Toward a Generalized Marxian Approach: A Synthesis of Heterodox Economic Approaches*

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Introduction

The aim of this paper is to synthesize three heterodox economic approaches to the problem of how accumulation is determined. I will therefore look at the Marx-Morishima, Keynes-Robinson and Marris-Wood approaches, and provide a generalized Marxian approach, from which the three approaches can be derived.

In section 1, I first provide a basic model. It contains the common features which three approaches share. They have three unknown variables: the rate of

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profit, growth and real wage, and two equations: the Cambridge equation and the wage-profit frontier. In order to complete the model, it is necessary to add one more equation. In the heterodox tradition, this can take the form of one of the three above mentioned approaches. Second, I will outline the three approaches. In the case of the Marx-Morishima approach, the wage-profit frontier is introduced with the constant conventional wage. According to the Keynes-Robinson approach, the model should be closed by an investment function. The Marris-Wood approach, to which little attention has been given, insists that it should be closed by what Marris calls the "growth-profitability function." Thus before us are three different approaches in a somewhat disjointed state. Therefore, a simple question is raised: which is correct?

In section 2, I will lay out a generalized Marxian approach in a constructive way, which takes explicitly into account the fact that it takes time for capital to move through its circuit. This is an expanded model of Foley (1982) approach, inspired by Marx's circuit of capital. This model is composed of two equations. One is the Cambridge equation. The other describes how long it takes for capital to pass through the processes of production and circulation. We call this equation the generalized wage-profit frontier, which relates profit rate not only to wage rate but also to growth rate. This equation can be identified with the growth-profitability frontier, as well as the wage-profit frontier if we add a simplified assumption that the capital-turnover rate is unity. If the real wage is assumed to be a constant, those two equations in this model, which are named circuit conditions, determine the profit rate and the growth rate at which capitalists will be content with what they are doing. This growth rate is, therefore, interpreted as Harrod's warranted rate of growth. That means this rate is not the actual rate of growth, which one observes at a point of time. If circuit conditions determine the warranted rate of growth, then what does determine the actual rate of growth?

In section 3, the paper will aim to derive a Marxian investment function, which determine the actual growth rate. Note that it is through the continuous metamorphosis that the self-expansion of the capital-value takes place. We can interpret the self-expansion of the capital-value as its objective, and metamorphosis as its constraint. Therefore capitalists maximize the capital-value, which is interpreted as the present value of net profits, discounted at interest rate, subject to the circular movement of capital, which can be translated as the circuit conditions. The first order condition indicates that the marginal rate of profit equals the promoter's profit per capital. This condition is nothing more than Marxian investment function. In this model, therefore, the four equations: Cambridge equation, generalized wage-profit frontier, the conventional wage and Marxian investment function can be deemed to determine our four variables of: profit, growth, real wage and interest rate.

1 The Basic Model

According to Marglin (1984b) and Dutt (1990), we construct a basic model.

Production is characterized by one-commodity model. We assume that the output is produced by means of two factors of production: input and labor input. Technology is assumed to exhibit fixed coefficient of input a and labor input l.

Price formation is

$$p = (1+\pi)pa + wl.$$

This equation says that the nominal price of the commodity p is the sum of the cost of the input pa, the profit πpa (in which π is the profit rate), and the cost of wage wl (in which w is the nominal wage rate).[1]

This equation should be expressed in real terms, not in nominal term. Let us introduce ω as the real wage w/p. Dividing through by p, and after some manipulation, we get:

$$\pi = (1 - a - \omega l)/a. \tag{1}$$

We call equation (1) "wage-profit frontier."

Turn now to the quantity side. It is assumed that workers' wage should not be saved, and capitalists save a fraction, s of their profit. s is called the capitalist's propensity to save, and assumed to be a constant. These assumption imply that capitalist's savings equal their investment, so that

$$s(p - pa - wl)x(t) = pa\dot{x}(t),$$

where $x(t)$ is output level at time t, $\dot{x}(t) \equiv \mathrm{d}x(t)/\mathrm{d}t$ is the increment of output at time t. This equation says that capitalists' savings from the profit at time t must equal their investment.

