

Marxian Social Accounting and Modeling: A Social Accounting Matrix Approach

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1. Introduction

This paper presents a Marxian approach to macroeconomics from a class analytic perspective. The objective of the paper is to clarify and revive Marx's unique contribution to this area of economic thought, as well as to provide a new basis for pursuing a distinctly Marxian macroeconomics. In this paper I rely heavily on two different innovations in economic theory, one from inside the Marxian tradition and one from outside of it. The first is the Marxian class theory first put forward by Resnick and Wolff in *Knowledge and Class* (1987). Their work initiated a new line of inquiry in Marxian thought, and the growing list of topics that have been the subjects of class analysis is a testament to the richness of this contribution.¹ But new ideas answer some questions while posing others. *Knowledge and Class* and *Bringing It All Back Home* (Fraad, Resnick, and Wolff, 1994) present innovative class analytic theories of enterprise, the state, and the household but no clear statement of how these disparate sites relate to one another as an aggregate that is capable of economic reproduction. Providing that statement is one of the objectives of this paper.

The second source that I draw on here is the literature on Social Accounting Matrices (SAMs). SAMs are not themselves models *per se*, but rather can be used as frameworks for organizing macroeconomic data, as the basis for computable general equilibrium models and simulations, or as a format for presenting economic theory (Pyatt, 1988). While SAM's are used in this paper primarily for the latter purpose, using them to illustrate theory also results in a social accounting framework for data collection and analysis. The class analytic Marxian SAM presented herein makes it possible to resolve a known problem with Marxian social accounts based on an input-output (IO) approach. IO based accounts are unable to explicitly distinguish the purchase of input commodities by productive enterprises or sectors used as inputs for activities that are considered unproductive in Marxian theory (sales, marketing, certain managerial and

clerical functions, etc.). Previous studies of the U.S. economy have either assumed that this problem is not likely to be significant (Wolff, 1987, p. 80, n. 19) or they ignore it (Shaikh and Tonak, 1994). Contrary to these studies I argue that there is strong evidence to suggest that this omission is indeed large and that an alternative approach is necessary.

At a very general level a SAM can be said to bring together the production, circulation (income distribution and commodity exchange), and disposition (consumption or accumulation) of goods and services in an economy into a consistent framework. Production, circulation and disposition are the three basic moments in what I will call economic activity. When properly specified a SAM based model of economic activity has many similarities with Marx's reproduction schemes from volume two of *Capital*, though the SAM contains significantly more information and constraints. Because this technique presents these processes as mutually constitutive, in the sense that each of the three moments are specified with reference to the others, it is consistent with Marx's emphasis on the macroeconomy as a "unity" (1973, "Introduction", esp. pp. 99-100).

One of the basic contributions of this paper is to show how the production, appropriation and distribution of surplus value can be made explicit in an extended model of economic reproduction. Resnick and Wolff (1987, ch. 3) refer to the production, appropriation, and distribution of surplus value as a set of class processes. Individuals figure in the theory as occupants of various class and nonclass positions that are defined in their relation to these class processes (p. 146). In so doing they make a radical break from conceptions of class which define it in terms of 'social class' (e.g. in terms of income levels, hereditary status, etc.). Constructing as SAM using these class positions places the class processes at the center of a model of economic reproduction, and maps an institutional framework for economic reproduction in the form of an aggregate class structure.

The paper consists of two main sections. Section I provides the reader with a brief introduction to the construction of SAMs, with particular emphasis on how a SAM is influenced by theories of value and distribution. Section II develops and presents a Marxian alternative.

2. A Primer On Social Accounting Matrices And Economic Models

Table 1 presents the most basic conceptual SAM.² It is conceptual or schematic because it lists what activity (consumption, investment, saving, etc.) the entries in the matrix signify rather than specific numeric entries, and serves here as a purely heuristic device. A SAM can contain either numeric or algebraic entries and can be presented in either an aggregated or disaggregated form. In the aggregated form the elements of Table 1 would be sums representing the total for that activity in the population. In a disaggregated version the intersection of each account of Table 1 would contain a matrix, representing the actions of different sub groups in that population.

[INSERT TABLE 1 ON PAGE FOLLOWING PRECEEDING PARAGRAPH]

Taken as a whole every SAM is a matrix. Like an IO transactions table, each entry in the matrix registers transactions between the accounts or sites listed on the entry's column and row heading. Each entry represents expenditure when considered in its column context and revenue in its row context. However, a SAM differs from an IO transactions table in important ways and contains significantly more information. In particular a SAM contains additional information describing income distribution and transfers, and relates the final uses of commodities to the generation and circulation of income in ways that an IO model does not. An IO model only requires consistency between total output, final output and value added; a SAM presents data for the entire 'circular flow' of income and products and requires consistency at each step of the

process. The information necessary for an IO model is contained in only the first row and column of the SAM in Table 1. Every SAM contains within it an IO transactions table but represents a significant extension of this model by explicitly accounting for the circulation of income and thereby making consumption endogenous.³

For simplicity Table 1 has only three accounts: Production, Current Income/Expenditure, and Accumulation. These are sufficient for a rudimentary representation of the three basic moments of the circular flow of economic activity (production, circulation, disposition of output). Value added in production forms the current income of the economy, and is shown in the matrix as “Wages and Profits” at the intersection of row two and column one. This is the revenue source for expenditures described by the Current Expenditures column (Consumption, Transfers of Current Income, and Saving). Saving represents revenue into the Accumulation account (row three), which finances Investment. Consistency in the matrix is achieved by imposing the constraint that the sum of the i th row must equal the sum of the j th column. Additional accounts, such as for government and the foreign sector, can be added as needed for a more detailed depiction of an economy.

2.1 From ‘Society’ to ‘Sites’

The first step towards a more theoretically specific model is to introduce the concept of sites. As a matter of convention economic models typically recognize three broad categories of sites: business enterprises, households, and governments. In order to minimize the degree of complexity only enterprises and households are considered in this paper.

