\vec{u} + \Delta\vec{S}') = ({}^t\mathbf{Inv} - {}^t\mathbf{Inv'})\vec{u} + (\Delta\vec{S} - \Delta\vec{S}') = \vec{P}_A - \vec{P}_V$$

## 5. Application to the French economy for the year 2007

We give in annex a table giving the calculus of the profit in accounting-value and in income-value for an I.O.T. in 41 industries. Instead of the whole income paid by firms to households we have only include the wages, so we get profits measured in wage-values not in income-value, but this method is perfectly well-founded as we will see in §6.

The table 2 gives the list of the products and activities (in French), while the table 3 gives the different components of the profit for each activity.

One can observe that:

- the totals of the 6th (2nd component, profit earned from saling intemediate consumption) and of the 7th columns (3rd component, profit lost when buying intemediate consumption) are equal;

- the totals of the 8th (4th component, profit earned from saling investment) and of the 9th colums (4th A component, profit lost when buying investment) are equal;

- the totals of the 5th (1st component, profit earned before any exchange between the activities) and of the penultimate columns (profit in value except from the rest of the world) are equal.

### 6. Why it is necessary to build a sna in icome value on a world level

In order to get the equation 11 we have supposed that for each imported product the coefficient ki was the same as the coefficient of the similar domestic product. This hypothesis is absolutely unrealistic: to get correct results one must take the true coefficient for each imported product.

To calculate these coefficients one must build a SNA on a word level.

Let us suppose that the world is divided in m countries (or in m areas grouping countries of similar structure). Each area of the world is called by a letter from a to m. The exports from the area d (d varying from a to m) evaluated in prices can be represented by a matrix $\mathbf{Exp_d}$ of n lines (one by product) and (m-1) columns (one by area receiving the exports).

For the area d the "horizontal" equation 6 becomes:

$\vec{\Sigma}_d = \mathbf{IC_d}\,\vec{u} + \vec{C}_d + \mathbf{Inv_d}\,\vec{u} + \overrightarrow{\Delta S}_d + \mathbf{exp_d}\,\vec{v}$ in which $\vec{v}$ is the "vertical" unit vector of m- 1 elements.

The imports in prices to the country d can be represented by a matrix $\mathbf{Imp_d}$ of (m-1) lines (one for each exporting area and n columns. Let us notice that each matrix $\mathbf{Imp_d}$ can be derived from the m matrices $\mathbf{Exp_d}$ since the imports to one area are the exports from the other.

The "vertical" equation 7 of the area d becomes:

$$^t\vec{u}\,\mathbf{IC_d} + {}^t\overrightarrow{VA}_d + {}^t\vec{v}\,\mathbf{Imp_d} \;=\; {}^t\vec{\Sigma}_d$$

and the equation 7' using income-values becomes:

$$^t\vec{u}\,\mathbf{IC'_d} + {}^t\overrightarrow{VA'}_d + {}^t\vec{v}\,\mathbf{Imp'_d} \;=\; {}^t\vec{\Sigma'}_d$$

The way from $\mathbf{IC_d}$ to $\mathbf{IC'_d}$ and from ${}^t\vec{\Sigma}_d$ to ${}^t\vec{\Sigma'}_d$ is always through the diagonal matrix $\mathbf{K}$ whose elements are only the n unknowns $k_i$ of the area d. Besides to get matrix $\mathbf{Imp'_d}$ from matrix $\mathbf{Imp_d}$, one must multiply each line of $\mathbf{Imp_d}$ by the line of $k_i$ relative to the area exporting the products.

There are nm unknowns $k_{id}$ for n products and m areas. There are m matrix equations 7' (one for each area) and each of them represents n algebrical equations (one by product), so there are nm algebrical equations for nm algebrical unknowns.

## 7. Conclusion : a Keyneso-Marxian "detransformation" theory

In this paper we have built the values from the prices: the prices are the data, while the values are the result of a calculus done by the economist. In her preface of her *Essay on Marxian economy*[12], Joan Robinson noted that *"Transformation occurs from prices to values and not in reverse way"* , so it is a kind of "detransformation". At the opposite Marx thought that the labour-value was a natural value and that a natural process will transform these values in prices. This natural process supposes two hypotheses : 1. there exists between the entrepreneurs a competition which will lead them to move capital in order to get the same rate of profit ; 2. to do this calculus, the entrepreneurs have to know the labour-values.

