# The Circuit of Bank Capital and the Possibility of Crisis: from the Perspective of Marxian Economic Theory

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**Abstract**

The aim of this paper is to investigate how bank capital moves with changing its forms and why it potentially triggers the crisis. In section 1, I argue the modified formulation of usurer's capital which is the theoretical background of bank capital. In section 2, I introduce two typical views of bank capital to formalize the motion of bank capital. In section 3, I formulate the motion of bank capital in order to argue the relationship between the circuit of bank capital and the possibility of crisis. We point out 3 possibilities; over-conversion, the failure of "credit-creation," and the failure of "credit-realization" on the basis of the mathematical formulation.

## 1 Modification of the Formula of the Usurer's Capital

Capital — It is an ongoing process oriented to the expansion of its value, passing through the circuit of its metamorphoses. It is noteworthy that it has two crucial conditions in this process. One is the expansion of the value, and the other is the metamorphosis. The formula of merchant capital, M — C — M′, gives us a typical example. The expansion of the value is represented as the positive gap between both poles, M and M′. In other words, the motion of capital has the aim that $\Delta M = M' - M$ should be created and maximized. In addition, it is necessary that the motion of capital is changing the forms.

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In the case of the merchant capital, it is constituted by both buying in order to sell dearer, M – C, and selling in order to reinvent, C — M′. The formula of the merchant capital, M — C — M′, is one phase of the everlasting processes in which the form of money M and the form of commodity C should appear alternately.

It is difficult to admit that the formula of usurer's capital should be one of the formulas of capital if we follow the above definition of the capital. Because Marx viewed the formula of usurer's capital as M — M′; it lends its own money M, and it collects more money M′. Indeed it is clear that usurer's capital increases its initial value M to M′. However there is no entity without money in the process. In other words, The form of the usurer's capitals have unchanged in the whole lifetime. It should be concluded that this formulation does not fulfill the condition of the metamorphosis, though it fulfills the expansion of the value. Does this mean that the motion of the usurer's capital is not one of the formulas of capital? Our answer is, of course, negative.

Susumu Hidaka, who is one of the famous unoist economists in Japan, criticized Marx's formulation and insisted the modified formalization of usurer's capital.[1] He proposed that the modified formula for usurer's capital should be;

$$\mathrm{M} — \mathrm{B} — \mathrm{M}' \tag{1}$$

instead of instead of M — M′, where B is bill. It lends its own money in exchange for bill B in the first phase: M — B. It claims the right to collect more money in the second phase: B — M′. Marx's formulation, M — M′, is interpreted as an "abridged form" (Marx (1990) p. 257) of the modified formula M — B — M′ [2].

Marx noticed the existence of bill in the motion of the usurer's capital even though he did not formalized it with the motion of the bill. It is well exemplified by the following quotation from *Capital* Volume III.

> *Prima facie*, loan capital always exists in the form of money, later as a claim to money (Marx (1991) p.641-642)

Usurer's capital takes a form of money, M, in the first position of the modified formula. In the second position of the modified formula, it takes a claim of money, B. Bill is a promise to pay from the point of payer's view,

[1]See Hidaka (1983).

[2]We can interpret the formula M — M′ as the form that the part of "— B —" is abbreviated and displaced by " — ."

but at the same time, it is a claim of money from the point of payee's view. Therefore, the modified formula is consistent with Marx's notion.

We admit Hidaka's proposal and treat the formation of usurer's capital as M — B — M′.

# 2 A Formulation of the Bank Capital

## 2.1 Two Views of the Bank Capital

It is undoubted that the bank capital is a variant form of the usurer's capital, but there are at least two formulations of the bank capital. One is Deposit-Comes-First (DCF), and the other is Issue-Comes-First (ICF). Let us explain in turn.

The first formulation is called Deposit-Comes-First (DCF). According to DCF, the bank capital behaves as to "borrow low and lend high." It collects cash in deposit from some industrial capitalists at the lower deposit interest rate. At the same time, it lends the borrowed money to other industrial capitals at the higher loan interest rate. It takes the roll to concentrate idle money reserves and place them on the money market.

The motion of the bank capital in DCF can be formulated as;

$$-\mathrm{D} — \mathrm{M} — \mathrm{B} — \mathrm{M}' — -\mathrm{D}'. \tag{2}$$

D is deposit and the sign of minus indicates that it is liabilities in the balance sheet of the bank capital. The phase of $-$D — M indicates that bankers offer the right to demand, and they receive in exchange to the money in which industrial capitalists handed. The next processes of M — B — M′ are the same activities of the usurer's capital. M′ — $-$D in the final phase indicates that bankers pay the interest from their profit to the industrial capitalists. It is clearly a variant of usurer's capital M — B — M′ because it's embedded in the DCF's formulation.