Both Marxian and non-Marxian theories agree that the interaction between households and enterprises is the basis of a theory of economic reproduction, but the structure of this relationship is, as I hope to show here, a matter of fundamental disagreement and contestation

between them. The contradictory positions that these theories take on this issue results from their different and antagonistic theories of value and distribution. This difference is a fissure running through the history of economic thought at least since Adam Smith's *Wealth of Nations* (1776). As is well known the *Wealth of Nations* contained two different and incompatible theories of value (Dobb, 1973, p.p. 112-120), a labor theory of value accepted and developed by Ricardo and then Marx, and one that Sraffa (1951) named the "Adding-up" theory of value which viewed land, labor and capital as independent sources of value. The 'Adding-up' theory of value formed the basis of the economics of Say, Malthus, Bastiat, Carey, and was later developed by Jevons, Walras, Menger, and the followers of their marginalist economics. In this paper I refer to all theories deriving from the 'Adding-up' approach to value, whether marginalist or not, as trinitarian.

The two different and incompatible theories of the *source* of value give rise to two different and incompatible *claims* to the value created in production, and it is these claims that provide a basis for the different theories of distribution. The labor theory of value gives rise to the concept of a surplus produced by labor over and above what they receive in compensation, which is subsequently appropriated and distributed to the groups who consume or accumulate this portion of the social income. Resnick and Wolff (1987, pp. 118-120) refer to the production and appropriation of surplus as the "fundamental class process", and to the subsequent distribution of the appropriated surplus as the "subsumed class process". These class processes may or may not be exploitative. Under exploitative class processes the individuals or groups who appropriate and distribute the surplus are different from those who produce it, and they distribute it in order to secure the conditions enabling this exploitation to take place. While the existence of surplus and exploitation have been the source of continuing debate between Marxian and non-Marxian theories, a key argument of this paper is that these issues also have implications for a theory of

economic activity and reproduction which has not generally been recognized. Value and distribution theories are the basis for statements establishing the categories and concepts that inform any model of economic activity, and hence they are also the basis for the organization and construction of a SAM. Any presentation beyond the rudimentary one presented thus far requires an explicit recognition of the underlying theories of value and distribution. The remainder of this section develops this idea by examining the specific ways that the dominant non-Marxian model of economic activity derives from the trinitarian theory of value and distribution.

Central to the trinitarian understanding of the relationship between the enterprise and the household is the argument that the enterprise is nothing more than the place where the various productive factors are brought together and put to use producing commodities. Harris (1975) describes the neoclassical vision of the relationship between enterprise and the household in this way,

The capitalist firm is seen merely as an intermediary between the individuals as suppliers of factors and the individuals as *rentiers* consuming their lifetime income (p. 329).

The factors are the property of households and the enterprise becomes their custodian when employing them in the production process. New value is created by the factors as a result of their factor services, the total of which represents value added in production. Since, as the theory contends, value added is the contribution of the different factors, their owners should receive it as compensation for their use—remuneration to each in proportion to their contribution. In a market based capitalist economy the revenues from the sale of the commodities accrue to the enterprise, which then distributes them to the owners of the various factors of production as factor incomes: wages for labor; dividends or interest for equity and finance capital; rents for real and intellectual property.

The key point is that in this vision of economic activity the enterprise plays no role other than that of facilitator; it benignly brings together and stewards the productive resources of numerous individuals that would otherwise remain uncoordinated and unproductive, and expedites the realization of the product of these resources for the various owners. Pre-marginalist theories of this type (Say, Bastiat, Carey, etc.) took the value adding capacity of the non-labor factors as a premise and appealed variously to fairness, natural rights, reason, pragmatism etc. to justify the claim to non-labor factor incomes. Marginalist theory likewise assumes the value adding capacity of capital but does not rely on such ‘normative’ claims for its theory of distribution—indeed one of marginalism’s principal contributions is exactly on this point (Stigler, 1968). It simply proposes that in a system of free markets and property rights this *will* be the case, albeit only if certain conditions are present (perfect information, costless recontracting, etc.). Euler’s Theorem, and additional conditions, solve the trinitarian “Adding-up problem” (Robinson, 1978) and allows marginalist theory to conclude that the total product is completely exhausted by paying each factor according to its marginal contribution. Furthermore, in a system of competitive markets the only equilibrium solution is one in which each factor is remunerated precisely in accordance with its marginal contribution. There can be no other durable outcome. Under such a system, according to marginalist theory, technology and the market mechanism provide the logic for income distribution, and enterprise is afforded no discretionary role. Enterprise simply mediates between the owners of productive factors and the consumers of goods and services by implementing the available technology, and serving as the conduit whereby revenues from the consumers are distributed to the rightful recipients.

In either case the basic argument remains the same. There is no portion of the revenues from production which is not the direct result of the productive service provided by one of the different factors, hence the role of the enterprise in income distribution is simply to return the

product of the different factors to their owners in the form of factor payments. The conclusion to be drawn from marginalist theory is that income distribution is a technical matter governed by engineering datum and nothing more. This is the logical extension of the pre-marginalist trinitarian approach which Marx (1967c, p. 830) characterized as “the conversion of social relations into things”. In either approach the enterprise is reduced to the passive instrument of the household. It employs such productive factors as the household chooses to provide; it responds to the household’s desire for consumption by providing products; it absorbs household savings and uses them to form new capital assets by accumulating investment goods.

It is a curious result that marginalist theory, whose *sine qua non* is choice, should reduce enterprise to the level of a mechanism in the general case, but this finds its antithesis in the role which is attributed to the household—this is the repository of choice and thereby causality. Since all current income belongs to households they make all expenditure decisions, including the decision to either consume or save. Households may transfer a portion of their income to another household, either voluntarily or under duress, but this does not change to total quantity of income available. The disposition of the economic product falls solely within the purview of the household.