But in the facts the mobility of capital is not perfect, and the rates of profit of the different industries may differ. Besides Bortkiewicz has shown that the entrepreneurs cannot use values to determine prices and Sraffa has built a theory of prices depending on the production process but not on the values.

Indeed entrepreneurs have no knowledge of values and cannot calculate them. Now to calculate values one need a complete knowledge of the economy which can be given only by an IOT. Further in a global economy one need a knowledge of the world economy. In the facts private accountants have very well understood the problem when they consolidate the accounts of a group of firms. To do this consolidation they must have a perfect knowledge of each firm consolidated, and they have it, but they have no knowledge of the firms outside the group, so they consolidate only the firms of the group. To do it, they eliminate the profits made by each firm of the group on the other and so introduce the equivalent of our "fourth A component" and get of profit measured in value. So what we want to do is only a consolidation but on the national level, even on the world level.

We have exposed the algorithm to calculate the income-value, i.e. the value in the monetary revenue going from the firms to the households. But this algorithm may be used for any kind of value, e.g. wage, labour-time, energy-units, carbon footprint, etc... and in our §6 we have used the wage value, i. e. in $\vec{C}\,'$ and $^t\overrightarrow{VA}\,'$ we have only included the wages paid by each industry and from this value we have succeeded to calculate a profit which is perfectly agregeable. Indeed the profit to be agregeable has only to be always measured in the same system of measure, which is not the case in the business accounting system for which the prices depend on the degree of consolidation.

[12]second edition of Essay *on Marx's economy,* p. XIII

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| | |
|---|---|
| AGRICULTURE SYLVICULTURE PECHE | FA0 |
| INDUSTRIES DE LA VIANDE ET DU LAIT | FB1 |
| AUTRES INDUSTRIES AGRICOLES ET ALIMENTAIRES | FB2 |
| HABILLEMENT CUIR | FC1 |
| EDITION IMPRIMERIE REPRODUCTION | FC2 |
| PHARMACIE PARFUMERIE ENTRETIEN | FC3 |
| INDUSTRIES DES EQUIPEMENTS DU FOYER | FC4 |
| INDUSTRIE AUTOMOBILE | FD0 |
| CONSTRUCTION NAVALE AERONAUTIQUE ET FERROVIAIRE | FE1 |
| INDUSTRIES DES BIENS D EQUIPEMENTS MECANIQUES | FE2 |
| INDUSTRIES DES EQUIPEMENTS ELECTRIQUES ET ELECTRONIQUES | FE3 |
| INDUSTRIES DES PRODUITS MINERAUX | FF1 |
| INDUSTRIE TEXTILE | FF2 |
| INDUSTRIES DU BOIS ET DU PAPIER | FF3 |
| CHIMIE CAOUTCHOUC PLASTIQUES | FF4 |
| METALLURGIE ET TRANSFORMATION DES METAUX | FF5 |
| INDUSTRIE DES COMPOSANTS ELECTRIQUES ET ELECTRONIQUES | FF6 |
| PRODUCTION DE COMBUSTIBLES ET DE CARBURANTS | FG1 |
| EAU GAZ ELECTRICITE | FG2 |
| BATIMENT | FH1 |
| TRAVAUX PUBLICS | FH2 |
| COMMERCE ET REPARATION AUTOMOBILE | FJ1 |
| COMMERCE DE GROS INTERMEDIAIRES | FJ2 |
| COMMERCE DE DETAIL ET REPARATIONS | FJ3 |
| TRANSPORTS | FK0 |
| INTERMEDIATION FINANCIERE | FL1 |
| ASSURANCES ET AUXILIAIRES FINANCIERS | FL2 |
| PROMOTION GESTION IMMOBILIERE | FM1 |
| LOCATION IMMOBILIERE | FM2 |
| POSTES ET TELECOMMUNICATIONS | FN1 |
| CONSEILS ET ASSISTANCE | FN2 |
| SERVICES OPERATIONNELS | FN3 |
| RECHERCHE ET DEVELOPPEMENT | FN4 |
| HOTELS ET RESTAURANTS | FP1 |
| ACTIVITES RECREATIVES CULTURELLES ET SPORTIVES | FP2 |
| SERVICES PERSONNELS ET DOMESTIQUES | FP3 |
| EDUCATION | FQ1 |
| SANTE | FQ2 |
| ACTION SOCIALE | FQ3 |
| ADMINISTRATION PUBLIQUE | FR1 |
| ACTIVITES ASSOCIATIVES | FR2 |