Similarly, the scale of quantity is arbitrary and then needs to be normalized. We set the volume of capital as numéraire, which is $pax(t)$ in the basic model. Dividing by $pax(t)$, we get:

$$g = s\pi, \tag{2}$$

where $g \equiv \dot{x}(t)/x(t)$, which is the rate of growth under the stationary state. This is so-called Cambridge equation.[2]

[1] We assume that wage is paid at the end of period.

[2] More precisely, we should distinguish between the rate of demand-side growth and supply-side growth, denoted by g^{d} and g^{s}, respectively. If we define $g^{\mathrm{d}} \equiv \dot{x}(t)/x(t)$ and $g^{\mathrm{s}} = s\pi$, then Cambridge equation in the basic model means that $g^{\mathrm{d}} = s\pi$, not $g^{\mathrm{s}} = s\pi$. In other words, Cambridge equation represents an equilibrium condition: $g^{\mathrm{d}} = g^{\mathrm{s}}$.

In the basic model, there are three unknown variables: π, ω, g, but there are only two equations: the wage-profit frontier (1) and Cambridge equation (2). One equation should be added in order to close the basic model. What kind of a equation should be added?

The answer is not the only one. We have, at least, three answers in the heterodox tradition: Marx-Morishima Approach, Keynes-Robinson Approach and Marris-Wood Approach. We examine these approaches in the following subsections.

1.1 Marx-Morishima Approach

In Marx-Morishima approach, the real wage is conventionally determined, in short, exogenously given.[3] This real wage is called the conventional wage, denoted by b. We get:

$$\omega = b. \tag{3}$$

In Marx-Morishima approach, there are three unknown variables: π, ω, g, and three equations: the wage-profit frontier(1), Cambridge equation(2), and the conventional wage(3). We assume that $1 > a + bl$, then this system has a unique solution.

1.2 Keynes-Robinson Approach

In Keynes-Robinson approach, the system is closed the by long-run investment function[4]:

$$g = g(\pi), \qquad g'(\pi) > 0. \tag{4}$$

This investment function shows that the rate of growth is determined by the rate of profit. More precisely, this equation should be written as $g = g(\pi^e)$ where π^e is the expected rate of profit. Because the higher the expected profit rate is going to be, the higher the growth rate is going to be. In order to close the model, the static profit expectations are employed, then $\pi^e = \pi$.

Therefore, three equations (1), (2), and (4) determine three unknown variables: π, ω, g. This equation system is completed.

[3] See Marx (1977), Morishima (1973), and also see Marglin (1984a,b)

[4] This equation is formulated by Robinson (1962). Also see Roemer (1981, chap. 9).

1.3 Marris-Wood Approach

In the place of the investment function, Marris-Wood approach is closed by so-called growth-profitability function.[5]

$$\pi = \pi(g), \qquad \pi'(g) < 0, \quad \pi''(g) < 0. \tag{5}$$

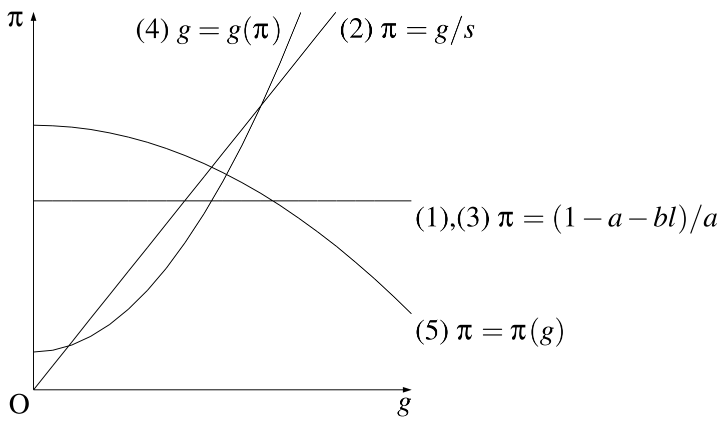
This function states that there is negative relationship between the profit rate and the growth rate. This is because, in order to achieve faster growth rate, the firm must spend higher "development expenditure," which is, for example, the cost for R & D and/or advertisement.