In keeping with the preceding discussion, Table 1a is a revised version of the SAM in Table 1 and now incorporates the role that the different sites play in economic activity. The SAM presented here is premised on a trinitarian theory of value and distribution. Enterprise appears twice: once in its role as the site of Production, and once as the site of Accumulation. It is excluded categorically from having a current income and hence from making expenditures thereof. Households are identified with receiving Current Income and disposing of it (Current Expenditure). They supply and hence receive the incomes of labor and non-labor factors, and choose between consumption, income transfers, or saving. Household saving forms the revenue

for the Accumulation account for Enterprise and, as mediated by the Flow-of- Funds, is used by Enterprise for Investment. Table 1b reproduces Table 1a in matrix notation. Letting the number of enterprises or enterprise sectors be n and the number of households or household types be k , the matrices in 1b are defined as:

Z = $[z_{ij}]$ an $n \times n$ matrix of inter-industry flows.

Y = $[y_{ij}]$ a $k \times n$ matrix listing the payment of wages and profits from the enterprises to the households.

K = $[k_{ij}]$ an $n \times k$ matrix listing the purchase of commodities by the households from the enterprises.

T = $[t_{ij}]$ a $k \times k$ matrix listing the transfers of income between household groups.

H = $[h_{ij}]$ an $n \times k$ matrix listing the savings from households to financial intermediaries.

I = $[i_{ij}]$ an $n \times n$ matrix listing the purchase of commodities for investment purposes by enterprises.

F = $[f_{ij}]$ an $n \times n$ matrix listing the flow of funds between financial intermediaries and enterprises.

[INSERT TABLES 1a AND 1b AS FIRST FULL PAGE AFTER LIST]

Household's receipt of current income from Enterprise is registered as the first entry of the Current Income row; their expenditures on either Consumption, Transfers, or Saving is shown in the Current Expenditure column. The i th household in Table 1 or 1a faces the following constraint,

$$\sum_{j=1}^n y_{ij} + \sum_{j=1}^k t_{ij} = \sum_{i=1}^n k_{ij} + \sum_{i=1}^k t_{ij} + \sum_{i=1}^n h_{ij} \quad (1)$$

Its expenditures for consumption, transfers, and saving (RHS of eq. 1) are constrained by its revenues from factor incomes and transfers from other households.

The constraint facing the enterprise is somewhat more complex than the household because it is the site of both production and accumulation. From row one of Table 1 we see that Production revenues accrue to the enterprises from the sale of Intermediate and Investment goods to other enterprises, and from the sale of Consumption goods to households. Row three indicates that Accumulation revenues result from the inflow of Savings from the Household sector, or from Flows of Funds transfers from other enterprises. The Production expenditures of the enterprise consist of purchases of intermediate goods and primary factors as inputs for production. Accumulation expenditures consist of Investment through the accumulation of final commodities and Flow of Funds transfers. Setting total Enterprise revenues equal to expenditures and rearranging gives the Enterprise constraint,

$$\left[\sum_{j=1}^n (z_{ij} + i_{ij}) + \sum_{j=1}^k k_{ij} \right] + \sum_{j=1}^k h_{ij} + \sum_{j=1}^n f_{ij} = \sum_{i=1}^n (z_{ij} + i_{ij}) + \sum_{i=1}^k y_{ij} + \sum_{i=1}^n f_{ij} \quad (2)$$

In short, the enterprise is constrained to the buying and selling of commodities in order to produce other commodities and to form capital assets. It has no other function, and it

discriminates amongst households based solely on their endowment of productive assets. Even if an enterprise wanted to engage in some other activities it would be unable to for the simple reason that payments to the owners of the factors exhausts its net revenue leaving no resources to support other activities. Marginal productivity theory solved the “Adding-up problem” by putting the enterprise into Euler’s straightjacket—a virtuous straightjacket according to this theory, but a straightjacket nonetheless.

This concludes the arguments for this section. One objective was to show that accounting matrices have a unique pedagogic value in illustrating economic activity. The second point that I have tried to make here is that theories of value and distribution impact how we understand economic activity and, consequently, our model and representation of this activity in a SAM. The next section develops a model and accounting matrix that is constructed according to Marxian theory. The objectives are to create a uniquely Marxian class analytic theory of economic activity, cast it in the form of an accounting matrix, and contrast it with the one developed thus far, as well as with some other well-known Marxian social accounting systems.

3. A Marxian Model

Following Resnick and Wolff, I take class as Marx’s distinctive insight and the distinguishing feature of Marxian theory. Notice that the trinitarian model developed previously categorically denies the existence of surplus, and in so doing precludes any notion of class as a process involving the production of surplus or the subsequent exploitation of the producers. The objective here then is to produce a different economic model, one that not only recognizes and makes class relations based on surplus labor explicit, but also makes it the organizing principle of a Marxian model of economic activity. Doing so enables us to ‘see’ class at individual sites, in the relations between sites, and in the functioning of economic activity as a whole.

The most basic version of the model proposed here is exemplified by the SAM in Table 2 “A Marxian Class Analytic Accounting Matrix”. In order to allow for a direct comparison between the SAM in Table 2 and those in the previous section assumptions 1-4 from Tables 1 and 1a are retained:

[INSERT TABLES 2 AND 2a AS THE FIRST AND SECOND FULL PAGES FOLLOWING THE PRECEDING PARAGRAPH]

- 1) Production occurs only in enterprises.
- 2) All factors (labor, industrial and financial capital, real and intellectual property) used in production are the property of households.
- 3) This economy is ‘closed’ in the sense that it excludes transactions with sites beyond its borders.
- 4) All savings are used to finance capital accumulation by enterprise.

To simplify exposition additional assumptions are added,

- 5) No financial intermediation, (i.e. savings are passed directly from savers to borrowers) which results in the elimination of the Flow of Funds.
- 6) No inter-household income transfers.
- 7) Only capitalism, one of the different class processes potentially present in a social formation, is considered here.

Assumption seven is added for several reasons. Perhaps most importantly is to maintain continuity with the earlier model, which is a model of a capitalist economy. In order to be clear how the Marxian model is different from the trinitarian one it must set out to describe the same class form. The last explicit simplifying restriction placed on the Marxian model is that the previous assumption that all enterprises are productive is modified in the following way:

- 8) There are no wholly ‘unproductive’ enterprises (i.e. no purely financial, merchant, etc. enterprises), but unproductive activities may be present within otherwise productive enterprises.