On the other side, the second view, the so-called Issue-Comes-First (ICF), criticizes DCF. According to ICF, the bank in the 19th century mainly operated to issue its own bank notes in exchange to the bill issued by the industrial capital so that it could create credit. The important point to note is that the bank never lend the cash of its own or the cash borrowed from others. However, DCF assume explicitly that banks lend the borrowed cash. That is the point that ICF can not accept DCF.

The simplest formulation of the bank capital in DCF can be written as;

$$-\mathrm{N} — \mathrm{B} — -\mathrm{N}', \tag{3}$$

where N is bank note. The bank capital moves as $-$N — B in the first phase of its circuit, say loan, because it issues the bank note which is substituted by a banker for the private bill, B. In the next phase, it collects the bank note N$'$ in exchange for the private bill, in which the bank handed in. In this case, the bank note can take the role of money. This formulation can be interpreted as the same formula of usurer's capital, because the bank note, which is the debt of the bank, $-$N, is substitutes of money, M.

Indeed both camps have a weak point. ICF is criticized by DCF because it does not treat money. But DCF is criticized because it does not treat credit. There are fundamental differences of opinion between two camps. However, we try to establish a synthesis between DCF and ICF. In order to do so, let me point out what is the same and difference between DCF and ICF.

## 2.2 A Modified Formulation of the Bank Capital

At the first glance, the difference is in the first position in the both formulae; the deposit comes first in DCF and the issue of the bank note comes first in ICF. But it doesn't make much difference whether the starting point is D or N, because both formulae start from the sign of the "$-$." It means that both start from a debt account of the bank capital. A bold theoretical abstraction can allow us to identified $-$D and $-$N because both are the debt of the bank and take the role of substitutes of money. Therefore, let us N be replaced by D, which means the debt, and deposit's D can also be interpreted as the debt in the balance sheet of the bank capital. Let us compare two formulations.

$$
\begin{array}{l}
-\mathrm{D} — \mathrm{M} — \mathrm{B} — \mathrm{M}' — -\mathrm{D}' \\
-\mathrm{D} \qquad\quad — \mathrm{B} — \qquad\quad -\mathrm{D}'
\end{array}
$$

According to the above comparison, it may be clear that another difference is whether the money, M, exists or not. But this is not the case, because ICF can easily incorporate the element of money as follows,

$$-\mathrm{D} — \mathrm{B} — \mathrm{M}' — -\mathrm{D}'. \tag{4}$$

Let us explain in turn. In the first phase ,$-$D — B, the bank capital accepts the bill of the industrial capital by crediting the amount into the bank account of the industrial capital. In this phase, the bank lend the money substitutes through the discount of the bill. It doesn't matter whether D or I is in the first position. In the next phase, B — M$'$, the bank collects the cash in exchange for the bill. The bank doesn't care about which money or money substitutes are collected. The important thing is whether the amount of money or money substitutes exceed the initial amount of the debt. In the final phase, the bank pays out money if it is demanded to convert deposit/note into cash. This phase is the same behavior of the final phase of DCF.[3] Therefore, ICF can corporate the money into the formula.

We can reach the stage to point out the crucial difference between DCF and ICF. The point is whether the bank capital lend the money or lend the debt. In other words, the difference is whether the process of −D — M — B is admitted or not. There is no any other differences between DCF and ICF.

We reject the process of the −D — M — B, not only because it admits the lend of the borrowed money, but also because it can not treat the credit economy. Therefore, we adapt the modified formulation (4) as the circuit of bank capital.

# 3 A mathematical formulation of the Bank Capital

In this section, I introduce a mathematical formulation of the bank capital in order to argue the relationship between the circuit of bank capital and the possibility of crisis.

## 3.1 A Diagrammatic Form of the Bank Capital

The whole motion of the bank capital is illustrated as Figure 1.

$$-D \xrightarrow[-d^{\mathrm{o}}]{+b^{\mathrm{i}}} B \xrightarrow[-b^{\mathrm{o}}]{+m^{\mathrm{i}}} M' \xrightarrow[-m^{\mathrm{o}}]{+d^{\mathrm{i}}} -D'$$

Phase 1 Phase 2 Phase 3

Figure 1: A Diagrammatic Form of the Bank Capital

The phase 1 represents the phase of loan, in which the debt of the bank, $-D$, is transformed into the bill, $B$, in which the industrial capital handed.

[3]For the simplicity, we ignore the saving account in the bank account and take the cash account only into consideration.

The bank makes the own debt in order to obtains the discounted bill. Let the total amount of the inflow of the discounted bill denoted by $(1+i)^{-1}b^{\mathrm{i}}$, where $i$ is the interest rate and $b^{\mathrm{i}}$ is the bill inflow. It is exchanged for the debt outflow, $-d^{\mathrm{o}}$. In Figure 1, the element above the arrow represents the amount of flow increasing the right-side stock, and the element below the arrow represents the amount of flow decreasing the left-side stock[4]. Naturally, the sum of the two elements above and below the arrow must equal zero. We call this rule "bookkeeping rule."

