

A Critique of Gossen's Fundamental Theorem of the Theory of Pleasure

Klaus Hagendorf

eurodos@gmail.com

Université Paris Ouest – Nanterre, La Défense, June 2010

Abstract: Hermann Heinrich Gossen is generally known as the predecessor of Walras, Menger and Jevons in preparing modern economic analysis. What is generally not known is his *Fundamental Theorem* as a marginal approach to the labour theory of value. This paper presents his Fundamental Theorem in terms of modern terminology and extends Gossen's hedonistic approach of pleasure and pain to the analysis of marginal labour values based on technical production functions. It is shown that the marginal approach to the labour theory of value underlies modern economic analysis. Gossen's introduction of the concept of marginal pain or labour in his Fundamental Theorem of the Theory of Pleasure is really the revolutionary part of the Marginalist Revolution and it appears to be rather natural that bourgeois and Marxist vulgar economists have ignored it.

Key words: Hermann Heinrich Gossen, Microeconomics, Marginalist revolution, Labour theory of value, Marginal analysis, Value theory, Marginal labour value, Marxian economics, Adam Smith, Karl Knies, W. S. Jevons, John Bates Clark, N. Georgescu-Roegen,

JEL classifications: B14, B31, B51, D00, D46

I. Introduction

Bourgeois economists seem to derive a certain pleasure from the idea that Marxian economics has received a fatal blow by a revolution, the Marginalist Revolution in economics. We shall first emphasize that the Marginalist Revolution is very much in support of the *Theory of Historical Materialism*, which understands the development of human society and science as phases which succeed each other in revolutionary ways. We may therefore see the general agreement of economists on the revolutionary character of the change from Classical economics to Modern economics as a strong support for the *Theory of Historical Materialism*, the primary achievement of Marx and Engels.

We shall see in the development of the argument in this paper that conspiracy also in this revolution played a role and that the bourgeois presentation of it is totally distorted as the introduction of marginal reasoning has not replaced the labour theory of value but on the contrary has improved it very much¹. Although one must admit that Marxists were indeed for some time incapable to appreciate the marginal reasoning properly there were developments in the USSR who had overcome this as is most notable seen in the works of Leonid Kantorovich.

If there was a Marginalist Revolution than Hermann Heinrich Gossen (1810-1858) was the Revolutionary who had indeed been recognized as such by the Prussian establishment². It is him, who, in his only published work - *Die Entwicklung der Gesetze des menschlichen Verkehrs und der daraus fliessenden Regeln für menschliches Handeln*. Braunschweig: Friedrich Viehweg &

1 The marginal concept of labour value is analogous to the concept of marginal cost; it is the reciprocal of the marginal productivity of labour.

2 The following remark of one of his examiners is only one indication that Prussian authorities were indeed hostile towards Gossen for political reasons: "I can regard the last half [of the paper, dealing with the nationalization plan] only as the outgrowth of an unbridled fantasy, an empty daydreaming, and therefore I must conclude that the author is completely unqualified for civil service in a government board. In any case, I cannot consider the paper as sufficient, but rather as quite objectionable." (Georgescu-Roegen, 1983, p. cxxiv).

Sohn; 1854 - developed with the aid of very simple graphical means a whole system of analysis allowing to determine the optimal conduct of man in order to achieve the maximum of pleasure in life. His *Theory of Pleasure* leads him to some results close to what we now call the *Pareto-Optimality Conditions*. Gossen's Theory of Pleasure was not intended to refute the labour theory of value³ as was the objective of his successors but on the contrary, by taking into account both sides of the coin, *marginal utility* and *marginal labour or pain*, he tried to devise an economic policy on the basis of the principle 'To each according to his work', a goal never called by its name but underlying all his proposals⁴, in particular his demand for the abolishment of privileges and for the nationalization of land, a theme which remains at the top of economic policy agendas up to the present day in the form of the socialisation of natural resources. It is obvious that Gossen was considered by the state officials as dangerous for the Prussian state which may be the main reason why his work was not referred to until Jevons and Walras recognized Gossen's importance publicly not without selfish interests⁵.

We are in this paper not concerned with Gossen's economic policy proposals⁶ nor with his clearly idealistic general social philosophical outlook. In fact, he appears as a sort of a Moses of the

3 He rejects the notion of labour value only in a sense of physical labour embodied, ignoring the social conditions. This view, although it is often imposed upon the Marxian concept of labour value, is of no relevance anyway as also the Marxian concept defines labour values as socially necessary labour. (see Appendix B).

4 Hermann Riedle (1953, p. 122 ff.) offers a very fair critique of Gossen's economic policy proposals from a Keynesian point of view.

5 The general neglect of Gossens 'Entwicklung' in Germany is part of a well cultivated saga promulgated by the counter-revolutionaries of the 1870ies and later. However, Pantaleoni has accused Carl Menger of plagiarism of Gossen. Jevons claim, not to have had any knowledge of Gossen's book is doubtful as the Library of the British Museum holds a copy since 1865 (Hayek, 1927, XII). In my research I found striking similarities in John Bates Clark's writings with those in Gossen. And this offers me the opportunity to add a further hypothesis to the explanation of the mystery surrounding Gossen's work. Robert Anderson, who apparently established the link between Gossen and Jevons, as well as John Bates Clark had studied at Heidelberg University in the 1870ies. This university is well-known for its hospitality and was certainly one of those places where students could practice Gossen's *Theory of Pleasure*. This not only because this university hold a copy of the first edition of Gossen but also the style of his theorems invited them to be published as decorations in the famous wine bars of the town. When Clark honours Karl Knies for having led him to seek to discover a unit of measurement of Wealth in 'Festgaben zu Karl Knies' (Clark, 1896, p. 3) this is only of secondary importance. Incidentally another one of the 'Festgaben zu Karl Knies' is Boehm-Bawerk's "The Close of the Marxian System". Clark defines his unit of wealth as: "this commodity will promote me by one degree in the scale of happiness. It is worth one unit of my collective labor."(ibid.) and more explicitly he demands "taking *marginal labor* as the test of cost"(ibid. p. 6) .

6 See footnote 4.

economists presenting to the profession the commandments of the Creator. On the other hand Gossen is clearly a materialist. For him, pleasure and pain are results of human activities which are nothing else but movements. Gossen regards even the acts of speaking or breathing as a form of movement. This aspect of pre-marxist materialism, without referring directly to Gossen, is well criticised in the chapter “Matter and Its Forms of Existence” in “Grundlagen der marxistisch-leninistischen Philosophie (Akademie der Wissenschaften der UdSSR and Ministerium für Hoch- und Fachschulwesen der UdSSR, 1972, p. 60 ff.):

“The pre-Marxist materialists, i. e. the French materialists of the 18. century defended the proposition of the unalienable connection of matter and movement, movement being the most important property of matter, its form of existence. All appearances were regarded as being subject to natural and unchangeable laws. The materialist philosophers of the past hold the point of view of the absolute perceptibility of matter, of the ability of man to comprehend its properties and laws whatever complicated and unusual they might be.

All these principles of the materialist view of the world were taken over by the dialectical materialism and modern natural science ...

The atoms were regarded as the original and indestructible component elements of the world which only merge or divide, change their position in space and by this create the whole qualitative multitude of the appearances of the world. In this way the idea of the unchanging nature of the atoms was identified with the idea of matter as the substantial fundamental of the world as well as the philosophical principal of the preservation of matter with the principal of the indestructibility of atoms. ... From this was inferred, if one could succeed to break down the object to be analysed (body) into its atoms, to determine their position and velocity and to specify their movement in an equation, it would be possible to comprehend any possible phenomenon, even the properties of our consciousness and of the

social processes." (p. 69-70 translated by the editor).⁷

In view of these comments Gossen's *Theory of Pleasure* can be seen as a pre-Marxian study in materialism with a clearly idealistic deist foundation.⁸ We limit our discussion to the core of his theory, his *Fundamental Theorem*, in order to present the first scientific proof of the labour theory of value based on the utility maximization of the economic agent.

Gossen's basic axiom of analysis is, human life is characterized by its time consuming activities which are movements and yield pleasures or pains of various intensities. Gossen takes it as a natural law that an enjoyment, because it is necessarily a movement, yields less and less pleasure over time up to the satiation point where it does not yield any pleasure at all. This is known as *Gossen's First Law*. (Wieser, 1893, p. 8).