These assumptions allow for the construction of the most rudimentary, but probably the most common, Marxian depiction of a capitalist economy. The simplicity allows for emphasis to be placed on what I wish to foreground here, namely the class process. I would likely agree with those who will criticize this approach for the injustice that it does to one or another of their particular concerns. But at this early state of development the emphasis on class as the *differentia specifica* of this model makes this narrowly circumscribed characterization a necessary choice.

Turning now to what is distinctive about the Marxian SAM in Table 2, an important difference from the earlier approach is the role of the enterprise in economic activity. Instead of simply catering to the desires of households, Enterprise—in this case capitalist Enterprise—is now understood as no less an active participant in economic activity than is the household. In Marxian theory capitalist enterprises contain an individual or small group that are the socially recognized appropriators and distributors of the surplus produced at that firm. Resnick and Wolff refer to this as the “fundamental class position” of the appropriator of surplus (1987, pp. 166-70). In Marx’s words, the capitalist who occupies the fundamental class position as appropriator of surplus is,

... the person who at first holds the whole surplus-value in his hands no matter how it may be distributed between himself and other people under the names of rent, industrial profit, and interest (1972, p. 471)

This is one of the roles traditionally ascribed to ‘the capitalist’ within a capitalist enterprise. At times Marx refers to a “functioning capitalist” and at others to an “industrial capitalist” as the occupant of the fundamental class position within the enterprise as the appropriator and distributor of surplus; throughout the remainder of this paper I use these two terms interchangeably. The industrial capitalist receives the surplus generated by capitalist production at the enterprise and subsequently distributes it to secure the conditions necessary to enable this exploitation to take place. While the industrial capitalist in the enterprise initially receives one form of the income generated by the production process, households receive another. Wages are paid directly from the enterprise to households that provide labor to the enterprise—a simple exchange of labor power for a wage—and this registers in the SAM as a payment from the Production account of Enterprise and an income into the Current account of the household.

The capital/labor relation therefore results in the occupants of two different class positions *initially* receiving the incomes generated by production: (1) wages are paid directly to households for providing productive labor to enterprises, and (2) surplus value realized for the enterprise by the sale of commodities is appropriated by the industrial capitalists. The appropriator of surplus stands at the juncture between the production and disposition of a portion of the social income. They are the first to receive it and they distribute it to subsequent recipients. Throughout much of *Capital* Marx’s discussion of the recipients of surplus focuses on the groups which classical political economy paid almost exclusive attention to: landlords as recipients of rents, the owners of enterprise as the recipients of industrial profits, and bankers or moneylenders as the recipients of interest payments. He also devotes considerable attention to merchants and

“commercial workers”, as well as discussing supervisors, managers, and others to a lesser extent, describing them as unproductive laborers whose incomes derive from distributed shares of the surplus from industrial capitalists. He lists the services that these unproductive workers provide as among the “*faux frais*” (false costs) of capitalist production, and treats these activities as forms of consumption rather than production (1967b, ch. 6, esp. p. 149; 1967a, pp. 330-335; 1972, pp. 355-358, and pp. 504-505).

Because the enterprise is the first to receive and expend the surplus value, an Enterprise sector is listed in the Current income and expenditure sections of the SAM. This depicts the movement of surplus value from its appropriation at an enterprise (an outflow from the Production account of Enterprise and an income into its Current account) to its distribution to the subsequent recipients (the Current expenditures of Enterprise in column two), thereby making this aspect of the circulation of surplus value explicit. Current expenditures of this revenue—distributions of surplus—can be grouped according to three broad types and are described in the SAM accordingly: (i) the consumption of commodities by the enterprises for the purposes of carrying on those unproductive tasks that Marx identifies as the *faux frais* or false costs of the economic system (K^c); (ii) the payment of dividends, interest and rents to equity holders, financiers, and the owners of real or intellectual property (S^p); and (iii) wages for laborers engaged in the type of unproductive (*faux frais*) activities discussed above (S^p). The expenditure of surplus value for commodities associated with the *faux frais* of capitalist production is an element of the final demand for goods and services. The expenditures of surplus value by an enterprise to households for providing capital, property, or unproductive labor represent a transfer of surplus value appropriated by the industrial capitalist from the productive laborers to the households who receive it as income. Productive laborers produce the surplus that is appropriated by the industrial capitalists, and, therefore, these two groups occupy “fundamental

class positions”; the income transfers to households by the distributions of surplus from industrial capitalists are referred to as “subsumed class payments”, and the recipients of these distributions are said to occupy “subsumed class positions” (Resnick and Wolff, 1987, pp. 118-120).

The quantity of surplus value realized by the enterprise is determined by its revenues and costs. The Production activities of an enterprise generate total revenues equal to the quantity of intermediate and final commodities it produces and sells; their production requires payments for intermediate goods and labor power. The demand for final commodities consists of three parts: total social consumption includes both the consumption of commodities by enterprises (\mathbf{K}^e) in order to carry out unproductive activities and the household’s demand for consumer goods (\mathbf{K}^h); the third element of final demand in this model economy is the accumulation of capital goods by the various enterprises (\mathbf{I}^e). Surplus value (\mathbf{S}) can be defined in the SAM as a residual of the revenue of the enterprise from the sale of commodities minus their direct production cost. Direct production cost for the enterprise is the sum of its inter-industry purchases (\mathbf{C}) and wage payments (\mathbf{V}) to productive laborers.

Presenting the Marxist theory of capitalist enterprise in this way has several advantages. Any SAM is a type of a model that describes the way a theory understands the production and distribution of commodities as well as the associated monetary flows in an economy. This is what differentiates it from a simple accounting statement. The most basic requirement of a Marxian model is to represent the class processes in its conception of economic activity and this requires making the production, appropriation, and distribution of surplus explicit. No existing Marxian model of the macroeconomy that I am aware of has accomplished this.

Another advantage of the model of economic activity presented in Table 2 and 2a is that it locates the production, appropriation and distribution of surplus as moments in the larger process of economic activity and thereby offers the potential to explore the relationship between

the class and nonclass (i.e. consumption, accumulation, saving, etc.) moments of this activity. Presenting the relationship between the enterprise and the household this way also separates the income that the household receives from Enterprise into two distinct components: income received for the sale of labor power for productive purposes (V), and income received which is a distribution of appropriated surplus value (S^P). Any household potentially has either or both of these income sources. Separating them makes possible an explicit analysis of the effects on the household from changes in either the capital/labor relation or the relative distributions of surplus by the enterprises.