Table 2
List of products and activities with their code

| industry | coefficient k | vector C | vector C' | profit earned before the exchanges between activities | profit earned when saling IC | profit lost when buying IC | profit earned from saling I | profit lost when buying IC | profit earned from inventories | profits earned from exports | profits lost when importing | final profit in value except from the rest of the world | final accounting profit except from the rest of the world |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| FA0 | 0,4 | 31 | 12,48 | 18,52 | 33,46 | 24 | 1,19 | 4,82 | 3,58 | 7,17 | 5,17 | 24,35 | 32,75 |
| FB1 | 0,45 | 58 | 26,07 | 31,93 | 9,36 | 21,62 | 0 | 0,97 | 0 | 4,95 | 2,95 | 18,69 | 19,66 |
| FB2 | 0,44 | 91 | 39,73 | 51,27 | 28,73 | 28,45 | 0 | 2,45 | 0 | 14,65 | 9,65 | 49,1 | 51,55 |
| FC1 | 0,46 | 42 | 19,4 | 22,6 | 1,08 | 2,99 | 0 | 0,33 | 0 | 6,46 | 13,46 | 20,35 | 20,68 |
| FC2 | 0,63 | 13 | 8,24 | 4,76 | 8,43 | 7,26 | 0 | 0,78 | 0 | 0,73 | 1,73 | 5,15 | 5,93 |
| FC3 | 0,51 | 54 | 27,49 | 26,51 | 10,8 | 18,49 | 0 | 0,79 | 0 | 16,2 | 4,2 | 18,03 | 18,82 |
| FC4 | 0,58 | 57 | 32,92 | 24,08 | 3,38 | 6,56 | 2,53 | 0,19 | 0 | 5,91 | 19,91 | 23,24 | 23,43 |
| FD0 | 0,59 | 61 | 36,28 | 24,72 | 15,4 | 31,91 | 10,54 | 2,3 | 0,41 | 20,67 | 19,67 | 16,46 | 19,16 |
| FE1 | 0,56 | 5 | 2,8 | 2,2 | 15,37 | 18,91 | 2,63 | 0,63 | 0,44 | 14,93 | 0,93 | 0,66 | 1,73 |
| FE2 | 0,62 | 4 | 2,48 | 1,52 | 21,67 | 21,55 | 15,58 | 1,2 | 0 | 14,06 | 13,06 | 16,02 | 17,22 |
| FE3 | 0,62 | 14 | 8,69 | 5,31 | 12,13 | 9,03 | 7,96 | 1,26 | 0 | 9,86 | 19,86 | 15,12 | 16,37 |
| FF1 | 0,55 | 4 | 2,19 | 1,81 | 20,36 | 8,93 | 0 | 0,86 | 0 | 2,71 | 5,71 | 12,38 | 13,24 |
| FF2 | 0,47 | 9 | 4,19 | 4,81 | 4,81 | 3,41 | 0 | 0,81 | 0 | 3,2 | 5,2 | 5,39 | 6,2 |
| FF3 | 0,51 | 6 | 3,07 | 2,93 | 17,58 | 10,61 | 0 | 1,65 | 0 | 4,4 | 8,4 | 8,26 | 9,91 |
| FF4 | 0,55 | 11 | 6,1 | 4,9 | 42,28 | 27,94 | 0 | 1,68 | 0,45 | 20,92 | 23,92 | 17,56 | 19,68 |
| FF5 | 0,58 | 5 | 2,92 | 2,08 | 43,61 | 28,45 | 1,66 | 1,58 | 0 | 14,95 | 18,95 | 17,31 | 18,89 |
| FF6 | 0,62 | 5 | 3,08 | 1,92 | 10,35 | 8,65 | 1,15 | 0,28 | 0 | 8,05 | 5,05 | 4,49 | 4,77 |
| FG1 | 0,44 | 44 | 19,48 | 24,52 | 61,29 | 27,32 | 0 | 1,03 | -0,56 | 8,91 | 55,91 | 57,46 | 57,93 |
