FUZZY CONCEPT OF VALUE

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1. Introduction

One of fundamental problems of economic theory and market practice is the problem of measuring of commodity value. In a theoretical plan it was first formulated by Adam Smith, who selected two sides of commodity value. These are they utilities of consumption and labour production inputs. It was pre-flows of two directions in an economic theory.

The founders *of labour theory of the value* David Ricardo and Karl Marks and their followers within the framework of Marxian Economics were not able to decide the problem of reduction of labour, namely: taking of difficult labour to simple, economy of past labour (general funds) and future labour (quality of commodity) in real one. Founders *of theory of the marginal utility* William Jevons, Karl Menger, G. Gossen, L. Walras and their followers within the framework of Austrian Economics were not able to decide the problem of authentication of commodity value out of area of market equilibrium.

For this reason the attempt which was made by Alfred Marshall of synthesis of these directions appeared inadequate to the real economic relations. Among eccentric approaches to the decision of problem of measuring value, it is necessary to mark *the energetic theory of the value* S. Podolinsky within the framework of the Physical economy, determination of marginal value («shadow prices») within the framework of *theory of optimum allocation of the limited resources of the production* Leonid Kantorovich and Tjalling Koopmans, *theory of the expected utility* John von Neumann and Oskar Morgenstem within the framework of Economy of business decisions making and *a probabilistic approach to political economy* Emmanuel Farjoun and Moshe Machover.

2. Reasons of the value estimation vagueness

In our view, it is necessary to select three fundamental reasons of the vagueness of economic value estimation as the central category of the political economy:

1. *Duality problem*. Objective presence of two constituents of estimation of economic value is labour (input) and consumer utility (output), which are possible to define in the conditions of economic equilibrium only.
2. *Problem of the labour reduction*, which is taken to the necessity, at first, discounting of of past labour (general funds) and future labour (quality of commodity) in real one and, second, taking of difficult labour to simple. In the conditions of a new economy - knowledge economics it is interesting to consider the level of the labour activity vagueness (complications) in a role of the single (resulted) measuring device of the labour constituent – its entropy or informative material supply.
3. *The problem of the utility variety* arises up in connection with the variety of product consumption. Absence of complete set of the alternatives, availability of the contradictory information about the consumption results, as a rule, do not allow adequately to use probabilistic approach, which is based on complete and nonconflicting probability distribution of the possible events.

Therefore, for formal description of the economic value category is offered to use fuzzy approach and fuzzy measures, the special case of which are probabilities.

3. Formal uncertainty estimation

For formal uncertainty estimation of economic value we use conception of fuzzy measures [Sugeno 1977, Wang, Z. & Klir, G.J., 1992], which covers all known formal mechanisms of different types uncertainty description (measure of belief, plausibility, necessity, possibility, probability, fuzzy).

We will consider the universal set of events Ω from the knowledge base, which is named a sure event. The empty set $\varnothing$ equates with an impossible event. To every event $A \subseteq \Omega$ we will put in accordance the real number $\eta(A)$ being the fuzzy measure of level of authenticity A.

An fuzzy measure $\eta(A)$ names a function η: $A \rightarrow [0,1]$, which satisfies to the axioms of narrow-mindedness (1), monotony (2) and continuity (3):

$$\eta(0) = 0 ; \; \eta(\Omega) = 1 ; \tag{1}$$

$$\Omega \subseteq A \subseteq B \subseteq \varnothing \Rightarrow \eta(A) \le \eta(B); \tag{2}$$

$$\lim_{i \to +\infty} \eta(A_i) = \eta\left(\lim_{i \to +\infty} A_i\right), \tag{3}$$

where $\{A_i\}_n$ is sequence of the inlaid great numbers of kind $A_0 \subseteq A_1 \subseteq ... \subseteq A_n \subseteq ...$ or $A_0 \supseteq A_1 \supseteq ... \supseteq A_n \supseteq ...$.

Expression $\eta(A)$ represents the degree of event A fuzzy, which can be interpreted as judgment fuzzy estimation of "$A \in \Omega$" or degree of subjective compatibility of event A with a sure event Ω. Monotony (2) of fuzzy measure $\eta(A)$ draws implementation of the following inequalities system:

$$\eta(A \cup B) \ge \max\{\eta(A), \eta(B)\};$$

$$\eta(A \cap B) \le \min\{\eta(A), \eta(B)\}, \; \forall \, A, B \in \Omega . \tag{4}$$

We will generalize the system (5) in case of arbitrary t- and s-norms:

$$\eta(A \cup B) \ge s\{\eta(A), \eta(B)\};$$

$$\eta(A \cap B) \le t\{\eta(A), \eta(B)\}, \; \forall \, A, B \in \Omega . \tag{5}$$

Specification of the system (5) allows to get the different special cases of fuzzy measure $\eta(A)$. If in the system (5) the first inequality is strict, and the second grows into equality, we will get family of *superadditive measures* possessing property, the special cases of which are:

measures of belief [Shafer 1978] (lower probabilities [Dempster 1967]):

$$\mu(A \cup B) > \mu(A) + \mu(B) - \mu(A \cap B),$$

$$\mu(A \cap B) = \mu(A)\mu(B); \tag{6}$$

measures of necessity [Dubois and Prade 1988] (concerted functions of belief [Shafer 1978]):

$$\mu(A \cup B) > \max\{\mu(A), \mu(B)\},$$

$$\mu(A \cap B) = \min\{\mu(A), \mu(B)\}. \tag{7}$$

For superadditive measures $\{\mu(\bullet)\}$ with the values from $\{0,1\}$ are executed $\mu(A \cap \overline{A}) = 0$, $\mu(\mathrm{A}) + \mu(\overline{A}) \le 1$ and if $\forall A \in \Omega$. Therefore these measures allow adequately to estimate the economic value in the conditions of incomplete great number of possible alternatives.