Consider, for example, an increase in the rate of exploitation as measured by a decrease in wages received by the Household sector from Enterprise relative to the quantity of surplus value that the Enterprise sector appropriates. Because the Household sector receives income both in the form of wages and subsumed class payments this implies *prima facie* that the ability of the Enterprise sector to sell its entire output of final goods and services may not necessarily be constrained. A further elaboration of the composition of the household sector and the effects of income distribution on final demand is necessary to consider this question. An entire body of Marxist literature on economic crises derives from the assumption that wages are received by one portion of the population which *must* consume them to survive, while surplus value is received by another group who *must not* consume them (e.g. save and accumulate) to survive—leading to the conclusion that an inevitable tendency towards realization crises exists. This conclusion is only one of many possible outcomes that this approach suggests. Placing class in the context of economic reproduction provides the ability to characterize not only the specific ways that surplus value is produced, appropriated and distributed in a given economic formation but also the ways in which this shapes and interacts with the entire process of economic reproduction. This is a distinctive contribution of this approach to Marxian economics.

Isolating the costs for commodity and labor inputs for productive and unproductive activities in the SAM provides a method whereby both of these things can be measured explicitly. Previous Marxian social accounting systems based on an IO approach have been unable to do this. Edward N. Wolff (1987) acknowledges this when he states that his matrix of inputs for productive sectors,

. . . likely overstates the socially necessary input requirements. The reason is that sectors that produce productive output also engage in unnecessary activities. The labor input in the productive sectors is, in fact, segregated into a productive component l_{pp} and an unproductive component l_{up} . In principle the same separation of inputs should be done for intermediate inputs . . . However, because of data limitations, this is virtually impossible to do, and all the inputs in (the matrix of productive inputs) are considered productive. Though this overstates the amount of productive inputs, the error introduced is probably not excessive since the bulk of material inputs used in manufacturing, processing, and transportation, and the like are absorbed directly in production. (p. 80, n. 19)

Shaikh and Tonak (1994) do not address this problem, though their analysis clearly indicates that they are aware of the issue of unproductive activities within otherwise productive enterprises because, like Wolff, they distinguish between productive and unproductive labor in productive sectors. However, their data indicate that the magnitude of this omission is likely to be large, and that both Wolff's and their own failure to account for these costs leads to significant errors in their estimates. Table F.1 of Shaikh and Tonak (1994) calculates total employment in productive sectors of the U.S. economy over the period of 1948-1989, and then estimates the breakdown between productive and unproductive employment within these productive sectors.

Over the period the proportion of unproductive employment rises from 17 to 37 percent of the total employment in productive sectors, reaching almost 22 percent of all employment in the U.S. in 1989. If roughly one-third of the total employment in productive sectors is engaged in unproductive activity then a significant portion of the total purchased inputs of these sectors must be associated with these activities. Furthermore, failing to account separately for the input commodities consumed by 22 percent of the total U.S. employment is certain to lead to inaccurate estimates of the size and effects of unproductive activity.

Estimating what proportion of the total purchased inputs are used to support unproductive activities in this sector will require further study, but clearly the input costs associated with them can be expected to be large, both as a portion of the inputs of productive enterprises and relative to the economy as a whole. The modified IO framework that both Wolff (1987) and Shaikh and Tonak (1994) use does not isolate these costs and makes it difficult to estimate them separately. Treating the production, appropriation, and distribution of surplus value explicitly, as is done in the Marxian Social Accounting Matrix in Table 2, provides a relatively straightforward solution to this problem.

Finally, and perhaps most importantly, the Marxian SAM does not simply present a framework that can measure the flows of income and commodities in a given economy, though it does do that. It also establishes these flows as quantitative manifestations of the social relations that arise in the process of production and reproduction, with a particular emphasis on the class relations. In this way the Marxian SAM depicts an aggregate class structure. It also generates conventional economic categories—intermediate goods, consumption, investment, etc.—as aspects of a class analytic model of economic activity.

3.1 The Circular Flow in the Marxian SAM

The circulation of surplus value and the interrelation between the accounts in the SAM can be demonstrated using a technique developed by Thorbecke and Jung (1996, p 282-87). This requires that the SAM be partitioned into endogenous and exogenous accounts. Typically accounts for capital accumulation, government, and the foreign sector (when present) are considered exogenous. The Production and Current accounts are taken as endogenous. In the case of Table 2 the Accumulation account represents the sole exogenous sector, and is consolidated into a vector of exogenous injections \mathbf{x} . With these modifications Table 2 or 2a can be written as the following system of equations:

$$\begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{11} & \mathbf{T}_{12} & \mathbf{T}_{13} \\ \mathbf{T}_{21} & \mathbf{0} & \mathbf{0} \\ \mathbf{T}_{31} & \mathbf{T}_{32} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} \quad (3)$$

Where \mathbf{T}_{ij} is the matrix at the intersection of the i th row and the j th column of Table 2a (e.g. $\mathbf{T}_{11} = \mathbf{C}$, etc.). A coefficient matrix is formed using the endogenous portion of the SAM, with the coefficients are defined as,

$$\mathbf{A}_{ij} = \mathbf{T}_{ij} \hat{\mathbf{q}}_i^{-1} \quad (4)$$

Where $\hat{\mathbf{q}}_i^{-1}$ denotes the inverse of a diagonal matrix $\text{diag}(\mathbf{q}_i)$. This yields the following coefficient matrix:

$$\mathbf{A}_n = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \mathbf{A}_{13} \\ \mathbf{A}_{21} & \mathbf{0} & \mathbf{0} \\ \mathbf{A}_{31} & \mathbf{A}_{32} & \mathbf{0} \end{bmatrix} \quad (5)$$