This system is consist of three equation: (1), (2), and (5) and three variables: π, ω, and g.

1.4 What is the Relation between the Three Approaches?

Now we obtain three approaches to complete the basic model. But these approaches are incompatible. As Figure 1 illustrates, there are too many equations for the number of unknown variables. For example, Marx-Morishima approach and Keynes-Robinson approach are incompatible, for the conventional wage and the investment function overdetermine the system.

Figure 1: Incompatibility between Three Approaches



More importantly, each approach has its own entirely different closures. In $\pi \times g$ as Figure 1, Marx-Morishima, Keynes-Robinson, and Marris-Wood approaches are closed by the horizontal, upward, and downward curve, respectively. There is thus an enormous difference between their approaches, and each claims to be the

[5] See Marris (1967). And also see Marris (1971, chap. 1), and Wood (1975).

most fundamental approach in the construction of the theory of the dynamics of capitalism.

Before us are three different approaches in a somewhat disjointed state. Therefore, a simple question is raised: which is correct?

In next section, we provide a generalized Marxian approach, from which the three approaches can be derived.

2 A Generalized Marxian Model

The circuit of capital, in *Capital*, volume II, chapter 1 provides the analytical tool to construct a generalized Marxian model. Marx represents the circular motion of capital in the following formula:

$$\mathrm{M} - \mathrm{C} \cdots \mathrm{P} \cdots \mathrm{C}' - \mathrm{M}'.$$

For Marx, the word capital is not given the same meaning as in modern economic literature. Capital undergoes the metamorphosis which transforms M, money-capital, successively into P, productive capital, C′, commodity-capital yet again and finally money-capital when the cycle is completed.

But we would like to employ the following formula as the circuit of capital, slightly different from Marx's formulation:

$$M \longrightarrow P \dashrightarrow C' \longrightarrow M'.$$

The difference is that C is omitted. The reason of this omission is that C in Marx's formulation does not represent any capital: money, productive, and commodity-capital, but represents the transaction of commodities. In other words, M, P, and C' are stock variables but C is exceptionally a flow variable. In our formulation, we can distinguish between stock and flow variables: each of the nodes correspond to the stocks, and the arrows between each nodes correspond to the flows. Our formulation will leave less room for misunderstanding than Marx's formulation.

We provide our formula of capital circuit with the mathematical expression in the next subsection.

2.1 Circuit of Capital

Figure 2 gives us the entire picture of capital circuit.

The phase 1 represents the transformation from money-capital M into productive capital P, in short, the phase of purchase. Purchasing commodities, say, raw materials, means increasing the amount of productive capital and decreasing the

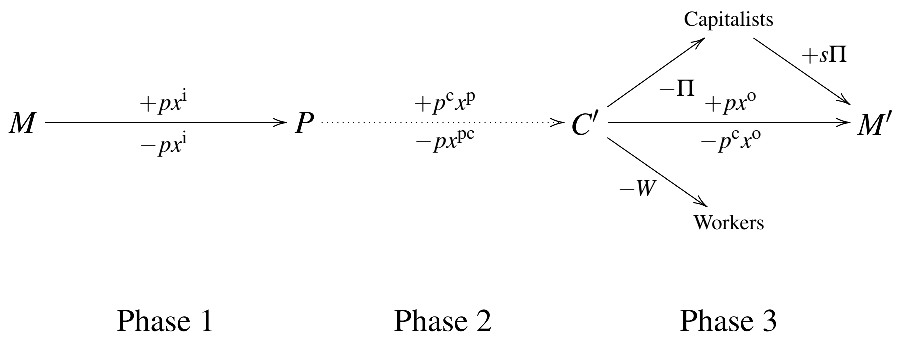


Figure 2: The formula of capital in diagrammatic form

amount of money-capital. Let the inflow of commodities denoted by x^{i}, so that the amount of productive capital is increased by $+px^{\mathrm{i}}$ in price term, and money-capital is decreased by $-px^{\mathrm{i}}$. In Figure 2, the element above the arrow represents the amount of flow increasing the right-side stock, and the element below the arrow represents the amount of flow decreasing the left-side stock. Naturally, the sum of the two elements above and below the arrow must equal zero. We call this rule "bookkeeping rule."