If in the system (5) the second inequality is strict, and the first grows into equality, we get family *of subadditive measures* $\{\nu(\bullet)\}$ possessing property $\nu(A \cup B) + \nu(A \cap B) < \nu(A) + \nu(B)$, the special cases of which are:

measures of plausibility [Shafer 1978] (upper probabilities [Dempster 1967]):

$$\nu(A \cup B) = \nu(A) + \nu(B) - \nu(A)\nu(B),$$

$$\nu(A \cap B) < \nu(A) + \nu(B) - \nu(A \cup B) = \nu(A)\nu(B); \qquad (8)$$

possibility measures [Zadeh 1978, Dubois and Prade 1988]:

$$\nu(A \cup B) = \max\{\nu(A), \nu(B)\},$$

$$\nu(A \cap B) < \min\{\nu(A), \nu(B)\}. \qquad (9)$$

For subadditive measures $\{\nu(\bullet)\}$ with the values from $\{0,1\}$ are executed $\nu(A \cup \overline{A}) = 1$, $\nu(\mathrm{A}) + \nu(\overline{A}) \geq 1$ for $\forall A \in \Omega$. Therefore these measures allow adequately to estimate the economic value in the conditions of contradictory (surplus) great number of possible alternatives.

If the system of inequalities (5) assumes a system of equalities form we will get that determination: probability measures ρ for uninteractive events (additive measure); probability measures for interactive events ($A \cap B \neq \varnothing$) at confluence of the systems (6) and (8); classical degree of fuzzy belonging at confluence of the systems (7) and (9).

For the possibility measures $\nu(A)$, probability measures $\rho(A)$, necessity measures $\mu(A)$ at $\forall A \in \Omega$ the following including is executed:

$$\nu(A) \geq \rho(A) \geq \mu(A). \qquad (10)$$

Probability describes the very narrow class of uncertainty, that explains narrow-mindedness of the methods, based on mathematical statisticians.

In axiomatic determinations of fuzzy measures (6)–(9) different concrete prospects are present t- and s-norm (see table 2), that specifies on the adequate operators of their treatment.

Breaking up on the classes of great fuzzy numbers of the economic value parameters in accordance with table 1.

Table 1 Classes of fuzzy of parameters and operators of their treatment

N	Classes fuzzy measures	Descriptions of uncertainty			Operators of treatment	
		plenitude	nonconflict	co-ordination	*t*-norms	*s*-norms
1	Clearness	1	1	1	$t = t_1$	$s = s_1$
2	Probability	1	1	0	$t = t_4$	$s = s_4$
3	Possibility	1	0	1	$t = t_4$	$s = s_5$
4	Plausibility	1	0	0	$t = t_3$	$s = s_4$
5	Necessity	0	1	1	$t = t_5$	$s = s_4$
6	Belief	0	1	0	$t = t_4$	$s = s_3$
7	Tie-up*	0	0	1	$t = t_2$	$s = s_2$
8	Complete fuzzy	0	0	0	—	—

∗this class is entered by author for the first time starting from description plenitude of fuzzy measures and is required additional research.

Classes t-norm and s-norm are used for simulation is represented in table 2. For any $\mu_1, \mu_2 > 0$ a next order takes place choice $s_1 \geq s_2 \geq s_3 \geq s_4 \geq s_5 \geq t_5 \geq t_4 \geq t_3 \geq t_2 \geq t_1$.

The offered accordance between the fuzzy measures classes and operators of treatment (table 1) is determinations of fuzzy measures (1)–(9) investigation of and rules of choice of the most informing operator from possible:

– choice of operator of t-*norms* for the measure of possibility of $t < t_5 \Rightarrow t \in \{t_i\}_4$, $t_1 \leq t_2 \leq t_3 \leq t_4 \Rightarrow t = t_4$;

– choice of operator of t-*norms* for the measure of plausibility of $t < t_4 \Rightarrow t \in \{t_i\}_3$, $t_1 \leq t_2 \leq t_3 \Rightarrow t = t_3$;

– choice of operator of s-*norms* for the measure of necessity of $s > s_5 \Rightarrow s \in \{s_i\}_4$, $s_1 \geq s_2 \geq s_3 \geq s_4 \Rightarrow s = s_4$;

– choice of operator of s-*norms* for the measure of belief of $s > s_4 \Rightarrow s \in \{s_i\}_3$, $s_1 \geq s_2 \geq s_3 \Rightarrow s = s_3$.