Using this coefficient matrix the system of equations given by (3) can be written as,

$$\mathbf{q} = \mathbf{A}_n \mathbf{q} + \mathbf{x} \quad (6)$$

Which is written out in full as,

$$\begin{aligned}
\mathbf{q}_1 &= \mathbf{A}_{11}\mathbf{q}_1 + \mathbf{A}_{12}\mathbf{q}_2 + \mathbf{A}_{13}\mathbf{q}_3 + \mathbf{x}_1 \\
\mathbf{q}_2 &= \mathbf{A}_{21}\mathbf{q}_1 + \mathbf{x}_2 \\
\mathbf{q}_3 &= \mathbf{A}_{31}\mathbf{q}_1 + \mathbf{A}_{32}\mathbf{q}_2 + \mathbf{x}_3
\end{aligned} \tag{7}$$

or

$$\begin{aligned}
\mathbf{q}_1 &= (\mathbf{I}-\mathbf{A}_{11})^{-1}\mathbf{A}_{12}\mathbf{q}_2 + (\mathbf{I}-\mathbf{A}_{11})^{-1}\mathbf{A}_{13}\mathbf{q}_3 + (\mathbf{I}-\mathbf{A}_{11})^{-1}\mathbf{x}_1 \\
\mathbf{q}_2 &= \mathbf{A}_{21}\mathbf{q}_1 + \mathbf{x}_2 \\
\mathbf{q}_3 &= \mathbf{A}_{31}\mathbf{q}_1 + \mathbf{A}_{32}\mathbf{q}_2 + \mathbf{x}_3
\end{aligned} \tag{7.1}$$

The system of equation (7.1) can be used to show the circular flow mechanism through which a change in the exogenous aspects of the system impacts the endogenous relations. An increase in spending on constant capital (\mathbf{x}_1), for example, generates an initial increase in production of $(\mathbf{I}-\mathbf{A}_{11})^{-1}\mathbf{x}_1$. This increased production generates surplus value equal to $\mathbf{A}_{21}\mathbf{q}_1$, and since $\mathbf{x}_2 = 0$ this is the initial impact on \mathbf{q}_2 . The surplus value generated by the initial increase in production is appropriated by the various industrial capitalists according to \mathbf{A}_{21} . They disperse $\mathbf{A}_{12}\mathbf{q}_2$ for commodities to carry out the unproductive activities and distribute $\mathbf{A}_{32}\mathbf{q}_2$ to the occupants of the subsumed class positions. The initial increase in production also increases wage payments by the amount $\mathbf{A}_{31}\mathbf{q}_1$. Both $\mathbf{A}_{31}\mathbf{q}_1$ and $\mathbf{A}_{32}\mathbf{q}_2$ are incomes for the Household sector and serve to increase the demand for final commodities through an increase in \mathbf{q}_3 . The increase in demand for final commodities by the Enterprise and Household sectors generates further increases in production via the Leontief matrix multiplier $(\mathbf{I}-\mathbf{A}_{11})^{-1}$, which leads to further increases and a series of dampening cycles. The ultimate cumulative effect of the initial exogenous change on the various accounts depends upon the rate at which income leaks from the system, which in the model economy given in Table 2 results only from savings by households.

3.2 From Class to Class Positions

Up to this point class has been examined in the context of aggregated Enterprise and Household sectors. The final step in illustrating this class analytic approach to Marxian analysis is to

formally introduce the multiple different class positions discussed thus far by disaggregating these sectors. Doing so adds several different dimensions to the understanding of class relations. In particular, explicitly distinguishing different sub-groups makes it possible to illustrate the issue of exploitation. All productive laborers are exploited in a capitalist economy, but a SAM based model can take the issue of exploitation one step further. It makes it possible to illustrate how the exploitation of the occupants of one class position (i.e. productive laborers) serves as the source of income for the occupants of other class positions (i.e. subsumed classes and unproductive laborers). This model of the macroeconomy exemplifies how in the course of economic activity some individuals perform surplus labor and produce surplus value while others ultimately receive it and obtain their subsistence this way—and the latter may even include some who also labor, but labor unproductively in the service of the capitalist economy. In this way exploitation forms the basis of a different way—a uniquely Marxian way—of understanding social relations between different groups in society. Disaggregating the household sector to allow for multiple different class positions also makes it possible to consider both the struggle between productive workers and the enterprise over surplus value, as well as the potential for struggle amongst different social groups competing for shares of the appropriated surplus value. Class struggle can therefore be understood as pervading the whole social structure, not simply the point of production:

Tensions and struggles between occupants of positions within the fundamental class process must be examined alongside of and in interaction with tensions and struggles between occupants of positions within the subsumed class process.

Class struggles are understood to occur over the dimensions and forms of the subsumed class as well as of the fundamental class process (Resnick and Wolff, 1987, p. 122).

A few simple equations will serve to make this disaggregation clearly, beginning with the Current

account constraint of the Enterprise. Setting the current revenues of the i th enterprise in Table 2 equal to its current expenditures forms this constraint equation:

$$\sum_{j=1}^n s_{ij} = \sum_{i=1}^n k_{ij}^e + \sum_{i=1}^k s_{ij}^p \quad (8)$$

As an initial formulation, limit the potential subsumed class payments to wages of unproductive laborers (s_{lab}^p), plus the three conventional forms of payment to property owners: dividend payments (s_{div}^p), interest payments (s_{int}^p), and rents (s_{rent}^p). The simplest definition of subsumed class payments consist of these four potential constituent parts. The disaggregated form of equation (8) is then written as,

$$\sum_{j=1}^n s_{ij} = \sum_{i=1}^n k_{ij}^e + \sum_{i=1}^k (s_{lab_{ij}}^p + s_{div_{ij}}^p + s_{int_{ij}}^p + s_{rent_{ij}}^p) \quad (8.1)$$

The subsumed class payments are received by the Household sector, and form one of two possible income sources for this sector. If the amount of surplus value is held constant, each of the different demands on this surplus compete for a share against the other. Presenting the enterprise in this way enables a new and fruitful way to understand social relations and economic behavior. Consider, for example, what this implies about some relatively recent episodes in the economic history of the United States. The phenomena of ‘downsizing’ of corporate middle management, which garnered much attention during the first half of the 1990’s⁴, or the late 1990’s exuberance of businesses for information technology to eliminate ‘back office jobs’ and reduce retailing or wholesaling costs can be explained as efforts to reduce expenditures on unproductive labor (s_{lab}^p) and the other costs associated with unproductive activities (k^e). These strategies can be seen as efforts by the occupants of one certain class positions (equity holders, and financiers) to improve their positions at the expense of others (managers and commercial workers).