[A. 1] *The magnitude [intensity] of pleasure decreases continuously if we continue to satisfy one and the same enjoyment without interruption until satiety is ultimately reached.* (Gossen, 1983, p. 6)⁹

7 The original German text is: „Die vormarxschen Materialisten, insbesondere die französischen Materialisten des 18. Jahrhunderts, vertraten die These vom untrennbaren Zusammenhang von Materie und Bewegung, von der Bewegung als dem wichtigsten Attribut der Materie, ihrer Daseinsweise. Alle Erscheinungen wurden als natürlichen und unerschütterlichen Gesetzen unterworfen betrachtet. Die materialistischen Philosophen der Vergangenheit gingen von der Idee der unbedingten Erkennbarkeit der Materie aus, von der Fähigkeit des Menschen, ihre Eigenschaften und Gesetze zu erfassen, wie kompliziert und ungewöhnlich sie auch sein mögen. Alle diese Prinzipien der materialistischen Auffassung der Welt wurden in den dialektischen Materialismus und in die moderne Naturwissenschaft übernommen. ... Die Atome wurden als die ursprünglichen und unzerstörbaren Bauelemente der Welt betrachtet, die sich nur vereinigen und teilen, ihre Lage im Raum verändern können und dadurch die ganze qualitative Vielfalt der Erscheinungen in der Welt hervorbringen. So wurde die Idee der Unveränderlichkeit der Atome mit der Idee der Materie als der substantiellen Grundlage der Welt und das philosophische Prinzip der Erhaltung der Materie mit dem Prinzip der Unzerstörbarkeit der Atome identifiziert. ... Daraus wurde geschlossen, wenn es gelänge, den zu untersuchenden Gegenstand (Körper) in seine Atome zu zerlegen, deren Lage und Geschwindigkeit zu bestimmen und ihre Bewegung in einer Gleichung zu erfassen, so könne man jede beliebige Erscheinung vollständig erkennen, selbst die Eigenschaften unseres Bewusstseins und der sozialen Prozesse.“. A similar but not identical English text is *Fundamentals of Marxist-Leninist Philosophy* (Konstantinov, 1982, p. 22 ff.)

8 He introduces his theory by presenting the imperative of the Creator: **“Man! Explore the laws of My creation and act in accordance with these laws!”** (Gossen, 1983, p. 6); in German: **“Mensch! Erforsche die Gesetze meiner Schöpfung, und diesen Gesetzen gemäß handle!”** (Gossen, 1854, p. 4).

9 The German original: **“Die Größe eines und desselben Genusses nimmt, wenn wir mit Bereitung des Genusses ununterbrochen fortfahren, fortwährend ab, bis zuletzt Sättigung eintritt.”** (Gossen, 1854, p. 4, 5). In the context of the wine bars of Heidelberg this is when you find yourself under the table. I leave it to the Reader to apply Gossen's other theorems to the conditions of student life in Heidelberg.

In modern theory we would deduce diminishing marginal utility as a consequence of an unlimited number of concurring wants and a limited amount of (life-)time so that the satisfaction of one particular want becomes more and more expensive, in terms of the other not fulfilled wants, the longer it takes. Whereas Gossen's activity analysis is of a subjective, physiological type, modern analysis is based on objective opportunity costs. This point needs to be emphasized as it is at the core of the development of economic theory away from psychological or physiological reasoning to an objective materialistic analysis. But obviously it is not always appropriate to ignore psychological, physiological or ergonomic aspects in the analysis of economic processes in particular in applied economic studies and certainly not in studies of the working or consumption processes.

II. The Fundamental Theorem of the Theory of Pleasure

The first author who has quoted the *Fundamental Theorem* in the context of the discussion on the labour theory of value was Tugan-Baranovsky in his “Theoretical Foundations of Marxism” (Tugan-Baranowsky, 1905, p. 158):

“This relationship between the labour effort for the production of a good and its value was very clear to the founder of the marginal utility school, Hermann Gossen: *'In order to maximize his life pleasure, man must distribute his time and energy among the preparation of various pleasures in such a way that the value of the last atom yielding*

each pleasure shall be equal to the magnitude of discomfort experienced by him if this atom had been created in the very last moment of the employment of force.” (Gossen, 1983, p. 53, translated by the editor)¹⁰ .

In Gossen this theorem is introduced with the following phrase: “If we take into account that the means of enjoyment must be produced by labour, *the fundamental theorem of the theory of pleasure* is that ...”(ibid, translated and emphasized by the editor).¹¹

This shows clearly that we have here the core of the *Theory of Pleasure*. It is then very curious that the followers, in particular Hayek in his introduction to the 3rd edition (Hayek, 1927), but also Recktenwald and Krelle (1987; 1988), don't mention this theorem at all and refer only to the so called 2 *Laws of Gossen*, terms created by the Austrian economist von Wieser¹². One must add that the so called *Second Law of Gossen* (Gossen, 1854, p. 93, 94, see below) is not even emphasized in Gossen¹³. The reason for the banning of the *Fundamental Theorem* is clear: The bourgeois economists have done everything to keep the concept of “*marginal labour*” secret as it is the pendant to the concept of “*marginal cost*” and therefore it is *the key to the correct understanding of the labour theory of value*.¹⁴ Bourgeois economists since Jevons always speak of marginal productivity of labour instead as “labour value” is considered a “dangerous” concept. This point has also escaped Georgescu-Roegen when he wonders about the banning of the *Fundamental Theorem*

10 The translation differs from (Gossen, 1983) to emphasize the labour theoretical meaning.

11 The original German text: „Mit Rücksicht auf die Nothwendigkeit der Beschaffung der verschiedenen Genüsse durch Arbeit lautet daher der oben gefundene Hauptgrundsatz der Genußlehre: *Um ein Größtes von Lebensgenuß zu erhalten, hat der Mensch seine Zeit und Kräfte auf die Bereitung der verschiedenen Genüsse der Art zu vertheilen, daß der Werth des letzten bei jedem Genuß geschaffenen Atoms der Größe der Beschwerde gleich kommt, die es ihm verursachen würde, wenn er dieses Atom in dem letzten Moment der Kraftentwicklung schaffte.*“ (Gossen, 1854, p. 45)

12 It is particularly humiliating for Gossen how the bourgeois economists, beginning with Wieser (1893, p. 8), over Hayek (1927) and Krelle (1988) express their admiration towards Gossen and at the same time suppress Gossen's labour theory of value and his basic goal: “To Each According to His Work (Pain)”.

13 Nicholas Georgescu-Roegen's remark in his very stimulating but controversial introduction to the English translation of Gossen's work on the non-italicizing of the Fundamental Theorem (Gossen, 1983, p. xvii) is not correct. Apparently he was confusing it with the non-italicizing of Gossen's Second Law.

14 An exception from this rule is John Bates Clark (for example (Clark, 1896) but his reasoning is not formulated in mathematical terms and somewhat contradictory.

(Georgescu-Roegen, 1983, p. xcvi).

The basic idea of the *Fundamental Theorem* is that a maximum of utility can be derived from the consumption of a good when one produces just so much of the good that the marginal pain or *marginal labour* is just equal to the *marginal utility* derived from its consumption. Relative prices are equal to relative marginal utilities and these are equal to the ratio of marginal labour values. So in the optimum relative prices are equal to relative marginal labour values which is the labour theory of value.

III. A Mathematical Presentation of Gossen's Theory of Pleasure

We present Gossen's *Theory of Pleasure* in modern terminology as a mathematical analyses of the optimal allocation of time to the activities of consumption and production of an individual, say Robinson Crusoe. Wilhelm Krelle (1987; 1988)¹⁵ has developed a mathematical presentation of Gossen's system but fails to present its central feature properly: Gossen's labour theory of value. In fact, no one speaks of Gossen's labour theory of value¹⁶ or if there is a discussion it is how to understand Gossen's concept of pain ('Beschwerde') in a physiological or ergonomic sense and the supply of labour power. Strange enough, the ergonomists do not care about Gossen. Even Georgescu-Roegen (1983) does not develop the labour theory of value in Gossen although at least he presents an analysis of Gossen's *Fundamental Theorem* (Georgescu-Roegen, 1983, p. xcv-cv)¹⁷.

15 Both texts are almost identical.

16 An exception is Kurz (2009) but see footnote 28.

17 G-R's analysis is very interesting but not always correct. For example his equation **(13)** can only be calculated after one has established the optimal solution (without time constraint) already. Also G-R's statement "the new economic theory, first crystallized by him [Gossen], adopted a purely psychological basis." (Gossen, 1983, p. xxi) is not quite correct neither as we shall see below by introducing the time constraint. But it is not the objective of this article to discuss the literature on Gossen but to present his labour theory of value and G-R has not done this properly. If he had done so he might have been awarded the Nobel Prize as he had hoped for.