| FG2 | 0,44 | 32 | 14,08 | 17,92 | 28 | 25,68 | 0 | 5,8 | 0 | 1,12 | 0,12 | 14,44 | 20,24 |
| FH1 | 0,54 | 97 | 52 | 45 | 19,48 | 46,46 | 32,47 | 1,67 | 0,46 | 0 | 0 | 48,82 | 50,96 |
| FH2 | 0,59 | 3 | 1,78 | 1,22 | 4,46 | 14,3 | 17,43 | 1,67 | 0 | 0 | 0 | 7,14 | 8,81 |
| FJ1 | 0,54 | 19 | 10,19 | 8,81 | 0,93 | 7,33 | 0 | 0,84 | 0 | 0 | 0 | 1,56 | 2,41 |
| FJ2 | 0,6 | 0 | 0 | 0 | 11,92 | 46,34 | 0 | 11,56 | 0 | 1,19 | 3,19 | -45,98 | -34,42 |
| FJ3 | 0,55 | 3 | 1,65 | 1,35 | 0 | 20,29 | 0 | 11,77 | 0 | 0 | 0 | -30,71 | -18,94 |
| FK0 | 0,56 | 32 | 18 | 14 | 32,82 | 37,02 | 0 | 11,5 | 0 | 9,19 | 2,19 | -1,69 | 9,81 |
| FL1 | 0,61 | 22 | 13,49 | 8,51 | 27,48 | 22,31 | 0 | 0,22 | 0 | 1,94 | -1,06 | 13,46 | 13,68 |
| FL2 | 0,53 | 37 | 19,47 | 17,53 | 23,22 | 21,38 | 0 | 0,05 | 0 | 0,47 | 0,47 | 19,32 | 19,37 |
| FM1 | 0,44 | 16 | 6,97 | 9,03 | 7,34 | 7,45 | 6,77 | 4,21 | 0 | 0 | 0 | 11,49 | 15,69 |
| FM2 | 0,08 | 207 | 17,38 | 189,62 | 45,8 | 17,09 | 0 | 2,4 | 0 | 0 | 0 | 215,93 | 218,34 |
| FN1 | 0,51 | 27 | 13,77 | 13,23 | 21,06 | 14,82 | 0 | 2,78 | 0 | 1,47 | 0,47 | 16,68 | 19,47 |
| FN2 | 0,59 | 26 | 15,47 | 10,53 | 84,25 | 52,53 | 21,47 | 3,04 | 0 | 5,27 | 7,27 | 60,68 | 63,72 |
| FN3 | 0,58 | 14 | 8,1 | 5,9 | 80,04 | 34,89 | 0 | 2,41 | 0 | 4,21 | 6,21 | 48,65 | 51,06 |
| FN4 | 0,72 | 9 | 6,45 | 2,55 | 6,51 | 8,48 | 0 | 2,31 | 0 | 1,13 | -0,87 | -1,73 | 0,58 |
| FP1 | 0,52 | 67 | 35,08 | 31,92 | 9,05 | 21,02 | 0 | 8,22 | 0 | 0 | 0 | 11,73 | 19,95 |
| FP2 | 0,61 | 55 | 33,6 | 21,4 | 5,06 | 16,07 | 1,17 | 3,99 | 0 | 0,78 | 0,78 | 7,57 | 11,56 |
| FP3 | 0,66 | 22 | 14,51 | 7,49 | 0,68 | 1,17 | 0 | 4,59 | 0 | 0 | 0 | 2,41 | 7 |
| FQ1 | 0,82 | 96 | 78,37 | 17,63 | 1,1 | 7,65 | 0 | 3,99 | 0 | 0 | 0 | 7,1 | 11,08 |
| FQ2 | 0,55 | 120 | 66,55 | 53,45 | 1,78 | 15,01 | 0 | 3,99 | 0 | 0,45 | -0,55 | 36,23 | 40,22 |
| FQ3 | 0,81 | 61 | 49,31 | 11,69 | 0 | 4,69 | 0 | 3,99 | 0 | 0 | 0 | 3,01 | 7 |
| FR1 | 0,79 | 157 | 124,18 | 32,82 | 0 | 21,82 | 0 | 3,99 | 0 | 0 | 0 | 7,01 | 11 |
| FR2 | 0,73 | 5 | 3,63 | 1,37 | 0 | 1,19 | 0 | 3,99 | 0 | 0 | 0 | -3,81 | 0,18 |
| totaux | | 1641 | 861,68 | 779,32 | 771,07 | 771,07 | 122,57 | 122,57 | 4,78 | 205,96 | 251,96 | 779,32 | 906,68 |

Table 3
Formation of the profits of the different activities