Turning next to the Household sector, the current revenues and expenditures of each household or household sector form the constraint equation:

$$\sum_{j=1}^n v_{ij} + \sum_{j=1}^n s_{ij}^p = \sum_{i=1}^n k_{ij}^h + \sum_{i=1}^n h_{ij}^\sigma \quad (9)$$

The left-hand or revenue side of this equation tells us that any individual household potentially has revenues from either the sale of labor power for productive purposes (v) to any of the n enterprises or enterprise groups, or from the distributions of surplus (s^p) from them. The class analytic interpretation of this equation is that the household potentially occupies a fundamental class position by laboring productively in return for wages, or a subsumed class position as the recipient of distributions of surplus. The right-hand or expenditure side of equation (9) indicates that the household may expend its income for the consumption of commodities provided by enterprise, a nonclass purchase of final goods and services, or may save it. Due to the assumption that the only potential use for savings is capital accumulation, saving would serve to secure either a new subsumed class position vis-à-vis one of the enterprises, or an expansion of an existing one.

As was done with equation (8), equation (9) can be expanded to make its class components explicit by subdividing the subsumed class revenues of the household into wages of unproductive labor, dividends interest, and rents. This yields,

$$\sum_{j=1}^n v_{ij} + \sum_{j=1}^n (s_{lab_{ij}}^p + s_{div_{ij}}^p + s_{int_{ij}}^p + s_{rent_{ij}}^p) = \sum_{i=1}^n k_{ij}^h + \sum_{i=1}^n h_{ij}^\sigma \quad (10)$$

This equation makes explicit the multiplicity of class positions that members of a household, or even an individual, may potentially occupy. The approach traditionally taken by Marxists is to identify different households with one of these class positions. An advantage of the traditional approach is to focus attention on exploitation as the source of the incomes for owners of capital goods, financial capital, and real or intellectual property. A convincing case can be made that this

is also the approach that Marx takes throughout much of *Capital*, though he largely abandons it in volume three as well as in *Theories of Surplus Value*. But this can also be seen as a particular assumption and not as a requirement to theorize society along class lines. One may choose to theorize in this way for political or empirical reasons, out of a perceived fidelity to Marx's texts, or other reasons, but these reasons should be recognized and acknowledged. Consider, for example, a productive laborer who also owns equity in capitalist industrial corporations through a pension plan. In such a case this person, and by extension their household, occupies both a fundamental class position as an exploited productive laborer and a subsumed class position as a recipient of distributions of surplus value by virtue of their pension fund. Over the course of their life their primary source of income will likely change from one source to the other. In this case—not an improbable one I might add—it would be a choice by the analyst to identify this individual with one of these different class positions. Critics of Marxism have also ignored the potential variability of class positions, and in so doing reached perverse conclusions regarding what a progressive economic agenda might entail. Consider, for example, the claims by the business writer Peter Drucker (1976) of an “Unseen Revolution” of “Pension Fund Socialism” resulting from the substantial holding of corporate equity by employee pension funds and the resultant flow of equity payments to retired workers. For Drucker the idea that individuals associated with productive labor might come to own a significant portion of corporate equity and receive the resultant equity payments constitutes a transition to socialism. What the approach presented here suggests, however, is that Drucker's transition to socialism is merely a change in the composition of the households receiving equity payments and not a change in the basic class structure of society itself.

4. Conclusion

The best points in my book are: 1. . . . the two-fold character of labour according to whether it is expressed in use-value or exchange-value, which is brought out in the very First Chapter; 2. the treatment of surplus-value regardless of its particular forms as profit, interest, ground rent, etc. . . . The treatment of the particular forms in classical political economy, where they are for ever being jumbled up together with the general form, is an olla potrida.' (Letter from Marx to Engels regarding *Capital*, 24 August 1867).

This paper demonstrates how to use class theory as the basis for a uniquely Marxian description of macroeconomic value flows. The objective is to show how the Marxian concepts of class and class exploitation can be modeled explicitly at the macroeconomic level as an aggregate class structure. This has the effect of emphasizing and clarifying the second point that Marx emphasizes in his own assessment of *Capital* reproduced above. What I have sought to bring out is that it is possible to provide a relatively simple multi-sector model of economic reproduction—which is admittedly still in a rather schematic form here—that both makes the relationship between surplus value and its appearance as “profit, interest, ground rent, etc.” explicit and locates this circulation of surplus value in a reproduction schema that includes a reasonably complete description of economic activity. This provides a basis to develop a more complex Marxian macro model that retains Marx’s class theory.

The obvious next step in the progression of the theory presented in this paper is to use the Marxian class analytic SAM as the basis for more formal modeling. Explicitly casting the relationships that are used in constructing the SAM in mathematical form would allow for a structuralist-type Marxian macroeconomic model, and for simulations. Another obvious

progression in the work presented here would be to reduce the number of restrictions placed on it. Removing these in particular would be especially productive: (i) the assumption of a closed economy, (ii) the absence of a government sector, and (iii) restriction of the analysis to only a capitalist class process. If the population is a nation then opening the model to transactions with sites other than the population under consideration allows a class analytic theory of international trade. This would likely be a fruitful way to pursue a Marxian theory of imperialism and globalization. Of particular interest to the author is what might follow from relaxing restrictions on the number of forms of the class process potentially present in a population. This would open the way to a class analytic theory of complex social formations.

FOOTNOTE TEXT

¹ Gibson-Graham, Resnick, and Wolff (2000) and (2001) compile some of these class analytic studies. Resnick and Wolff's first book length application of their class theory is Resnick and Wolff (2002).