Due to the fact that Gossen uses linear marginal functions of pleasure, utility and pain his system can be represented rather easily and this is also why Gossen had chosen these forms, not because he believed them to be realistic, but to simplify analysis, a common feature of modern economics.

Robinson, living on his island, tries to optimize pleasure in life. There are different enjoyments which take time and the problem is, how much time to spend for the various enjoyments. Georgescu-Roegen points out that time is the ultimate scarce resource and even in Paradise there is an economic problem: Gossen's problem of deciding how long one should enjoy a particular pleasure in order to maximize life's pleasure. Only if one lives forever in a Paradise there is no economic problem. This particular observation is very nice indeed for economics as a profession as there is always a future for economists which defies Sir Roy Harrod's position that the economic problems would be overcome one day.

Gossen observes that the marginal pleasure of one particular enjoyment diminishes with time, because it is a movement and therefore necessarily leads to fatigue, and he assumes that the *marginal pleasure function*¹⁸ is a straight downward sloping line with a satiation point. It relates marginal pleasure to the time of enjoyment.

Usually the enjoyment activities involve some use of goods. We simplify and relate one good to one enjoyment. Another simplifying assumption is that the amount of the good used up per time, remains constant over the enjoyment. The *marginal utility function* relates marginal pleasure to the quantity of the good consumed.

In an analogous manner the production of the good takes time and follows similar rules. As also

¹⁸ These terms for the functional relationships are mine. They are not found in Gossen but they ease the exposition enormously. The functional concepts are very close to Krelle's but in the decisive point, introducing objective marginal labour value, they differ.

production is understood as a movement, it first yields pleasure but less and less up to the satiation point. But now it may be necessary to continue the working process in order to produce a sufficient quantity of the good and so Robinson accepts some pain to achieve his plan. Here again we speak of a *marginal pain function* relating marginal pain to time and of a *marginal labour function* when marginal pain is related to the quantity of the good produced.

From this we get already 8 types of functions: the *pleasure function*, the *utility function* the *pain function*, and the *labour function* and their marginal counterparts. In addition we need a consumption function as well as a production function to explain the Theory of Pleasure.

We start with the *marginal pleasure functions* as straight lines. There are i enjoyments consuming i goods. They give the *marginal pleasure functions* (G from German 'Genuß')

$$\frac{dG_i}{dt} = a_{i0} - a_{i1}t; \text{ for } i=1,2,\dots \quad (1)$$

Integration (omitting the integration constants) yields the *pleasure functions*

$$G_i = a_{i0}t - \frac{1}{2}a_{i1}t^2; \text{ for } i=1,2,\dots \quad (2)$$

We introduce the goods via *consumption functions* with a constant Q/t ratio. Here the quantity of a good consumed, Q , is a function of time, t . The marginal consumption of the good with respect to time is assumed to be constant.

$$Q_i = f_i(t) = c_i t; \quad \frac{dQ_i}{dt} = c_i \text{ constant}; \quad \text{for } i=1,2,\dots \quad (3)$$

Substituting time in the *pleasure functions* (2) by the inverses of the *consumption functions*

$$t = \frac{1}{c_i} Q_i \quad \text{we get the utility functions}$$

$$U_i = a_{i0} \frac{1}{c_i} Q_i - \frac{1}{2} a_{i1} \left(\frac{1}{c_i}\right)^2 Q_i^2; \text{ for } i=1, 2, \dots \quad (4)$$

We replace the constant parameters by new terms

$$\bar{a}_{i0} = a_{i0} \frac{1}{c_i}; \bar{a}_{i1} = a_{i1} \left(\frac{1}{c_i}\right)^2 \quad (5)$$

and the *utility functions* become

$$U_i = \bar{a}_{i0} Q_i - \frac{1}{2} \bar{a}_{i1} Q_i^2; \text{ for } i=1, 2, \dots \quad (6)$$

Differentiation yields the *marginal utility functions*

$$\frac{dU_i}{dQ_i} = \bar{a}_{i0} - \bar{a}_{i1} Q_i; \text{ for } i=1, 2, \dots \quad (7)$$

In an analogous manner we define the functions for the working process.

The marginal pain functions are

$$\frac{dB_i}{dt} = b_{i0} + b_{i1} t; \text{ for } i=1, 2, \dots \quad (8)$$

Integration (omitting the integration constants) yields the *pain functions*

$$B_i = b_{i0} t + \frac{1}{2} b_{i1} t^2; \text{ for } i=1, 2, \dots \quad (9)$$

We introduce now the *production functions*. This is a central point in our analysis. Gossen does not know the concept of a production function. And here again he chooses simplicity, that is a constant ratio of output of the good to labour time. We would speak of a constant marginal productivity of labour, or its reciprocal, constant marginal labour values. It is exactly here where Krelle (1988, p. 29) does his *faux pas*: in order to hide the concept of *marginal labour value* he imposes upon Gossen a Leontief-type production function. It is true, a Leontief function fits also to Gossen's

model but it is only one special case ($\frac{Q}{t} = \frac{dQ}{dt}$; $\epsilon = \frac{dQ}{dt} \frac{t}{Q} = 1$); the general case - we are discussing here in the context of the Marginalist Revolution - is to assume production at constant returns to scale and a *constant marginal productivity of labour*, which results also in a constant output-labour time ratio. This more general method to define a constant output-labour time ratio allows for marginal analysis properly. We define *labour value* as the reciprocal of the *marginal productivity of labour*. Production at constant returns to scale corresponds to production at minimum average total cost. So our definition assures that Gossen's assumption of a *constant output - labour time ratio* is production at minimum average cost. Of course this goes well beyond Gossen's work, but this is very important to understand modern marginal analysis of labour values properly.¹⁹

The quantity of a good produced, **Q**, is a function of labour time, **L**, capital, **K**, and land, **T**.

$$Q_i = f_i(K, T, L); \quad L = t; \quad K - \text{capital}; \quad T - \text{land}; \quad L - \text{labour time};$$

$$\frac{\partial Q_i}{\partial L} = \mu_i \text{ constant}; \quad \text{for } i=1, 2, \dots \quad (10)$$

As we assume constant returns to scale, Euler's theorem applies and so we have

$$Q = \frac{\partial Q}{\partial L} L + \frac{\partial Q}{\partial K} K + \frac{\partial Q}{\partial T} T \quad (11)$$

As the marginal productivities are assumed to be constant and the ratios of the quantities of the factors of production are necessarily also constant we get

$$\frac{Q}{L} = \frac{\partial Q}{\partial L} + \frac{\partial Q}{\partial K} \frac{K}{L} + \frac{\partial Q}{\partial T} \frac{T}{L} = \mu + s; \quad s = \frac{\partial Q}{\partial K} \frac{K}{L} + \frac{\partial Q}{\partial T} \frac{T}{L} = \text{constant} \quad (12)$$

We simplify to

$$Q = (\mu + s) L \quad \text{or} \quad L = \frac{1}{(\mu + s)} Q \quad (13)$$

Notice that the reciprocal of the marginal productivity of labour is $1/\mu_i$ and is the *marginal labour*

¹⁹ The practice of former SS-Obersturmbannführer and MIT student Wilhelm Krelle is typical for bourgeois economists who systematically separate the analysis of labour values from marginal analysis, Sraffian economics being the incarnation of it, its *raison d'être*. (See also footnote 28, p. 27).

value of a technical production function.

$$\frac{\partial L}{\partial Q_i} = \frac{1}{\mu_i} = v_i \text{ constant ; for } i=1,2,\dots \quad (14)$$

This needs to be carefully distinguished from Gossen's physiological labour functions of the working process where we have marginal pain. Marginal pain is a function of labour time. This leads Georgescu-Roegen to the remark: “Gossen never renounced the struggle to push the analysis of the supply of hours of work as far as possible ... In particular, he clearly distinguished between labor power and labor time long before Marx's *Das Kapital* came off the press.” (Georgescu-Roegen, 1983, p. xcvi). Labour power in a Marxian sense is the ability of the labourer to work. The labourer sells this labour power to the capitalist for a period of time. However, the labour power is consumed in the working process, that is the worker's ability to work diminishes with the duration of the working process. Or, alternatively, the pain suffered from work increases with the duration of the working process. So Gossen's *physiological pain function* is important as an expression for the destruction of labour power in the working process and matters in the negotiations of labour contracts.²⁰

The *marginal pain function* with the explicit argument of labour time is:

$$\frac{dB_i}{dL} = b_{i0} + b_{i1} L; \text{ for } i=1,2,\dots \quad (15)$$

and this integrated gives the *pain function*:

$$B_i = b_{i0} L + \frac{1}{2} b_{i1} L^2; \text{ for } i=1,2,\dots \quad (16)$$

Pain is an increasing function of labour time.