The phase 2 represents the phase of collection in which the bill $B$ is transformed into cash, $M'$. In this process, the bank claims the right to call in loans. Let $-b^{\mathrm{o}}$ be the outflow of the bill, and $m^{\mathrm{i}}$ be the amount of the collection of claim in the money form.

In the phase 3, Cash $M'$ is transformed into the debt $D'$. We call this phase conversion[5]. Let $-m^{\mathrm{o}}$ be the outflow of the cash, and the amount of the debt inflow is $d^{\mathrm{i}}$. This process makes the debt disappear. And then the circuit of bank capital will restart without stopping.

In next subsection we will derive mathematical expressions of bank capital circuit from Figure 1.

## 3.2 A mathematical form of the Bank Capital

### 3.2.1 Flow Relations of the Bank Capital

At first, we investigate how the stocks of bank capital increase in the circuit. The Stocks in the circuit, $-D$, $B$ and $M$, are governed by the following rule:

$$
\begin{aligned}
\dot{B}(t) &= b^{\mathrm{i}}(t) - b^{\mathrm{o}}(t), \\
\dot{M}(t) &= m^{\mathrm{i}}(t) - m^{\mathrm{o}}(t), \\
-\dot{D}(t) &= d^{\mathrm{i}}(t) - d^{\mathrm{o}}(t).
\end{aligned} \tag{5}
$$

These equations (5) follow from the bookkeeping rule. They represent that each capital is increased by the inflow and decreased by the outflow, and then each capitals is accumulated when the inflow is more than the outflow. If we define the self-owned capital of the bank as $K(t) \equiv M(t) + B(t) - D(t)$,

[4]Note that the debt in the balance sheet of the bank capital is the quantity of minus, because it is not asset. The negative number minus the negative number equal the negative number, and the absolute value is increasing.

[5]This name is clear in case of the convertible gold note, though it may be misleading in the case that the debt is deposit. We ignore saving accounts, and assume that there are only checking accounts open. And then we identify the process of the conversion as the drawing process.

then the increment of capital is denoted by $\dot{K}(t) = \dot{M}(t) + \dot{B}(t) - \dot{D}(t)$. By (5) and bookkeeping rule, we can easily get:

$$\dot{K}(t) = \dot{M}(t) + \dot{B}(t) - \dot{D}(t) = id^{\mathrm{o}}(t). \tag{6}$$

In short, the increment of bank capital is the interest revenue through the discount process. At the same time, this is also the profit of the bank capital because there are no wage, dividend and interest payment.[6] In this case, the profit rate, defined by $\pi$, is as follows.[7]

$$\pi = g. \tag{7}$$

In other words, this growth path is nothing more than Von-Neumann process. Therefore, the existence of profit is also the condition of the positive growth rate. The existence of increment of bank capital depends on the positive growth rate.

### 3.2.2 Stock Relations of the Bank Capital

In the previous subsection, we evaluate the increasing amount of the total volume of bank capital, $\dot{K}(t)$. In this subsection, we evaluate the amount of the total volume of bank capital, $K(t)$. The inflow and outflow are related by the convolution:

$$\begin{aligned} b^{\mathrm{o}}(t) &= \int_{-\infty}^{t} b^{\mathrm{i}}(t')\beta(t-t')\,\mathrm{d}t', \\ m^{\mathrm{o}}(t) &= \int_{-\infty}^{t} m^{\mathrm{i}}(t')\mu(t-t')\,\mathrm{d}t', \\ d^{\mathrm{i}}(t) &= \int_{-\infty}^{t} d^{\mathrm{o}}(t')\delta(t-t')\,\mathrm{d}t'. \end{aligned} \tag{8}$$

$\beta$ represents a distributed lag in the process of collection, interpreted as the proportion of bill inflow at time $t$, that are collected at time $t+t'$. $\mu$ is a distributed lag in the process of conversion, interpreted as the proportion of money inflow at time $t$, that are converted at time $t+t'$. $\delta$ is also a distributed lag in the process from the birth to death of the bank debt, which is also interpreted as the proportion of debt outflow set by loan at time $t$, which are disappeared to be withdrawn at time $t+t'$. $\beta$, $\mu$ and $\delta$ are nonnegative and integrate to 1 over the positive half-line.

[6]These are assumptions which we can relax.