² Table 1 and the SAM's in the remainder of this paper, are organized somewhat differently from the conventions which have become established in the more recent SAM literature. But the construction employed here is analagous to the early formulation developed by Stone (1961) and the first SAM produced by Stone and Brown in Cambridge University (1962)—the work which coined the term “Social Accounting Matrix”. It is also similar to the earliest published applied study outside of Cambridge done by Pyatt, *et. al.* (1973). The interested reader can follow the development of the SAM conventions to where they now stand in Pyatt and Thorbecke (1976), Pyatt, Roe and Associates (1977), and Pyatt and Round (1979 and 1985). Early applications and development of the SAM approach is surveyed in Pyatt and Round (1977).

³ Pyatt, Roe and Associates (1977, ch. 3) discuss the relationship between a SAM and an IO model.

⁴ Neither Gordon (1996) or Baumol, Blinder, and Wolff (2003) find evidence to support the claim that enterprises who announced “downsizing” initiatives actually reduced the size of their firm in the long run. Baumol, Blinder, and Wolff do, however, conclude that “. . . a central effect of downsizing has apparently been a transfer of income from labor to capital—that is, from workers to the owners” (p. 261), and that the net effect seems to have been a shift in income away from wage and salary earners to the recipients of income from property (p. 262).

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Table 1: Schematic Aggregated Social Accounting Matrix (closed economy)

	PRODUCTION	CURRENT: EXPENDITURE	ACCUMULATION	Σ
PRODUCTION	Intermediate	Consumption	Investment	Total Industrial Output
CURRENT: INCOME	Wages + Profits	Transfers of Current Income	0	Wages + Profits + Transfers
ACCUMULATION	0	Saving	Flow of Funds	Savings + Financial Transfers
Σ	Total Industrial Outlay	Consumption + Transfers + Saving	Investment + Financial Transfers	

Table 1a: Schematic Aggregated Social Accounting Matrix (closed economy)

		PRODUCTION	CURRENT: EXPENDITURE	ACCUMULATION	Σ
		ENTERPRISE	HOUSEHOLD	ENTERPRISE	
PRODUCTION	ENTERPRISE	Intermediate	Consumption	Investment	Total Industrial Output
CURRENT: INCOME	HOUSEHOLD	Wages + Profits	Transfers of Current Income	0	Wages + Profits + Transfers
ACCUMULATION	ENTERPRISE	0	Saving	Flow of Funds	Savings + Financial Transfers
Σ		Total Industrial Outlay	Consumption + Transfers + Saving	Investment + Financial Transfers	

Table 1b: Schematic Aggregated Social Accounting Matrix (closed economy)

		PRODUCTION	CURRENT: EXPENDITURE	ACCUMULATION	Σ
		ENTERPRISE	HOUSEHOLD	ENTERPRISE	
PRODUCTION	ENTERPRISE	Z	K	I	y_1
CURRENT: INCOME	HOUSEHOLD	Y	T	0	y_2
ACCUMULATION	ENTERPRISE	0	H	F	y_3
Σ		y_1'	y_2'	y_3'	

Notes:

Tables 1 and 1a assume:

- 1) Production occurs only in enterprises.
- 2) All factors (labor, industrial and finance capital, real and intellectual property) used in production are the property of households.
- 3) This economy is 'closed' in the sense that it excludes transactions with sites beyond its borders.
- 4) All savings are used to finance capital accumulation by enterprise.
- 5) All activity by enterprises is assumed to be 'productive'.

Throughout this paper the following notational conventions are employed: scalars are rendered as uppercase, no bold, or lowercase, no bold, italic; matrices as uppercase, bold; vectors as lowercase, bold.

Table 2: A Marxian Class Analytic Accounting Matrix (Closed Economy)

	PRODUCTION	CURRENT		ACCUMULATION	Σ	
	ENTERPRISE	ENTERPRISE	HOUSEHOLD	ENTERPRISE		
PRODUCTION	ENTERPRISE	Inter-Industry Flows	Commodities for 'Faux Frais'	Consumer Goods Consumption	Capital Goods Accumulation	Total Production Output
CURRENT	ENTERPRISE	Surplus Value	-	0	0	Enterprise Current Income
	HOUSEHOLD	Wages of Productive Laborers	a) Dividends, Interest, Rent b) Wages of Unproductive Laborers	-	Variable Capital Accumulation	Household Current Income
ACCUMULATION	ENTERPRISE	0	0	Household Saving	-	Sources of Accumulation Funds
Σ		Total Production Outlay	Enterprise Current Expenditure	Household Current Expenditure	Uses of Accumulation Funds	

Table 2a: A Marxian Class Analytic Accounting Matrix (Closed Economy)

		PRODUCTION	CURRENT		ACCUMULATION	Σ
		ENTERPRISE	ENTERPRISE	HOUSEHOLD	ENTERPRISE	
PRODUCTION	ENTERPRISE	C	K^e	K^h	I^e	q₁
CURRENT	ENTERPRISE	S	-	0	0	q₂
	HOUSEHOLD	V	S^p	-	I^v	q₃
ACCUMULATION	ENTERPRISE	0	0	H^{σ}	-	q₄
Σ		q₁'	q₂'	q₃'	q₄'	

These matrices are defined as:

C = $[c_{ij}]$ an $n \times n$ matrix of inter-industry flows.

S = $[s_{ij}]$ an $n \times n$ diagonal matrix recording the appropriation of surplus by the enterprises.

V = $[v_{ij}]$ a $k \times n$ matrix listing wage payments by the enterprise to the households.

K^e = $[k_{ij}^e]$ an $n \times n$ matrix listing the purchase of commodities by enterprises as part of the *faux frais*.

S^p = $[s_{ij}^p]$ a $k \times n$ matrix recording the distribution of surplus from the enterprises to the households.

K^h = $[k_{ij}^h]$ an $n \times k$ matrix listing the purchase of commodities by households for consumption.

H ^{σ} = $[h_{ij}^{\sigma}]$ an $n \times k$ matrix listing the savings by households.

I^e = $[i_{ij}^e]$ an $n \times n$ matrix listing the purchase of commodities for accumulation.

I^v = $[i_{ij}^v]$ a $k \times n$ matrix listing the purchase of labor power from households to expand employment.