²⁰ An article on this physiological, energetic aspect of the labour process is (Berthe & Renault, 2001). The physical aspect of labour and the marginalist revolution is also mentioned in chapter 12 of (Ayres, 1994). This work is a good criticism of the intrinsic failures of a market system in contrast to Gossen's ultra-liberal position.

In order to get a relationship between pain and the good we use the technical production function at constant returns to scale.

Substituting labour time in the *pain functions* by using (13) we get the *labour functions*

$$B_i = b_{i0} \frac{1}{(\mu_i + s_i)} Q_i + \frac{1}{2} b_{i1} \left(\frac{1}{\mu_i + s_i} \right)^2 Q_i^2; \text{ for } i=1, 2, \dots \quad (17)$$

And again we replace the constant parameters by the new terms

$$\bar{b}_{i0} = b_{i0} \frac{1}{\mu_i + s_i}; \bar{b}_{i1} = b_{i1} \left(\frac{1}{\mu_i + s_i} \right)^2 \quad (18)$$

and the *labour functions* become

$$B_i = \bar{b}_{i0} Q_i + \frac{1}{2} \bar{b}_{i1} Q_i^2; \text{ for } i=1, 2, \dots \quad (19)$$

Differentiation yields the *physiological marginal labour functions* or *marginal pain functions* as functions of the goods

$$\frac{dB_i}{dQ_i} = \bar{b}_{i0} + \bar{b}_{i1} Q_i; \text{ for } i=1, 2, \dots \quad (20)$$

With this we have developed the analytical tools in order to present Gossen's *Fundamental Theorem* in modern terminology. In fact, we are going to demonstrate the labour theoretical logic underlying the intersection of the supply and demand curves in economics. The downward sloping demand curve had been developed already by Cournot and Jules Dupuit. Gossen, although he never uses the term as such, introduces the upward sloping supply curve up to the point of contact with the demand curve. It remains Léon Walras' eternal fame to have extended it further above the demand curve.²¹

21 The Reader should not take this to literally as there have been others who had used diagrams of supply and demand curves before, i. e. Hans von Mangoldt.

IV. Gossen's Physiological Labour Theory of Value

Figures 1a and 1b show a *marginal utility function* and a *marginal pain function* for a good. The difference of the marginal pain function is simply that it also continues into the negative quadrant whereas the marginal utility function ends at the abscissae.



Figure 1a: Marginal Utility of a Good

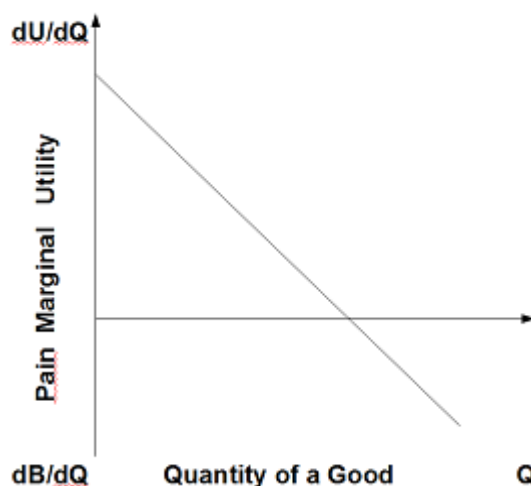


Figure 1b: Marginal Utility and Pain Producing a Good

In order to easily equalize marginal utility and marginal pain, Gossen redefines pain as negative utility and so the marginal pain functions become upward sloping as we have defined them above.

Now the marginal pain function starts in the negative quadrant, pain being negative means it yields utility, it then reaches the abscissae and becomes positive that is it becomes pain. This is demonstrated in **Figure 2**.²²

The intersection of the two curves, where marginal utility equals marginal pain, represents the optimum point. If Robinson would produce more of the good, he would feel more pain from longer working and this additional pain would be greater than the additional utility gained from the greater

²² Gossen's presentation (Gossen, 1854, p. 41) is here more elegant than Jevons' (Jevons, (1871) 1965, p. 173).

quantity of the good produced.

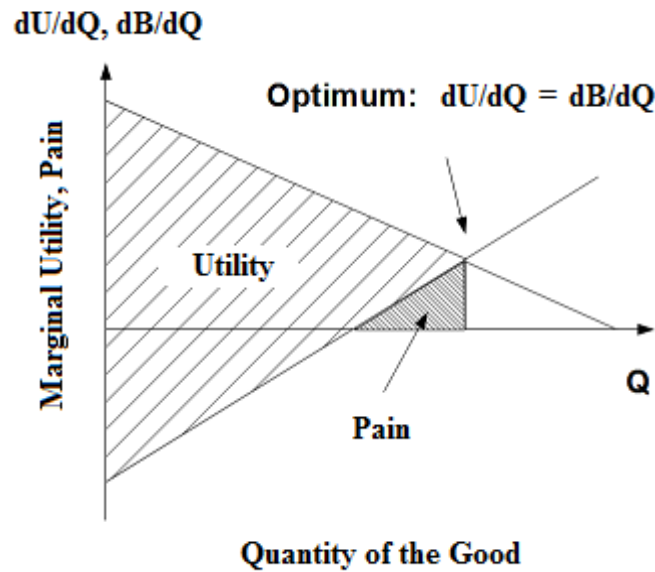


Figure 2: Optimum of Production of a Good

When we interpret pain as labour proper and utility as value then *marginal labour* (labour per unit of the good) equals the *marginal value* (the price) of the good. Therefore, in the optimum we have the validity of the labour theory of value.

Mathematically, the problem is to optimize utility minus pain or net utility, U^* .

$$\max U^* = U(Q) - B(Q) \quad (21)$$

The First Order Condition to maximize net utility is:

$$\frac{\partial U^*}{\partial Q} = \frac{dU}{dQ} - \frac{dB}{dQ} = 0 \quad (22)$$

and therefore

$$\frac{dU}{dQ} = \frac{dB}{dQ} \quad (23)$$

At the optimum amount produced, yielding the maximum net utility, the marginal utility is equal to marginal pain, and this is graphically the intersection of the marginal utility curve with the marginal

pain curve, in other words the intersection of Gossen's demand with the supply curve.²³ After all, Gossen's work is a graphical and mathematical analysis of Adam Smith's view:

"The real price of every thing, what every thing really costs to the man who wants to acquire it, is the toil and trouble of acquiring it. What every thing is really worth to the man who has acquired it, and who wants to dispose of it or exchange it for something else, is the toil and trouble which it can save to himself, and which it can impose upon other people. What is bought with money or with goods is purchased by labour, as much as what we acquire by the toil of our own body."

(Smith, 1776; book I, ch. V)

Now we must say this is a subjectivist analysis based on purely individualistic physiological relationships. And indeed this type of analysis is used by the vulgar economists to emphasize that value and price is purely psychological and has nothing to do with the labour time involved. That this is not quite correct is obvious as there is a relationship between pain and labour time, or in Marxian terms, between labour and labour time. So, even in this simplest case the labour theory of value holds, if understood as physiologic subjectivist. And more important, Gossen's analysis is a type of activity analysis, opening up the possibility to base the *Theory of Historical Materialism* on the analysis of the behaviour of individuals and the labour theory of value.

But it is clear that the subjectivist, individualistic character of the functions involved are not satisfactory for economic analysis and it remains to be shown how these functions can be used to derive objective functions which can be observed. We get a first hint of how to proceed, when we consider Robinson. When we asked Robinson to explain to us his utility preferences he would not be able to do so. But if we asked him in the following way: "How much more time would you be

²³ To be precise, the demand and supply curves are derived in a somewhat different manner (see below), but for the moment this will do.

prepared to work for an additional unit of the good?”, Robinson would well be able to give us a whole schedule of his preferences defined in units of marginal labour time. This gives us the perspective to replace the units of marginal utility of the vertical axis in the figures with marginal labour values of an objective kind derived from the production functions.

However, before we do this, we generalize the analysis above for many goods and enjoyments as Gossen has done also. In the analysis above we have presented the proof only for the case of one good, that is one enjoyment, but it is easy to extend it to the case of many goods and enjoyments.