Table 2 Base operations t-norm and s-norm

Classes	Properties	Operation t- norm & s- norm
Idempotent norms	$t(\mu,\mu)=\mu$ $s(\mu,\mu)=\mu$	minization $t_5(\mu_1,\mu_2)=\min\{\mu_1,\mu_2\}$ maximization $s_5(\mu_1,\mu_2)=\max\{\mu_1,\mu_2\}$
Strictly monotonous Archimedean operations	$t(\mu,\mu)<\mu$ $s(\mu,\mu)>\mu$	$t_\gamma(\mu_1,\mu_2)=\dfrac{\mu_1\mu_2}{\gamma+(1-\gamma)(\mu_1+\mu_2-\mu_1\mu_2)}$ $s_\gamma(\mu_1,\mu_2)=\dfrac{\mu_1+\mu_2-(2-\gamma)\mu_1\mu_2}{1-(1-\gamma)\mu_1\mu_2}$ $\gamma=1$ probabilistic multiplication $t_4(\mu_1,\mu_2)=\mu_1\mu_2$ probabilistic sum $s_4(\mu_1,\mu_2)=\mu_1+\mu_2-\mu_1\mu_2$ $\gamma=2$ $t_3(\mu_1,\mu_2)=\dfrac{\mu_1\mu_2}{2-(\mu_1+\mu_2-\mu_1\mu_2)}$ the Lorents operator $s_3(\mu_1,\mu_2)=\dfrac{\mu_1\mu_2}{1+\mu_1\mu_2}$
Nil- Idempotent norms	$t(\mu,\mu)<\mu$ $s(\mu,\mu)>\mu$ $s(\mu,1-\mu)=1$ $t(\mu,1-\mu)=0$	limited difference $t_2(\mu_1,\mu_2)=\max\{0,\mu_1+\mu_2-1\}$ limited sum $s_2(\mu_1,\mu_2)=\min\{1,\mu_1+\mu_2\}$ strongly limited difference $t_1(\mu_1,\mu_2)=\begin{cases}\mu_1, & \text{if} \quad \mu_2=1;\\ \mu_2, & \text{if} \quad \mu_1=1;\\ 0, & \text{if} \quad \mu_1,\mu_2\neq 1\end{cases}$ strongly limited sum $s_1(\mu_1,\mu_2)=\begin{cases}\mu_1, & \text{if} \quad \mu_2=0;\\ \mu_2, & \text{if} \quad \mu_1=0;\\ 1, & \text{if} \quad \mu_1,\mu_2\neq 0\end{cases}$

4. Fuzzy approach for the economic value estimation

The fuzzy model of measuring of commodity value, which allows to synthesize labour expense and utility (effectiveness) its constituents, is offered:

1. The labour constituent of value of commodity V_L is determined taking into account the level of complication of expenses of past (capital), real (living labour) and future (quality) labour:

$$V_L = \underset{i}{S_N}\left[T_N\left(\log_2 \eta_i,\, \tau_i\right)\right], \qquad (11)$$

where τ_i - public necessary expenses of time of *i*–types of labour;

η_i - it is index of variety of *i*–types of labour;

S_N, T_N are operators *of* s*-norms* and t*-norms* (table 2);

N — class of uncertainty (table 1).

2. Utility making the value of commodity V_U is determined by unclear generalization of index of the expected utility J.Neumann and O.Morgenstem, that allows to estimate consumption of commodity from risk positions (additive distributing), incompleteness (subadditive distributing) and contradiction (superadditive distributing) of information:

$$V_U = \underset{j}{S_N}\left[T_N\left(\mu_j,\, u_j\right)\right], \qquad (12)$$

where μ_j - fuzzy measures (superadditive - belief measures, subadditive - plausibility measures, additive - probability measures);

η_j it is utility of alternative of consumption of commodity;

S_N, T_N are operators *of* s*-norms* and t*-norms* (table 2);

N — class of uncertainty (table 1).

3. Synthesis of labour and utility making the value of commodity is determined by their equilibrium:

$$V_L \times (1+\gamma) \equiv V_U, \qquad (13)$$

where $\gamma > 0$ is parameter of rising productivity of labour (the law of falling labour-content).

4. Оценка эластичности условия равновесия (13):

$$E_{UL} = \frac{dS_U}{dS_L} \times \frac{S_L}{S_U}, \qquad (14)$$

where E_{UL} is a elasticity coefficient, which shows on how many percents changes utility making the economic value V_U at the change of the labour constituent of economic value V_L on 1%.

Thus it is possible to define the development phases of complex economics systems (see table 3):

Table 3 Development phases of complex economics systems (CES)

Development phases CES	Elasticity coefficient
Growth	$E_{UL} > 1$
Equilibrium	$E_{UL} \cong 1$
Recession	$0 \le E_{UL} < 1$
Crisis	$E_{UL} < 0$

Adequate stochastic estimation of the economic value is possible only on the phase of equilibrium, when the labour and utility constituents are balanced (13).

5. Conclusion

This article must be examined, as debatable attempt to formulate bases of the fuzzy approach to the economic value estimation.

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