The phase 2, indicated by the dotted arrow, represents the transformation from productive capital P into commodity-capital C', in other words, the phase of production. In this process, the products are produced and the raw materials are productively consumed. Let x^{p} be the amount of production , and x^{pc} be the amount of productive consumption, so that $x^{\mathrm{pc}}/x^{\mathrm{p}} = a$ by definition of Leontief coefficient. The products should be measured at cost price, which is denoted by p^{c}. Then P is decreased by px^{pc}, and C' is increased by $p^{\mathrm{c}}x^{\mathrm{p}}$. According to bookkeeping rule, it must hold that $p^{\mathrm{c}}x^{\mathrm{p}} = px^{\mathrm{pc}}$, so that we get $p^{\mathrm{c}} = pa$.

In the phase 3, there are two different kinds of arrows. One is a horizontal arrow, and the others are diagonal arrows. The horizontal arrow represents the phase of sale, and the diagonal arrows represent the phase of distribution. The sale of commodities is the transformation commodity-capital C' into money-capital M'. It decreases C', the merchandise inventory and increases M', the fund reserve. Let x^{o} be the outflow of commodities, then the total amount of commodities' outflow is $-p^{\mathrm{c}}x^{\mathrm{o}}$, and the gross amount of cash inflow is $+px^{\mathrm{o}}$. The amount of these difference, $px^{\mathrm{o}} - p^{\mathrm{c}}x^{\mathrm{o}}$, is called income.

The income is distributed to workers as wages, W, and to capitalists as profits, Π. W can be expressed as wlx^{o}, where w is the nominal wage rate and l is the labor coefficient. From bookkeeping rule, we get $\Pi = px^{\mathrm{o}} - p^{\mathrm{c}}x^{\mathrm{o}} - W =$

$(p-pa-wl)x^{\mathrm{o}}$. The some fraction of the profits is recommitted in the case of expanded reproduction. This fraction is the capitalists' propensity to save, or rather, to accumulate. we denote this retained fraction as s, then the amount of recommitted profit is $s\Pi$.[6] And then the circuit of capital will restart without stopping.

In next subsection we will derive relational expressions of capital circuit from Figure 2.

2.2 Accumulation of Capital

At first, we investigate how the capital stocks increase in the circuit. The Stocks in the circuit, M, P and C,[7] are governed by the following rule:

$$\begin{aligned} \dot{P}(t) &= px^{\mathrm{i}}(t) - pax^{\mathrm{p}}(t), \\ \dot{C}(t) &= pax^{\mathrm{p}}(t) - pax^{\mathrm{o}}(t), \\ \dot{M}(t) &= px^{\mathrm{o}}(t) - W(t) - \Pi(t) + s\Pi - px^{\mathrm{i}}(t), \end{aligned} \tag{6}$$

where

$$W(t) = wlx^{\mathrm{o}}(t), \tag{7}$$

$$\Pi(t) = (p-pa-wl)x^{\mathrm{o}}(t). \tag{8}$$

These equations (6) follow from the bookkeeping rule. They represent that each capital is increased by the inflow and decreased by the outflow, and then each capitals is accumulated when the inflow is more than the outflow. If we define the total volume of capital as $K(t) \equiv P(t) + C(t) + M(t)$, then the increment of capital is denoted by $\dot{K}(t) = \dot{P}(t) + \dot{C}(t) + \dot{M}(t)$. We can easily get:

$$\dot{K}(t) = \dot{P}(t) + \dot{C}(t) + \dot{M}(t) = s(p-pa-wl)x^{\mathrm{o}}(t). \tag{9}$$

We concentrate on analyzing the stationary state, then the growth rate is $g = \dot{K}(t)/K(t)$ and the profit rate is $\pi = \Pi(t)/K(t)$, which are independent of time t.

The scale of output is arbitrary. Dividing (9) by the volume of capital $K(t)$, we get:

$$g = s(p-pa-wl)x^{\mathrm{o}}(t)/K(t) = s\pi.$$

Cambridge equation (2) is also hold in a generalized Marxian model. But the interpretation differs from that of the basic model. Cambridge equation in the

[6]Therefore, $(1-s)\Pi$ represents the dividends in the case of joint stock company.