Mathematically we simply index the functions involved. The net utility to be maximize is now a function of several goods:

$$\max U^* = \sum_i (U_i(Q_i) - B_i(Q_i)) \quad (24)$$

The First Order Conditions to maximize net utility, U^* , are:

$$\frac{\partial U^*}{\partial Q_i} = \frac{dU_i}{dQ_i} - \frac{dB_i}{dQ_i} = 0 \quad ; \quad \text{for } i=1,2,\dots \quad (25)$$

and therefore

$$\frac{dU_i}{dQ_i} = \frac{dB_i}{dQ_i} \quad ; \quad \text{for } i=1,2,\dots \quad (26)$$

Here we have the mathematical expression of Gossen's *Fundamental Theorem*. Graphically it is nothing else but the intersection points of all demand and supply curves: the solution of General Equilibrium. But here we are limited to the case of Robinson. And furthermore the assumptions of the utility functions are very restrictive: utility is cardinal, additive and independent and the functions are physiological, subjectivist. There is one very important point which has been overlooked by all the commentators: if wants are unlimited and i reaches infinity then there is necessarily a time constraint. Labour time is the ultimate constraint in economic activity because

life time of humans is finite. This is why the labour theory of value is so important as it is this time constraint of human life which gives value to commodities.

That the bourgeoisie, who does not work, considers this as nonsense is obviously understandable but one must insist that it can live without working only, because it exploits the work (life time) of others. Now we shall analyse the optimum solution under conditions of labour time constraint before we introduce other factors of production and exploitation.

V. The Objective Labour Theory of Value

It is surprising that Gossen did not analyse the case under time constraint. For the case of enjoyments, ignoring the production of goods, he did analyse the impact of the time constraint. Given that human wants are unlimited, there is never enough time to reach the optimal quantities of all goods as there is simply not enough labour time available. Labour time is the ultimate scarce resource and this changes the nature of the solutions fundamentally. Now, objectively measurable labour time becomes the price of a commodity; to be precise and this is really the most important aspect of the Marginalist Revolution in economics, it is marginal labour value which matters and not average labour value, which determines the price. The solution is straightforward. But first, we show the solution for the maximization of utility under time constraint, ignoring production.

The problem is to maximize total utility, U^* , derived from several enjoyments subject to a time constraint, T .

$$\begin{aligned} \max U^* &= \sum_i U_i(Q_i) \\ \text{s.t. } T &= \sum_i t_i \end{aligned} \tag{27}$$

We solve this problem with the Lagrange method. The Lagrangian is:

$$\mathcal{L} = \sum_i U_i(Q_i) + \lambda [T - \sum_i t_i] \quad (28)$$

The First Order Conditions are

$$\frac{\partial U_i}{\partial Q_i} = \lambda \frac{\partial t_i}{\partial Q_i}; \text{ for } i=1,2,\dots \quad (29)$$

$$T = \sum_i t_i$$

The $\delta t/\delta Q$ s are the reciprocals of the marginal intensities to consume of (3) and are constant, and the solution is

$$\frac{\partial U_1/\partial Q_1}{\partial t_1/\partial Q_1} = \frac{\partial U_2/\partial Q_2}{\partial t_2/\partial Q_2} = \frac{\partial U_3/\partial Q_3}{\partial t_3/\partial Q_3} \dots \quad (30)$$

This is one version (in terms of utilities which is analogous to that in terms of pleasure) of *Gossen's Second Law* which can be expressed also with an amount of money as the budget constraint (then the $\delta t/\delta Q$ s have to be replaced by the prices):

[1.2] *In order to maximize his total pleasure, an individual free to choose between several pleasures but whose time is not sufficient to enjoy all to satiety must proceed as follows: However different the absolute magnitudes of the various pleasures might be, before enjoying the greatest pleasure to satiety he must satisfy first all pleasures in part in such a manner that the magnitude [intensity] of each single pleasure at the moment when its enjoyment is broken off shall be the same for all pleasures. (Gossen, 1983, p. 14).²⁴*

24 The German original text: “Der Mensch, dem die Wahl zwischen mehreren Genüssen frei steht, dessen Zeit aber nicht ausreicht, alle vollauss sich zu bereiten, muß, wie verschieden auch die absolute Größe der einzelnen Genüsse sein mag, um die Summe seines Genusses zum Größten zu bringen, bevor er auch nur den größten sich vollauss bereitet, sie alle theilweise bereiten, und zwar in einem solchen Verhältnis, daß die Größe eines jeden Genusses in dem Augenblick, in welchem seine Bereitung abgebrochen wird, bei allen noch die gleiche bleibt.” (Gossen, 1854, p. 12). It is easy to imagine how to apply this theorem to the problems of a student enjoying the hospitality of Heidelberg, its girls and wine bars.

In a similar manner we find the solution for the case of consumption and production under time constraint.

The Lagrangian is

$$\mathcal{L} = \sum_i (U_i(Q_i) - B_i(Q_i)) + \lambda [L_0 - \sum_i L_i] \quad (31)$$

We name the time constraint L_0 instead of T .

The First Order Conditions are

$$\begin{aligned} \frac{\partial U_i}{\partial Q_i} - \frac{\partial B_i}{\partial Q_i} - \lambda \frac{\partial L_i}{\partial Q_i} &= 0; \text{ for } i=1,2,\dots \\ L_0 &= \sum_i L_i \end{aligned} \quad (32)$$

and this leads to

$$\begin{aligned} \frac{\frac{\partial U_i}{\partial Q_i} - \frac{\partial B_i}{\partial Q_i}}{\frac{\partial L_i}{\partial Q_i}} &= \lambda; \text{ for } i=1,2,\dots \\ L_0 &= \sum_i L_i \end{aligned} \quad (33)$$

We now simplify the expressions by substituting marginal net utilities for the differences of

marginal utilities and marginal pains, $\frac{\partial U_i^*}{\partial Q_i} = \frac{\partial U_i}{\partial Q_i} - \frac{\partial B_i}{\partial Q_i}$, and for the *technical marginal labour*

values, $\frac{1}{\mu_i} = \frac{\partial L_i}{\partial Q_i}$, as defined in (11). From this we get

$$\begin{aligned} \frac{\partial U_i^* / \partial Q_i}{1/\mu_i} &= \lambda; \text{ for } i=1,2,\dots \\ L_0 &= \sum_i L_i \end{aligned} \quad (34)$$

And therefore

$$\frac{\partial U_1^* / \partial Q_1}{1/\mu_1} = \frac{\partial U_2^* / \partial Q_2}{1/\mu_2} = \frac{\partial U_3^* / \partial Q_3}{1/\mu_3} \dots \quad (35)$$

From this the validity of the labour theory of value in a technical, objective sense follows as now

the ratio of net marginal utilities equals the ratio of *marginal labour values* as obtained from the technical production functions.

$$\frac{\partial U_i^*/\partial Q_i}{\partial U_1^*/\partial Q_1} = \frac{1/\mu_i}{1/\mu_1} \text{ for } i=2,3,\dots \quad (36)$$

To give it an even clearer expression we equalize the ratio of marginal net utilities with the ratio of

prices, $\frac{\partial U_i^*/\partial Q_i}{\partial U_1^*/\partial Q_1} = \frac{p_i}{p_1}$ for $i=2,3,\dots$, and write for the *technical marginal labour values*

$$v_i = \frac{1}{\mu_i}$$

$$\frac{p_i}{p_1} = \frac{v_i}{v_1} \text{ for } i=2,3,\dots \quad (37)$$

The result is striking as Robinson optimises his net utility taking into account the pleasure of enjoyments of consuming as well as the sufferings of the pain it causes him to produce the goods *and* he evaluates the goods according to the *marginal labour values* necessary for their production. These marginal labour values are the technically given ones based on the production functions.

The graphical demonstration shows this change from the subjective physiological solution to the objective technical solution most clearly. As marginal labour values are assumed to be constant (see **(10)** above) the supply curve becomes a horizontal line. In this case demand has no impact on the price and determines the optimal quantity only. But now we also have to develop a proper demand function for the good as the proper solution requires that the dimension of the vertical axis is now technical marginal labour value and not marginal utility or disutility.

The derivation of the demand curve is as follows: Robinson maximizes utility subject to a budget constraint. From **(37)** we know already that the prices are equal to marginal labour values (v_i). So we may express the budget constraint in terms of labour values. For simplicity we assume only 2

goods:

$$L_0 = v_1 Q_1 + v_2 Q_2 \quad (38)$$

The Lagrangian for this problem is:

$$\mathcal{L} = U_1 + U_2 + \lambda [L_0 - v_1 Q_1 - v_2 Q_2] \quad (39)$$

and the First Order Conditions are

$$\begin{aligned} \frac{\partial U_i}{\partial Q_i} &= \lambda v_i, \text{ for } i=1, 2; \\ L_0 &= v_1 Q_1 + v_2 Q_2 \end{aligned} \quad (40)$$

By eliminating λ and resolving for Q_2 , then substituting the solution into the budget constraint and solving for v_1 one obtains the demand function $v_1 = f(Q_1)$.²⁵

What matters is that we have now both functions, demand and supply functions expressed in terms of *marginal labour values*. The subjective notions of utility and pain have disappeared as this is shown in **Figure 3**.