[7]Note that the growth rate $g$ is defined as $\dot{K}(t)/K(t)$,

Under the stationary state, the initial conditions must satisfy the following equations.[8] Reasoning from three equations (8),

$$\begin{aligned} b^{\mathrm{o}} &= b^{\mathrm{i}}\beta^*(g), \\ m^{\mathrm{o}} &= m^{\mathrm{i}}\mu^*(g), \\ d^{\mathrm{i}} &= d^{\mathrm{o}}\delta^*(g), \end{aligned} \tag{9}$$

where

$$\beta^*(g) = \int_0^\infty \beta(t)\exp(-gt)\,dt$$

which is the Laplace transform of the lag function $\beta(.)$ and similarly for $\mu^*(g)$ and $\delta^*(g)$. The Laplace transform has specific properties as follows:$\beta^*(0) = 1$, $\mathrm{d}\beta^*(g)/\mathrm{d}g < 0$, $\lim_{g\to\infty}\beta^*(g) = 0$.

From these equations, we can drive the stock variables: $B$, $M$, and $-D$. Noting that all stock variables grow at the rate of $g$, and substituting (9) to (5), we get:

$$\begin{aligned} B &= d^{\mathrm{o}}\big(1-\beta^*(g)\big)\delta^*(g)/g\beta^*(g)\delta^*(g), \\ M &= d^{\mathrm{o}}\big(1-\gamma^*(g)\big)\delta^*(g)/g\gamma^*(g), \\ -D &= d^{\mathrm{o}}\big(\delta^*(g)-1\big)/g. \end{aligned} \tag{10}$$

Summing up (10), then we get

$$K = B + G - D = d^{\mathrm{o}}\big(\delta^*(g) - \beta^*(g)\gamma^*(g)\big)/g\beta^*(g)\gamma^*(g). \tag{11}$$

Substituting the bank profit (6) and the total capital (11) to the definition of profit rate $\pi = \Pi/K$, then we get:

$$\pi = i\big(\delta^*(g) - \beta^*(g)\gamma^*(g)\big)/g\beta^*(g)\gamma^*(g). \tag{12}$$

#### 3.2.3 Existence of the Positive Growth Rate

We have two expression of the profit rate; (7) and (12), which are solved as a set of simultaneous equations. By (7) and (12), we get:

$$\frac{1}{1+i} = \beta^*(g)\gamma^*(g)\delta^*(g)^{-1}. \tag{13}$$

[8]We omit the initial time subscript such as $x(0) = x$.

This is the characteristic equation of this system. This system has a positive unique solution $g^*$ if

$$\frac{\mathrm{d}\beta^*(g)\gamma^*(g)\delta^*(g)^{-1}}{\mathrm{d}g} < 0, \tag{14}$$

because $\beta^*(0)\gamma^*(0)\delta^*(0)^{-1} = 1$, $\lim_{g\to\infty} \beta^*(g)\gamma^*(g)\delta^*(g)^{-1} = 0$ and $\beta^*(g)\gamma^*(g)\delta^*(g)^{-1}$ is continuous. The equation (14) holds if

$$\eta_\beta + \eta_\mu - \eta_\delta > 0, \tag{15}$$

where $\eta_\beta$, $\eta_\mu$ and $\eta_\delta$ is the elasticity of the transfer function $\beta$, $\mu$ and $\delta$ with respect to the growth ratio respectively, in short,

$$\eta_\beta \equiv -\frac{\mathrm{d}\beta^*}{\mathrm{d}g}\frac{g}{\beta^*}.$$

To put it the other way around, if the condition (15) is violated, the capitalist system can not follow the stationary path. Therefore, we can deduce the possibility of crisis from the condition (15) violated.

# 4 The Possibility of Crisis and Concluding Remarks

We can find the possibility of crisis in each phases of its circuit.

First, crisis can happen if $\eta_\delta$ is sufficiently-large. It means that economic agents withdraw their deposit in excessive response to the downturn of the growth rate and triggers a run on the bank. This is a typical story of the monetary crises.

Second, crisis can happen if $\eta_\beta$ is sufficiently-small. It means that the downturn of the growth rate should make banks reluctant to issue new loans, because banks can not find healthy business partners to whom banks want to lend. Consequently, credit money is not created. We call the collapse of $\eta_\beta$ the failure of "credit-creation."

Third, crisis can happen if $\eta_\mu$ is sufficiently-small. It means that the downturn of the growth rate should make it difficult to collect loans. We call the collapse of the collection of claim the failure of "credit-realization."

These failure of the metamorphosis of bank capital implies that the crisis can happen. But this approach in this paper has some weak points which should be relaxed.

This approach should incorporate the motion of the industrial capital and central bank. In other words, we have to relax the assumption of a kind of the partial equilibrium analysis. This approach has another weakness. It concentrates on the analytical attention to the steady state growth. This eliminates the analytical domain of the instability of capitalism. In other words, we can only point out the possibility of crisis, not the reality of crisis.

The future direction of this study will be one that encompasses these topics.

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