[7]Henceforth in this paper we denote the commodity-capital as C, not C', for the symbol of the prime may be misunderstood as the derivation.

basic model means that saving equals investment. On the other hand, it does not hold true in a generalized Marxian model, because the increment of capital $\dot{K}(t)$ contains the increment of the money-capital $\dot{M}(t)$ which is not real investment. $s\Pi$ in this model represents the additional funds which increases the volume of capital and has not been expended yet. g in this model should be interpreted as the rate of supply-side growth, which is determined by the additional funds $s\Pi$.

2.3 Turnover of Capital

In the previous subsection, we evaluate the increasing amount of the total volume of capital, $\dot{K}(t)$. In this subsection, we evaluate the amount of the total volume of capital, $K(t)$. To that end, we consider the transfer process and formulate the ratio of capital turnover. The inflow and outflow are related by the convolution:

$$\begin{aligned} ax^{\mathrm{p}}(t) &= \int_{-\infty}^{t} x^{\mathrm{i}}(t')\alpha(t-t')\,\mathrm{d}t', \\ x^{\mathrm{o}}(t) &= \int_{-\infty}^{t} x^{\mathrm{p}}(t')\beta(t-t')\,\mathrm{d}t', \\ px^{\mathrm{i}}(t) &= \int_{-\infty}^{t} \big(px^{\mathrm{o}}(t') - W(t') - \Pi(t') + s\Pi(t')\big)\gamma(t-t')\,\mathrm{d}t'. \end{aligned} \tag{10}$$

α represents a distributed lag in the process of production, interpreted as the proportion of commodity inflow at time t, that are consumed productively at time $t+t'$. β is a distributed lag in the process of sale, interpreted as the proportion of products at time t, that are sold at time $t+t'$. γ is also a distributed lag in the process of purchase, which is also interpreted as the proportion of money inflow obtained by selling at time t, which are paid to get in a stock at time $t+t'$. α, β and γ are nonnegative and integrate to 1 over the positive half-line.

Under stationary state, the initial conditions must satisfy the following equations.[8] Reasoning from three equations (10),

$$\begin{aligned} ax^{\mathrm{p}} &= x^{\mathrm{i}}\alpha^*(g), \\ x^{\mathrm{o}} &= x^{\mathrm{p}}\beta^*(g), \\ px^{\mathrm{i}} &= \big(p-(1-s)(p-pa-wl)-wl\big)x^{\mathrm{o}}\gamma^*(g), \end{aligned} \tag{11}$$

where

$$\alpha^*(g) = \int_0^{\infty} \alpha(t)\exp(-gt)\,dt$$

[8] we omit the initial time subscript such as $x(0) = x$.

which is the Laplace transform of the lag function $\alpha(.)$ and similarly for $\beta^*(g)$ and $\gamma^*(g)$. The Laplace transform has specific properties as follows:$\alpha^*(0) = 1$, $\mathrm{d}\alpha^*(g)/\mathrm{d}g < 0$, $\lim_{g\to\infty}\alpha^*(g) = 0$.

From these equations, we can drive the stock variables: P, C, and M. Noting that all stock variables grow at the rate of g, and substituting (11) to (6), we get:

$$\begin{aligned} P &= pax^{\mathrm{o}}\big(1-\alpha^*(g)\big)/g\alpha^*(g)\beta^*(g), \\ C &= pax^{\mathrm{o}}\big(1-\beta^*(g)\big)/g\beta^*(g), \\ M &= pax^{\mathrm{o}}\big(1-\gamma^*(g)\big)/g\alpha^*(g)\beta^*(g)\gamma^*(g). \end{aligned} \tag{12}$$

Summing up (12), then we get

$$K = P + C + M = pax^{\mathrm{o}}/\tau(g), \tag{13}$$

where

$$\tau(g) \equiv g\alpha^*(g)\beta^*(g)\gamma^*(g)/\big(1-\alpha^*(g)\beta^*(g)\gamma^*(g)\big). \tag{14}$$

$\tau(g)$ represents the ratio of capital turnover, which is calculated by dividing the total cost pax^{o} with the total volume of capital K.