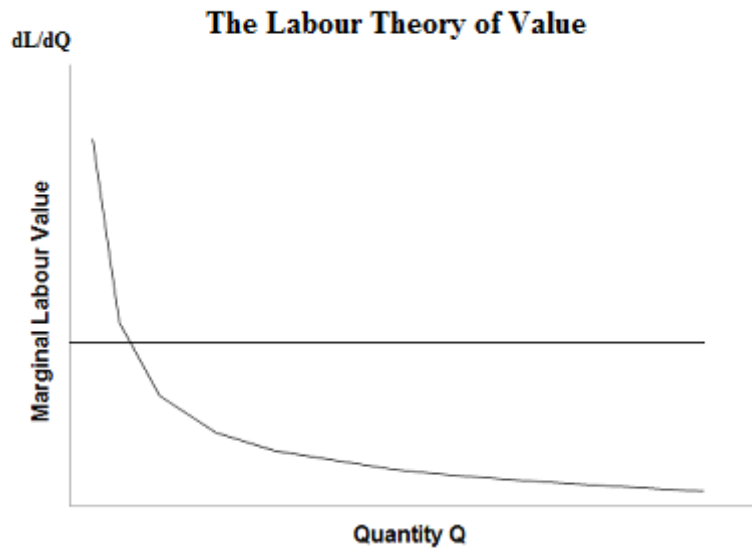


Figure 3: Demand and Supply curves based on objective marginal labour values

²⁵ The derivation of the demand function is shown in the Appendix A.

Now we understand why the bourgeois economists do not mention the *Fundamental Theorem* of Gossen.²⁶ It is very important to realize that this solution does not depend on a zero rate of interest as Kurz argues (Kurz, 2009, p. 485), as we are considering *marginal labour values* and these correspond to the *dated quantities of labour* in the context of a linear algebra analysis of an equilibrium system. Dated quantities of labour are calculated on the basis of the Sraffian inverse and not on the basis of the Leontief inverse (Hagendorf, 2008, p. 11 ff.).²⁷ Furthermore the result of our analysis is not limited to the case of fixed marginal labour values as in Gossen. If we allow for variable marginal labour values then the supply function becomes upward sloping. In any case we remain always in the positive quadrant.

It remains to be shown that the explicit introduction of other factors of production does not invalidate the labour theory of value.

VI. Gossen's Analysis of Rent

We get back to Gossen's physiological analysis of rent (Gossen, 1854, p. 102 ff.), which is very much a Ricardian approach. Gossen distinguishes the location of production as a factor influencing the productivity of labour and those locations which enable a higher return to labour than others, be it through their natural properties or artificially improvements (investments), yield rents. The *marginal location* does not yield any rent.

We may use our functions defined above to provide a simple analysis. The same good can be produced at 2 locations which are unequally suitable for production. The utility function for the good is unique but the pain functions of production differ. It is clear that the *Fundamental Theorem*

²⁶ But one should also remark that the *Fundamental Theorem* is less suitable as a decoration in a wine bar.

²⁷ For a detailed modern analysis of the demand and supply curves in terms of marginal labour values see (Hagendorf, 2009).

holds for both places but in the more suitable place net utility is greater and this difference of net utility, expressed in terms of the product or in terms of labour time, is the *payable rent* ('zahlbare Rente').

We denote with Q_x^* and Q_y^* , ($Q_x^* > Q_y^*$), the optimal quantities of one good to be produced on location x or alternatively on y . Net utility at y is the difference between utility derived from the quantity Q_y and the pain suffered producing this quantity Q_y .

$$U_y^* = U_y(Q_y) - B_y(Q_y) \quad (41)$$

The First Order Conditions are

$$\frac{\partial U_y^*}{\partial Q} = \frac{\partial U_y}{\partial Q} - \frac{\partial B_y}{\partial Q} = 0 \quad (42)$$

and therefore for location y at the optimum quantity produced

$$\frac{\partial U_y}{\partial Q} = \frac{\partial B_y}{\partial Q} \quad (43)$$

$$Q_y^* = \frac{a_0 - b_{0y}}{a_1 + b_{1y}} \quad (44)$$

The determination of the optimal quantity produced and the rent to be paid at x is almost identical but here we have to reduce the quantity constituting rent from the utility function.

Net utility for the worker at x is defined as

$$U_x^* = U_y(Q_x - Q_R) - B_x(Q_x) \quad (45)$$

and must be equal to net utility at y

$$U_x^*(Q_x - Q_R) = U_y^*(Q_y) \quad (46)$$

The First Order Conditions are

$$\frac{\partial U_x^*}{\partial Q} \Big|_{Q_x - Q_R} = \frac{\partial U_x}{\partial Q} - \frac{\partial B_x}{\partial Q} = 0 \quad (47)$$

The two unknowns are the quantity to be produced at x , Q_x and the payable rent, Q_R , and they can be derived from the two equations (46) and (47). In particular from (47) follows that Gossen's

hedonistic labour theory of value holds.

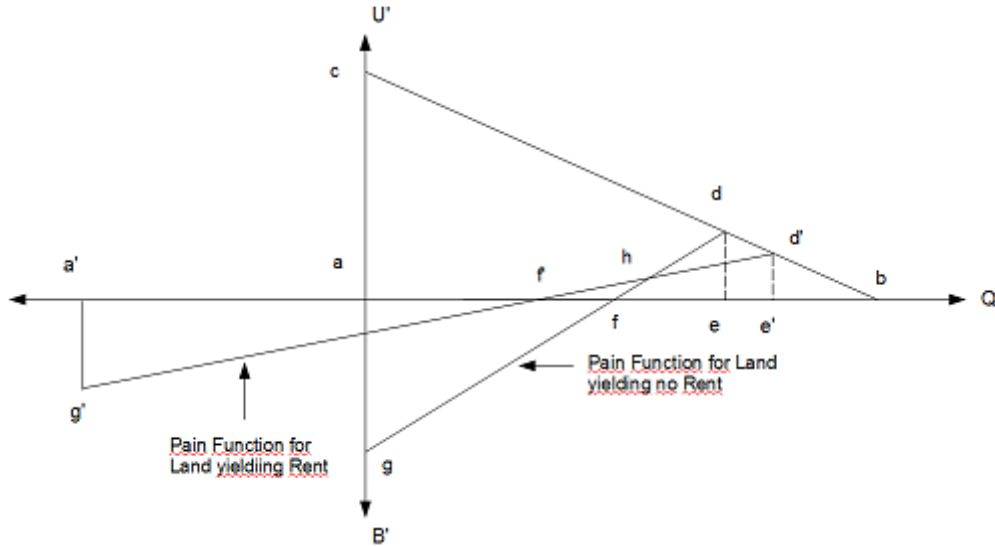


Figure 4: Gossen's rent analysis

Figure 4 shows Gossen's rent analysis (Gossen, 1854, p. 103, Fig. 21). The *utility function* is the same at two different places but their *marginal pain functions* differ. The distance $\overline{aa'}$ is the payable rent.

But now again we have to introduce the time constraint.

The Lagrangian for place **x** (and analogously for **y**) is

$$\mathcal{L}_x = U_x(Q_x - Q_R) - B_x(Q) + \lambda_x[L_0 - L] \quad (48)$$

and for **y**

$$\mathcal{L}_y = U_y(Q) - B_y(Q) + \lambda_y[L_0 - L] \quad (49)$$

The First Order Conditions are

$$\begin{aligned} \frac{\partial U_x}{\partial Q} - \frac{\partial B_x}{\partial Q} - \lambda_x \frac{\partial L}{\partial Q} = 0 & & \frac{\partial U_y}{\partial Q} - \frac{\partial B_y}{\partial Q} - \lambda_y \frac{\partial L}{\partial Q} = 0 \\ L_0 = L & & L_0 = L \end{aligned} \quad (50)$$

Using the same procedure as above to simplify the expression we introduce net utilities and the constants for marginal labour values and the First Order Conditions are respectively

$$\begin{aligned} \frac{\partial U_x^*}{\partial Q} - \lambda_x \frac{1}{\mu_x} = 0 & \quad \text{and} \quad \frac{\partial U_y^*}{\partial Q} - \lambda_y \frac{1}{\mu_y} = 0 \\ L_0 = L & & L_0 = L \end{aligned} \quad (51)$$

The optimal quantities are determined solely by the average productivities \mathbf{c}_x and \mathbf{c}_y and the amount of labour time, L_0 . In (15) we have defined

$$\frac{Q}{L} = \frac{\partial Q}{\partial L} + \frac{\partial Q}{\partial K} \frac{K}{L} + \frac{\partial Q}{\partial T} \frac{T}{L} = \mu + s; \quad s = \frac{\partial Q}{\partial K} \frac{K}{L} + \frac{\partial Q}{\partial T} \frac{T}{L} = \text{constant} \quad (15)$$

We define $c_x = \mu_x + s_x$; $c_y = \mu_y + s_y$ and so we have

$$Q_x^* = c_x L; \quad \text{and} \quad Q_y^* = c_y L \quad (52)$$

From the First Order Conditions (51) (assuming $\lambda_x = \lambda_y$) we get

$$\frac{\partial U_x^* / \partial Q_x^*}{\partial U_y^* / \partial Q_y^*} = \frac{1/\mu_x}{1/\mu_y} = \frac{v_x}{v_y} \quad (53)$$

The calculation of the payable rent at x involves now an additional step. From the First Order Condition (51) and the optimal quantity produced at y (52) we can determine λ_y . We equalize λ_x with λ_y and then we can determine the payable rent, \mathbf{Q}_R , at x so that the First Order Condition at x is satisfied. Equation (53) looks very much like the *Pareto-Optimum-Conditions*²⁸. Relative prices (relative net utilities) are equal to relative *marginal labour values* of the technical kind.

28 Here it becomes very clear why it is so important to define marginal labour values $1/\mu$ instead of Krelle's average labour values. Heinz Kurz comments in his article on Gossen show to what kind of errors that leads, in particular the abandonment of the labour theory of value in general. Kurz writes: "The proposition that relative prices determined by the marginal utility theory are directly proportional to the relative quantities of labour can astonishingly be found in almost all early marginalist authors, amongst them C. Menger, W. S. Jevons, E. v. Böhm-Bawerk and J. B. Clark... The fact, that with a positive rate of interest relative prices differ from labour values is known since Ricardo but is astonishingly overlooked by the early marginalist authors." (Kurz, 2009, p 485, footnote 19, transl. by the editor). Kurz' error is simple: he imposes upon the Marginalists Morishima's Cold War concept of labour value which ignores the norm of effectiveness (Kantorovich) instead of using the concept of *marginal labour value*, the revolutionary part of the Marginalist Revolution and the relevant concept for modern economics.

In modern theory we would do the analysis as follows: There is a perfect competitive market for a commodity which has a price p . The productivity of marginal land is zero and the production function of the firms using marginal land ignores land altogether.

$$Q_{mT} = f(K, L) \quad (54)$$

The optimal factor combination requires that the factor prices equal the value of their marginal product:

$$w = p \frac{\partial Q_{mT}}{\partial L} \quad \text{and} \quad r_K = p \frac{\partial Q_{mT}}{\partial K} \quad (55)$$

For firms using certain quantities of special land, T_i , this land enters as a fixed constant into their production functions:

$$Q_{Ti} = f(K, L, T_i); T_i = \text{const. for } i=1, 2, \dots \quad (56)$$

In their cost minimization problem enter also the factor prices for labour and capital, w and r_K .

The Lagrangian is:

$$\mathcal{L} = r_K K + wL + r_{T_i} T_i + \lambda [Q_{Ti}^0 - f(K, L, T_i)] \quad (57)$$

In this case the variables Q_{Ti} , K , L and r_{T_i} need to be determined.

The First Order Conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial K} &= r_K - \lambda \frac{\partial Q_{Ti}}{\partial K} = 0 \\ \frac{\partial \mathcal{L}}{\partial L} &= w - \lambda \frac{\partial Q_{Ti}}{\partial L} = 0 \\ \frac{\partial \mathcal{L}}{\partial T_i} &= r_{T_i} - \lambda \frac{\partial Q_{Ti}}{\partial T_i} = 0 \\ Q_{Ti}^0 &= f(K, L, T_i) \end{aligned} \quad (58)$$

Cost minimization will be at constant returns to scale and average cost will be equal to marginal cost, λ . and marginal cost equals price.

$$p = \lambda = \frac{dC}{dQ_{Ti}^*} = \frac{C}{Q_{Ti}^*} \quad (59)$$

Notice in particular that

$$\lambda = \frac{dC}{dQ_{T_i}^*} = w \frac{\partial L}{\partial Q_{T_i}} = p \quad (60)$$

From Euler's theorem we have

$$Q_{T_i} = \frac{\partial Q_{T_i}}{\partial K} K + \frac{\partial Q_{T_i}}{\partial L} L + \frac{\partial Q_{T_i}}{\partial T_i} T_i \quad (61)$$

This divided by L leads to (15).

$$\frac{Q_{T_i}}{L} = \frac{\partial Q_{T_i}}{\partial K} \frac{K}{L} + \frac{\partial Q_{T_i}}{\partial T_i} \frac{T_i}{L} + \frac{\partial Q_{T_i}}{\partial L} \quad (62)$$

$$\frac{Q_{T_i}}{L} = s_i + \mu_i; s_i = \frac{\partial Q_{T_i}}{\partial K} \frac{K}{L} + \frac{\partial Q_{T_i}}{\partial T_i} \frac{T_i}{L}; \mu_i = \frac{\partial Q_{T_i}}{\partial L}$$

The factor price for the land of type i is

$$r_{T_i} = p \frac{\partial Q_{T_i}}{\partial T_i} \quad (63)$$

The rent paid to i is

$$R_i = p \frac{\partial Q_{T_i}}{\partial T_i} T_i \quad (64)$$

It is clear that this rent is not earned by labour but a monopoly price. It occurs because T_i is fixed. It's magnitude depends mainly on the degree of social development and this is one of the reasons to socialise natural resources and land in particular.

VII. Conclusions

The *Theory of Pleasure* as developed by Hermann Heinrich Gossen is an early example of microeconomic analysis. It is a kind of activity analysis based on physiological functions. Taking Gossen's Fundamental Theorem we have sketched how to derive results of modern microeconomic analysis starting from Gossen's assumptions. In particular we have shown that his physiological

labour theory of value leads straight forward to the modern interpretation of the labour theory of value based on objectively determined marginal labour values. Moreover the labour theory of value holds also in the presents of surplus labour which yields rent or profits. This result we may call “The Old Solution”.

Coming to the end we should put some emphasize on a particular theorem of Gossen which is obviously true as long as there are people appreciating to having a good time together and it balances out a bit Gossen's ultra-liberal, free-market fetishism, Gossen's *Free-Round-Theorem*, a theorem ignored so far by all of his critics²⁹. The theorem goes as follows:

[4.1] “If a pleasure becomes available to man without his effort, his total life pleasure increases by the entire magnitude of this pleasure.”³⁰

(Gossen, 1983, p. 59)

In this sense, have a toast on the 200. anniversary of Gossen's birthday: Cheers!

Paris, 16.6.2010

Klaus Hagendorf

29 Presumably, because in the wine bars it is usually hidden behind the counters.

30 “*Durch den Genuß, der dem Menschen ohne sein Zuthun gewährt wird, vergrößert sich die Summe seines Genusses um die ganze Größe desselben.*” (Gossen, 1854, p. 50).

Appendix A: The Derivation of the Gossen Type Demand Function

The following procedure shows how the subjective utility function is used to derive a demand function with objective arguments for the determination of optimal price and quantity. For simplicity only 2 goods and a budget constraint are considered.

The Lagrangian for the maximization of utility, given the utility functions for 2 goods and a budget constraint with prices expressed in terms of marginal labour values, is:

$$\mathcal{L} = U_1 + U_2 + \lambda [L_0 - v_1 Q_1 - v_2 Q_2] \quad (\text{A-1})$$

The labour constraint, L_0 , is to be understood as “labour commanded” of the Classical economists. One obtains it by dividing a given amount of money by the ruling wage rate. For Robinson there is no money and so his budget constraint must be expressed in labour units. For him this means that his actual labour time constraint must be increased by the amount of labour time which he can save using his capital equipment. It is a very interesting problem if for Robinson, in long term equilibrium, there is a positive cost of using his equipment. This is equivalent to the problem of Schumpeter's zero-rate-of-interest theorem for the stationary state.