Substituting the total profit (8) and the total capital (13) to the definition of profit rate $\pi = \Pi/K$, then we get:

$$\pi = (1-a-\omega l)\tau(g)/a. \tag{15}$$

This equation (15) states that profit rate depends not only upon the wage rate but also upon the growth rate. We call this equation the "generalized wage-profit frontier." It is slightly similar to the wage-profit frontier in that it depends upon the real wage rate, and slightly similar to the growth-profitability function in that it depends on the growth rate. How does it relate to the other approaches? And that's a question we answer in the next subsection.

2.4 The Generalized Wage-Profit Frontier and Circuit Conditions

Marx-Morishima approach is a special case of a generalized Marxian approach. It is derived from a generalized model by the addition of the assumption that the ratio of capital turnover is identically unity: if $\tau(g) \equiv 1$, then (15) is reduced to (1). More precisely, it is assumed that the period of circulation is instantaneous, i.e., $\beta(g) = \gamma(g) = 1$, and the period of production is unity. In Marx-Morishima approach, in other words, the elasticity of the profit rate with respect to the growth ratio is assumed to be zero.

There is not much difference between a generalized Marxian approach and Marris-Wood approach, for the ratio of capital turnover in a generalized Marxian approach depends upon g, then the profit rate is expressed as $\pi = \pi(g)$ as in the case of Marris-Wood approach. There is a slight difference between both approaches in the assumption of the derivatives of profit rate. In Marris-Wood approach, it is simply assumed that both $\pi'(g)$ and $\pi''(g)$ are negative. In a generalized Marxian approach, on the other hand, these signs are generally indeterminate, for the sign of τ depends upon the shape of distributed lags. Hereafter we assume that they are negative for the sake of simplicity.[9]

Applying (2), (15) and the assumption of the conventional wage (3), we can obtain the unique growth rate without any investment function. From (2), (15), and (3), we derive:

$$1/\big(1+s(1-a-bl)/a\big) = \alpha^*(g)\beta^*(g)\gamma^*(g).$$

The value of left hand side is the constant less than unity. The value of the right hand side has some specific features: it is unity when $g = 0$ and zero when $g \to \infty$. And yet it is the continuous decreasing function on g. Therefore it has an unique solution even if our model lacks any investment function. We define (2) and (15) as the "circuit conditions" and denote this "supply-side" growth rate as g^s. Then as Figure 3 illustrates, the circuit conditions with the constant wage determine the supply-side growth rate.

Why is the growth rate determined without any investment function? The answer is that this rate of growth is not the actual growth rate balancing saving and investment. It is the rate of the steady-state growth at which capitalists feel they have the right level of capital and do not wish to increase or decrease their capital increment. When capital is behaving as in the case, this rate of growth is

[9] What is the significance of this assumption? In order to interpret this assumption, we define two concepts. One is the increment of capital, which is defined as $I \equiv gK$, and the other is the elasticity of the increment of capital with respect to the growth ratio, which is defined as $\eta \equiv gI'/I$.

First, the sign of the first derivatives of profit rate is negative if and only if we assume:

$$\eta - 1 > 0.$$

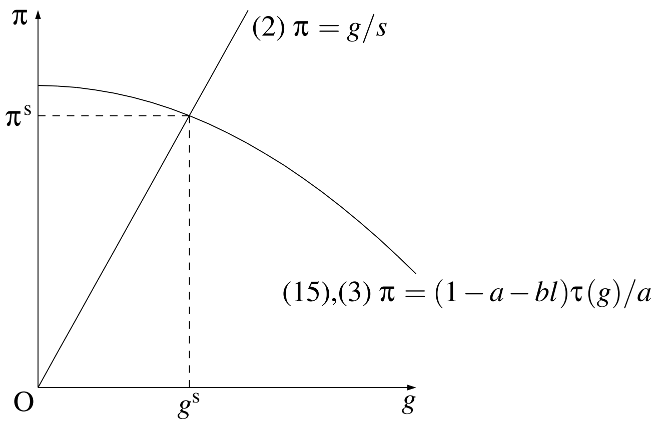
In other words, the elasticity is more than unity. The meaning of this condition is clear. It states that the rate of change of the growth rate is less than the rate of change of the capital increment. This assumption, therefore, eliminates the possibility of the increasing return to scale.