We define the budget constraint as *labour commanded*, that is Robinson's actual labour time constraint plus the cost, in terms of labour time, of using his equipment in addition to depreciation cost, the First Order Conditions are

$$\begin{aligned} \frac{\partial U_i}{\partial Q_i} &= \lambda v_i; \text{ for } i=1,2; \\ L_0 &= v_1 Q_1 + v_2 Q_2 \end{aligned} \quad (\text{A-2})$$

Eliminating λ and Q_2

$$\frac{a_{01} - a_{11} Q_1}{v_1} = \frac{a_{02} - a_{12} Q_2}{v_2} \quad (\text{A-3})$$

$$Q_2 = \frac{a_{02}}{a_{12}} - \frac{v_2}{v_1} \frac{a_{01} - a_{11} Q_1}{a_{12}} \quad (\text{A-4})$$

$$L_0 = v_1 Q_1 + v_2 \left(\frac{a_{02}}{a_{12}} - \frac{v_2}{v_1} \frac{a_{01} - a_{11} Q_1}{a_{12}} \right) \quad (\text{A-5})$$

and this resolved for Q_1 yields the demand function as a function of *marginal labour values*

$$Q_1 = \frac{v_1 L_0}{v_1^2 + \frac{v_2^2}{a_{12}} (a_{01} - a_{11})} \quad (\text{A-6})$$

or generally

$$Q_1 = f(v_1, v_2, L_0) \quad (\text{A-7})$$

A particularity of this Gossen type demand function is that it is not invertible which means it is somewhere backward bending. In **Figure 3** (page 22) this has been left out of the range.

Appendix B: Gossen's View on Value in Economics (See also Footnote 3, p. 3)

“Whoever has occupied himself with economics of the least scientific coloration knows that the disparate results obtained by various economists in their conclusions are merely the consequences of disparate definitions of value, and that in this science the endless number of controversies on this topic is the result of an equal variety of concepts of value. If I were to attempt to clarify the distinctions between these definitions, I would find myself involved in unlimited diffuseness since there is no good reason to give any one of these concepts a definite preference. Therefore, I will, limit myself here to calling attention to the following point: *According to my views of the external world, there exists nothing to which a so-called absolute value may be attributed.*

This is contrary to what is now assumed more or less explicitly by economists, for whom every object is conceived as having some definite inherent value. Nothing has induced more unfortunate

regulations than this fiction of an absolute value. The cause for all this undoubtedly has been the fact without the assumption of [an absolute] value, value seems to become subject to such tremendous fluctuations that it would appear difficult to make practical use of the term. In the matter of value, economists have found themselves in a still worse situation than the mathematicians were in regard to so many forces of nature before the invention of differential and integral calculus. Not only does mathematical analysis per se cause insuperable difficulties for most economists, but here they would have been obliged to work with a magnitude that constantly keeps changing in their hands and that would therefore slip too frequently away from them or disintegrate into nothing just when they thought they had taken hold of it. It was believed that one could overcome this elusiveness of value if one could postulate an absolute value. If such an absolute value existed, it would indeed simplify all calculations. It is regrettable, therefore, that such an absolute value does not exist and that all calculations without exception by economists have ended in error. It was believed that absolute value was determined by making inherent physical characteristics of objects an integral part of the concept of value – namely, those physical characteristics that render objects capable of satisfying pleasure to some significant degree, either directly or indirectly. Examples are food, wool, and, above all, gold and silver. In the pampas around Buenos Aires, however, the cattlemen let most of the meat rot, even though it contains all the same good qualities that give value to meat in our land. They are fully aware of these qualities, yet they let it rot and raise cattle only for the hides, horns, and hoofs. New settlers in North America, while fully aware of the qualities of wood, devote a great deal of effort to the destruction of whole forests. Also, Robinson, in full awareness of the qualities of gold, kicked in disgust a nugget he found. All this is perfectly comprehensible in terms of the situation just described, but could never occur if an absolute value were inherent in these objects. Indeed, it is a phenomenon of the greatest frequency, which could not go unnoticed, that

[3.1] *Only a limited quantity of any object has value for either the isolated individual or the majority of mankind. [italics added]*” (Gossen, 1983, pp. 54, 55; Gossen, 1854, pp. 46, 47).

References

- Akademie der Wissenschaften der UdSSR, and Ministerium für Hoch- und Fachschulwesen der UdSSR. 1972. *Grundlagen der marxistisch-leninistischen Philosophie*. 2nd ed., Berlin (DDR): Dietz Verlag.
- Ayres, Robert. 1994. Evolutionary economics and environmental imperatives. Chapter 12 in: *Information, Entropy and Progress. A New Evolutionary Paradigm*. Woodburg: AIP Press; 1994. p. 273 ff.
- Berthe, Bénédicte, and Renault, Michel. 2001. Economic analysis of human effort in organizations: a historical and critical perspective. In: Garrouste, Pierre; Ioannides, Stavros. *Evolution and path dependence in economic ideas: past and present*. Cheltenham (UK), Northampton (MA): Edward Elgar Pub., pp. 179-204.
- Boenigk, Otto Johannes Eugen Ferdinand Wilhelm (Freiherr von). 1896. *Staatswissenschaftliche Arbeiten. Festgaben für Karl Knies zur fünfundsiebzigsten Widderkehr seines Geburtstages in dankbarer Verehrung dargebracht von Eugen v. Böhm-Bawerk ... O. v. Boenigk ... J. B. Clark ...*
Berlin: O. Haering.
- Clark, John Bates. 1896. The Unit of Wealth. In (Boenigk, 1896, pp. 1-20)
- Georgescu-Roegen, Nicholas. 1983. Hermann Heinrich Gossen: His Life and Work in Historical Perspective. In: (Gossen, 1983).
- Gossen, Hermann Heinrich. 1983. *The Laws of Human Relations and the Rules of Human Action Derived Therefrom*, trans. Blitz, Rudolph C. Cambridge (Mass.) London: The MIT Press.
- Gossen, Hermann Heinrich. 1854. *Entwicklung der Gesetze des menschlichen Verkehrs und der daraus fliessenden Regeln für menschliches Handeln*. 1st ed. Braunschweig: Verlag von Friedrich Vieweg und Sohn.
- Gossen, Hermann Heinrich. 1927. *Entwicklung der Gesetze des menschlichen Verkehrs und der daraus fliessenden Regeln für menschliches Handeln*, 3rd ed. Berlin: R. L. Prager.
- Hagendorf, Klaus. 2008. The Labour Theory of Value: A Marginal Analysis. *Munich Personal RePEc Archive*, No. 10202.
- . 2009. Labour Values and the Theory of the Firm. Part I: The Competitive Firm. Available at: *SSRN: <http://ssrn.com/paper=1489383>*.
- Hayek, F. A. von. 1927. Einleitung. In: (Gossen, 1927, IX – XXIII).
- Jevons, William Stanley. (1871) 1965. *The Theory of Political Economy. With Preface and Notes and an Extension of the Bibliography of Mathematical Economic Writings*. 5th ed. New York: A. M. Kelley.
- Konstantinov, Fedor. V. 1982. *The Fundamentals of Marxist-Leninist Philosophy*. Trans. Dagish, Robert. Moscow: Progress Publishers.
Available at: <http://leninist.biz/en/1963/FML734/index.html>
- Krelle, Wilhelm. 1988. Über Gossens Gesetze des menschlichen Verkehrs. In: (Krelle, Recktenwald, 1987; 1988, pp. 25-47).
- Krelle, Wilhelm, and Horst Claus Recktenwald. 1987. *Gossen und seine "Gesetze" in unserer Zeit. Vademecum zu einem frühen Klassiker der ökonomischen Wissenschaft*. Verlag Wirtschaft und Finanzen.
- Krelle, Wilhelm; Recktenwald, Horst Claus. 1988. *Gossens Gesetze - Leitmuster moderner Nutzentheorie*. Mainz: Akademie der Wissenschaften und der Literatur.
- Kurz, Heinz D. 2009. Wer war Hermann Heinrich Gossen (1810–1858), Namensgeber eines der Preise des Vereins für Socialpolitik? *Schmollers Jahrbuch*, vol. 129, no. 3, pp. 473-500.
- Riedle, Hermann. 1953. *Hermann Heinrich Gossen 1810-1858: Ein Wegbereiter der modernen ökonomischen Theorie*. Thesis, University of Zurich, Winterthur: Keller.

- Smith, Adam. 1904. *An Enquiry into the Nature and Causes of the Wealth of Nations*. E. Cannan. London: Methuen & Co.
- Tugan-Baranowsky, Mikhail Ivanovich. 1905. *Theoretische Grundlagen des Marxismus*. Leipzig: Verlag von Duncker & Humblot.
- Wieser, Friedrich von. 1893. *Natural Value*. Edited with a preface and analysis by William Smart.