Second, the sign of the second derivatives is negative if and only if we assume:

$$\frac{g\eta'}{\eta} > \eta - 1.$$

In other words, "the elasticity of the elasticity" is more than the elasticity minus unity. This formulation is mathematically clear, but it is too complex to interpret.

Figure 3: Circuit Conditions Determine the Supply-Side Growth Rate



interpreted as the "warranted rate of growth" *à la* Harrod. It should be natural that this rate is not the actual rate of growth, or market clearing growth rate.

Therefore, the problem here is that we have to research the investment function which will determine the actual rate of growth, and the consistent relationship between the warranted and actual rate of growth.

2.5 The Logic of Capital and Marxian Investment Function

All we have to do here is that we investigate Marxian investment function. For that purpose we should reconsider the logic of capital.

What is capital? Capital is an ongoing process oriented to the expansion of its value, passing through the circuit of its metamorphoses: from money to productive to commodity capital, and back. We can point out two moments in this movement. One is the expansion of the value, and the other is the metamorphosis.

We can interpret the expansion of the value as the objective of capital movement. The expansion of value is generally identified with the profit maximization. That is true in case of statistic model, but it is not correct in dynamic model we employ here. In dynamic context, the aim of capital movement is to maximize the value of capital in the long run, which is measured by the capitalization of the dividends, in short, the discounted present value of the net profit, which equals the profit minus the capital increment.

On the other hand, what a role does the metamorphosis play in the capital movement? In the circuit of capital, the value of capital cannot grow directly from M to M' without passing through metamorphosis from M to P, P to C and C to M'. The movement of capital cannot eliminate such circular restrictions altogether. The metamorphosis of capital is, therefore, interpreted as the constrained conditions for the expansion of its value. We have already formulated this constraints as "circuit conditions."

Now we can formulate the motion of capital as constrained maximization problem: the objective of capital is to maximize the value of capital, and the constraints are the circuit conditions. Let V be the value of capital and i be the interest rate, capitalists maximize the following objective function:

$$V = \int_0^\infty (1-s)\pi K(t)e^{-it}\,dt$$

subject to the circuit constraints: (2) and (15). Substituting (2) and (15) to the above objective function and after some manipulation, we get:

$$V = K + \int_0^\infty (\pi(g)-i)K(t)e^{-it}\,dt \tag{16}$$

The value of capital, V, equals the sum of the volume of capital, K, plus the discounted value of the difference between the profit and the opportunity cost of capital. This discounted value is called promoter's profit by Hilferding.[10] Therefore, the value of capital equals the sum of the volume of capital plus the promoter's profit.

V should be normalized. Let v be the value of capital per unit of the volume of capital, V/K. Dividing V by K and after some manipulation,[11] we get:

$$v \equiv \frac{V}{K} = 1 + \frac{\pi(g)-i}{i-g} = \frac{\pi(g)-g}{i-g}. \tag{17}$$

Capitalists try to maximize this rate, which is called the valuation ratio by Richard Kahn.[12] Setting $v'(g)=0$, we get the following first-order condition for capital-value maximization:

$$-\pi'(g) = \frac{\pi(g)-i}{i-g} = v-1. \tag{18}$$

This equation states that the marginal profit rate with respect to growth rate (MPG) equals the promoter's profit per unit of capital. MPG represents the marginal opportunity cost incurred by increasing growth rate. Promoter's profit per unit of capital represents a kind of the marginal revenue that an additional growth will bring to capitalists.Then the growth rate is determined when this equation is hold.

[10] See Hilferding (2006, chap. 9).

[11] We assume that $\lim_{t\to\infty} K(t)e^{-it} = 0$. In other words, the discounted value of the volume of capital at infinite horizon approaches zero. And we also assume $i > g$.

[12] See Kahn (1972). This notion is the same as Tobin's q.

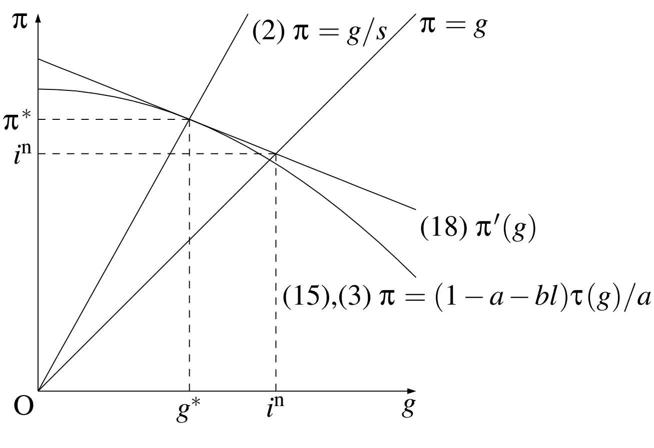
Therefore we can interpret this equation as Marxian investment function, which determines the rate of growth. We denote this "demand-side" growth rate as g^{d}.

We shall now look more carefully into how g^{d} is determined. It depends upon $v-1$, and v depends upon g, and i, noting that π depends upon g. Then g^{d} is a function of i: $i \rightarrow g^{\mathrm{d}}$. If i is an exogenous parameter, then g^{d} is determined as a constant value. But we interpret the interest rate as an unknown variable in this model.

According to Wicksell, we introduce two kind of interests. One is the natural rate of interest, at which supply and demand in the real market was at equilibrium. It is denoted by i^{n}. On the other hand, the money rate of interest is the interest rate in the capital market. It is used to discount the net profit to the present value. It is the same as shown in (16) and (18). It is obvious that if $i < i^{\mathrm{n}}$ then $g^{\mathrm{s}} < g^{\mathrm{d}}$, and vice versa. That is to say, an economy is overheating when the natural rate of interest is higher than the market rate. We cannot mention whether the scenario of Wicksell's famous "cumulative process" is correct or not, for our analysis concentrate on the steady state growth. In other words, it is difficult to derive the (in)stability of equilibrium under capitalism from our model. But it is natural that we ask whether the equilibrium exists or not. Does the market-clearing equilibrium exist?

The answer is, of course, "Yes." Because if $i = i^{\mathrm{n}}$, then $g^{\mathrm{s}} = g^{\mathrm{d}}$ as Figure 4 illustrates. A generalized Marxian model is constituted by four equations: (2), (3), (15), and (18), and four unknown variables: π, g, ω, and i.

Figure 4: Growth Rate in a Generalized Marxian Model



Concluding Remarks

This paper can provide a microfoundation for the growth model in the heterodox tradition. Wage-profit frontier is derived from the generalized wage-profit frontier if we assume that the rate of capital turnover is unity. Growth-profitability frontier is also derived from the generalized wage-profit frontier. Keynes-Robinsonian investment function lacks a microfoundation, but Marxian investment function has a microfoundation. It is derived from the constrained maximization problem: maximizing the capital-value subject to circuit conditions. Then this is the most general approach to solve the problem how to close the system.

A generalized Marxian approach is fully compatible with other heterodox approaches. For example, Marx-Morishima and Keynes-Robinson approaches are not compatible, but a generalized Marxian approach is compatible with both them. It allows both the conventional wage and investment function. This is because it introduces the new unknown variable: the interest rate. Then four variables are determined by the four equations: the generalized wage-profit frontier, Cambridge equation, conventional wage, and Marxian investment function.

But It can be thought that the introduction of the interest is both a strength and weakness of the approach. Indeed it makes the approach generalized, but it is not general that the interest rate is determined in the goods market. it accepts implicitly the loanable fund theory, thus rejects the liquidity preference theory. At any rate, this approach needs to incorporate the analysis of the financial market.

This approach has another weakness. It concentrates on the analytical attention to the steady state growth. This eliminates the analytical domain of the instability of capitalism. This elimination may be connected with the omission of the financial market.

The future direction of this study will be one that encompasses these topics.

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